ADDIS ABABA UNIVERSITY

COLLEGE OF NATURAL SCIENCES

DEPARTMENT OF PHYSICS

GENERAL PHYSICS MODULE

(PHYS 1011)

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ADDIS ABABA

Contents

1 Preliminaries

Learning Outcome

After completing this Chapter, students are expected to:

- define a physical quantity, unit of measurement and uncertainty in measurement.
- identify significant figures in measurements and calculations.
- define a unit vector and describe its purpose.
- distinguish between the vector components and the scalar components of a vector.
- apply dimensional analysis to determine the relation between different quantities.
- Solve problems related to the addition, components, magnitude and direction of vectors.

Introduction

This chapter introduces measurements of physical quantities and the uncertainties inherent to measurements. Accurate and precise measurements are important in the study of and research in physics. Since traditional ways of measurements do not give accurate and precise values, we use standard measurement techniques in science. For example, instead of saying that a string is 5 armlengths, we can specify its length using a standard measuring tape and say that the string is 2 meters long. In this chapter we use a standard known as the International System of Units (SI).

How do we know that our measurements are accurate and precise? What we know for sure is that all measurements have some degree of error or uncertainty. No measurement is exact! A measurement is only an estimation of the true value. Some factors that cause measurement uncertainty and determining the amount of uncertainty will be discussed in this chapter.

In physics experiments, measurements of different physical quantities are taken to verify or discover any relationship between them. Dimensional analysis is covered in this chapter to give students some ideas on how to check the dimensional consistency of equations and how new equation can be discovered. Also, geometric and algebraic methods of adding and resolving vectors will be discussed.

1.1 Physical Quantities and Units of Measurement

Learning Outcome

After completing this section, students are expected to:

- define a physical quantity and a unit of measurement.
- describe what measurement means in science.
- distinguish between quantities and units.
- solve problems related to units of measurement.

1.1.1 Quantities and Units

A quantity is a definite or indefinite amount or size of something.

Examples

Thing	Amount	Type
Desire	Strong	Indefinite
Mass of box	Heavy	Indefinite
Mass of box	5 kg	Definite
Love	Deep	Indefinite
Height of a person	Tall	Indefinite
Height of a person	2 m	Definite

Some of the quantities above are physical and some are non-physical.

1.1.1.1 Non-physical quantities

Non-physical quantities (qualitative) such as love, hate, fear and hope are not a concern in physics studies and experiments and cannot be measured in the sense described below. However, there are research disciplines, such as psychology, that study and quantify such quantities as *fear of exam* or *exam anxiety*.

1.1.1.2 A physical quantity

A physical quantity is a quantity that can be measured by defining its units of measurement or using a measuring instrument. A physical quantity is always expressed in terms of a numerical value (magnitude) and a unit.

Physical quantity = (numerical value) unit

Examples

1. speed of sound = 331 m/s,
2. mass of box = 5 kg

Exercise

Give other examples of physical and non-physical quantities.

1.1.2 A unit of measurement

A unit of measurement (defined and adopted by convention) is a standard by means of which the amount of a physical quantity is expressed. In the first example above the unit is “m/s” and the speed of sound is expressed as containing 331 such units. In the second example, the unit is “kg” and the mass of a box is expressed a containing 5 such units.

Experiments in physics involve taking measurements of quantities and calculating some results. For measurements and calculations to be meaningful, units must be introduced. Physics without units is meaningless.

Exercises

1. Can your heart beat be used as a unit of time? Discuss.
2. Can your arm length or your step be used as units of length? Discuss.

1.1.3 Measurement

A measurement is defined as the process of finding the size or amount of a physical quantity using the standard unit for that quantity (see below for standard units).

Exercises

1. Is every act of finding the amount/size of a physical quantity a measurement? Consider the following:
   a. Counting students in a class, the amount of Birr you have, the number of stars in the sky;
   b. Measuring the length of a table using your arm;
   c. Estimating the distance between two towns;
   d. Comparing the length of a meter stick with your height.

1.1.4 Fundamental and Derived Units

Physical quantities and their units are of two types: Fundamental (or *Basic*) and *Derived*.

1.1.4.1 Fundamental (basic) quantities and units

In the SI system, there are seven basic physical quantities and units, listed in Table 1-1 (below).

Table 1-1 The seven fundamental quantities and their SI units.

Basic physical quantity	Symbol for quantity	Basic unit	Symbol for unit
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A
Temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity	I_v	candela	cd

Some unit symbols are in upper-case letter because they are named after scientists; for example, the unit of temperature (kelvin, K) reminds us Lord Kelvin who contributed a lot in thermodynamics. Note that the full names of the units are all in lower-case letters.

Units of Time, Length, and Mass

To give the student some insight into how standards are adopted we briefly discuss the three fundamental units of mechanics; the second, the meter and the kilogram. See your lab manual for more information about standard units of measurement.

The Second

The SI unit for time, the second (abbreviated s) was first defined as 1/86,400 of a mean solar day. Since the solar day is getting longer due to the gradual slowing of the Earth's rotation, a new standard of the *second* was adopted in terms of a non-varying physical phenomenon for greater accuracy. One such phenomenon is the steady vibrations of Cesium atoms, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations (See https://physlibretexts.org).

The Meter

The SI unit for length, the meter (abbreviated m) was first defined as 1/10,000,000 of the distance from the equator to the North Pole. The meter was redefined more accurately to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. It was again redefined even more accurately in terms of the wavelength of light, so 1 m became 1,650,763.73 wavelengths of orange light emitted by krypton atoms. The present definition of the meter is the distance light travels in a vacuum in 1/299,792,458 of a second (See https://physlibretexts.org). The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass, the kilogram (abbreviated kg) was previously defined to be the mass of a platinum-iridium cylinder kept near Paris with exact replicas reserved at different parts of the globe. Since airborne contaminants slightly change the platinum-iridium mass over time, the scientific community adopted a more stable definition of the kilogram in May 2019. The kilogram is now defined in terms of the second, the meter, and Planck's constant, *h* (a quantum mechanical value that relates a photon's energy to its frequency).

Exercises

1. Why do scientists keep redefining standards?
2. How are the standards for the other SI units defined (Table 1.1)? See the references given at the end of the course outline.

1.1.4.2 Derived quantities

Derived quantities are combinations of two or more basic quantities. For example, volume is obtained by combining three lengths; speed is derived from length and time. Similarly, derived units are made by a combination of two or more of the fundamental units. The unit of volume is meter cubed (m^3); the unit of speed is meter per second (m/s). Table 1-2 (below). shows some examples of derived quantities and the corresponding derived units.

Table 1-2 Some derived quantities and their SI units

Derived quantity	Unit	Symbol	
Area	square meter	m^2	m^2
Volume	cubic meter	m^3	m^3
Frequency	Hertz	Hz	s^{-1}
Density	kilogram per cubic metre	kgm^{-3}	kgm^{-3}
Force	Newton	N	$kgms^{-2}$
Work, energy	Joule	$J\ (Nm)$	kgm^2s^{-2}
Power	Watt	$W\ (J/s)$	kgm^2s^{-3}
Velocity (speed)	metre per second	ms^{-1}	ms^{-1}

Note that in the rightmost column the derived units are expressed as powers of the fundamental units. These powers are called the *dimensions* of the physical quantity in the base units. The next sub-section discusses dimensional analysis.

Exercise

Express the following derived units in terms of the fundamental SI units given in Table 1-1 (above):

The unit of acceleration
The unit of linear momentum
The unit of angular speed
The unit of torque
The unit of moment of inertia
The unit of charge (coulomb, C)
The unit of potential difference (vlot, V)
The unit of magnetic field (tesla, T)

1.1.5 Dimension and dimensional analysis

Every physical quantity can be expressed in terms of some *powers* of the fundamental SI quantities as shown in the rightmost column of Table 1-2 (above). These powers are called the *dimensions* of the physical quantity in question. The square brackets [ ] stand for "dimension of". For example,

$[mass]$ or $[m]$ means "dimension of mass" and we write $[mass] = [m] = M$.

$[length] = [l] = L$ means "dimension of length" and we write $[length] = [l] = L$

$[time] = [t] = T$ means "dimension of time" and we write $[time] = [t] = T$

$[current] = I$ means dimension of electric current and so on.

In mechanics, a derived physical quantity x can be expressed as

$$[x] = [l]^a[m]^b[t]^c = L^aM^bT^c$$

Examples

1. The dimensions of volume: $[V] = [length] \times [width] \times [height] = L^3$
   Volume has a dimension of 3 in length

2. The dimensions of density: $[\rho] = [m]/[V] = ML^{-3}$
   Density has a dimension of 1 in mass and a dimension of -3 in length

3. The dimensions of force: $[F] = [m] \times [a] = MLT^{-2}$
   Force is said to have a dimension of 1 in mass, a dimension of 1 in length and a dimension of -2 in time

4. The dimensions of energy: $[E] = [F] \times [s] = ML^2T^{-2}$
   Energy has dimensions of 1, 2 and -2 in mass, length and time, respectively

Dimensional analysis is useful in deriving new formulas or checking existing formulas apart from dimensionless factors that may exist in the formulas.

Examples

1. Verify that the following relation is correct apart from dimensionless factors: $s = \frac{1}{2}at^2$

Solution

$[s] = L$

$[at^2] = [a][t^2] = LT^{-2}T^2 = L$

Therefore, $[s] = [at^2]$

The given equation is dimensionally correct.

2. Show that the following equation is dimensionally consistent.
   $Force \times distance = mass \times velocity\ squared$

Solution

$[Force \times distance] = [force][distance] = MLT^{-2}L = ML^2T^{-2}$

$[mass \times velocity\ squared] = [mass][velocity]^2 = M\left(\frac{L}{T}\right)^2 = ML^2T^{-2}$

The given equation is dimensionally correct.

In general, the dimension of any physical quantity can be written as $L^aM^bT^cI^d\Theta^eN^fJ^g$ for some powers a,b,c,d,e,f, and g. We can write the dimensions of a length in this form with $a = 1$ and the remaining six powers all set equal to zero: $L^1 = L^1M^0T^0I^0\Theta^0N^0J^0$. For a dimensionless quantity all seven powers are zero. Dimensionless quantities pure numbers. Table 1-3 (below) displays the symbols for all fundamental quantities.

Table 1-3 Fundamental Quantities and Their Dimensions

Base Quantity	Symbol for Dimension
Length	L
Mass	M
Time	T
Current	I
Thermodynamic temperature	Θ
Amount of substance	N
Luminous intensity	J

Exercises

Following the example above, check that the following equations are dimensionally correct.

1. $v^2 = v_0^2 + 2as$
2. $a = \frac{v^2}{r}$
3. $s = v_0 t + \frac{1}{2}at^2$
4. $mv^2 = kx^2$
5. $v = v_0 + at$
6. $P = \rho gh + 0.5\rho v^2$
7. $rF = ml^2$
8. $f = \mu N$

1.1.6 SI Prefixes and scientific notation

The International System of Units (SI) is a decimal system in which units are divided or multiplied by 10 to give smaller or larger units. For example, it doesn't make sense to give the length of a football field as 120000 millimetres or as 0.120 kilometer. A more appropriate unit is the metre. Telling people that the football field is 120 meters long gives them a better idea of the actual length of the field. It would be equally unsuitable to give the thickness of a human hair as 0.000 000 1 kilometer. In this case, the appropriate unit is the millimetre; Saying that the human hair is 1 millimetre thick would give people a far better idea of the actual thickness of the hair.

Table ***1-4*** lists the SI Prefixes in order with their names and symbols

Table 1-4 SI Prefixes

Prefix	Symbol	Base Unit Multiplier	In Words	Exponential
yotta	Y	1,000,000,000,000,000,000,000,000	septillion	10^{24}
zetta	Z	1,000,000,000,000,000,000,000	sextillion	10^{21}
exa	E	1,000,000,000,000,000,000	quintillion	10^{18}
peta	P	1,000,000,000,000,000	quadrillion	10^{15}
tera	T	1,000,000,000,000	trillion	10^{12}
giga	G	1,000,000,000	billion	10^{9}
mega	M	1,000,000	million	10^{6}
kilo	k	1,000	thousand	10^{3}
hecto	h	100	hundred	10^{2}
deca	da	10	ten	10^{1}
(base unit)		1	one	10^{0}
deci	d	0.1	tenth	10^{-1}
centi	c	0.01	hundredth	10^{-2}
milli	m	0.001	thousandth	10^{-3}
micro	µ	0.000001	millionth	10^{-6}
nano	n	0.000000001	billionth	10^{-9}
pico	p	0.000000000001	trillionth	10^{-12}
femto	f	0.000000000000001	quadrillionth	10^{-15}
atto	a	0.000000000000000001	quintillionth	10^{-18}
zepto	z	0.000000000000000000001	sextillionth	10^{-21}
yocto	y	0.000000000000000000000001	septillionth	10^{-24}

Similarly, it is not convenient to write the electron mass as 0.0000000000000000000000000000091 kg or the diameter of the observable universe as 880000000000000000000000000 m. We rather use a scientific notation to express too big or too small numbers:

$$electron\ mass\ =\ 9.1\times10^{-31}\ \mathrm{kg}$$
$$diameter\ of\ observable\ universe = 8.8\times10^{26}\ \mathrm{m}$$

Examples

1. Some illustrations of the use of Prefixes:
   a. Radius of hydrogen atom = 53 pm (picometers).
   b. Distance between Earth and Sun = 149.5 Gm (gigameters).
   c. Mass of electron = 0.000 91 yg (yoctograms).
   d. The mass of the earth = 5983 Yg (yottagrams).

2. Write the following physical quantities in scientific notation and using SI prefixes.

   3270 g, 0.128 m, 65 000 000 W, 0.000056 s

Solution

Given Physical Quantity	Scientific Notation	Using SI Prefixes
3270 g	3.27×10^3 g	3.27 kg
0.128 m	1.28×10^{-1} m	128 mm
65 000 000 W	6.5×10^7 W	65 MW
0.000056 s	5.6×10^{-5} s	56 μs

Exercises

1. Convert the following numbers into scientific notation:
   a. 27 000 000
   b. 101
   c. 007 12
   d. 81 250 000 000
   e. 821
   f. 000 002 05

2. Write the following numbers using SI prefixes:

a. 5.80×10^6	b. 2.52×10^{-3}
c. 6.32×10^{-5}	d. 6.10×10^{-11}
e. 8.56×10^4	f. 6.25×10^{-24}
g. 2.30×10^{10}	h. 1.5×10^{19}

3. Write the standard form of
   a. speed of light in a vacuum = 298 000 000km/s
   b. one light year = 10 000 000 000 000km

1.2 Uncertainty in Measurement and Significant Figures

Learning Outcome

After completing this section, students are expected to:

- define measurement uncertainty,
- distinguish between error and uncertainty,
- explain the difference between accuracy and precision,
- give order of magnitude estimation of physical quantities
- identify significant digits in a measurement value,
- analyse errors in data and report results,

1.2.1 Uncertainty in measurement

If a teacher overcounts the number of students in a class, she could easily correct the mistake by recounting the students two or more times. Miscounting is an example of error or a mistake that could easily be removed. On the other hand, an error made in physics measurements can never be removed no matter how the measurement is taken or how often it is repeated. Measurement errors in physics mean more than simple human mistakes; they are *uncertainties* inherent to the physical measurements.

Although the terms "error" and "uncertainty" are used interchangeably, they are a bit different.

1.2.1.1 Error

Error is defined as the difference between an *observed* value and a *true* value.

$$\boldsymbol{Error = observed\ value - true\ value}$$

The "observed" value is either a result of direct measurement or a calculated value using other measured values in a formula. The "true" value exists but is unknown. Then how can one determine the error in measurements? The goal of measurement is to *estimate* the true value of a physical constant using experimental methods.

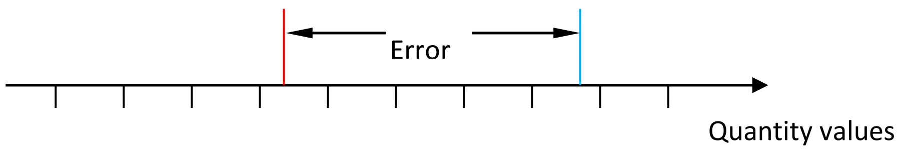


Figure 1-1 A schematic representation of error

1.2.1.2 Uncertainty

Uncertainty is a quantification of the *doubt* about the measurement result. This quantification gives the range of values within which the true value is believed to lie with some level of confidence. Uncertainty is determined by statistical analysis of many values of measurement.

$$\boldsymbol{measurement} = (\boldsymbol{best\ estimate} \pm \boldsymbol{uncertainty})\ \boldsymbol{unit\ of\ measurement}$$

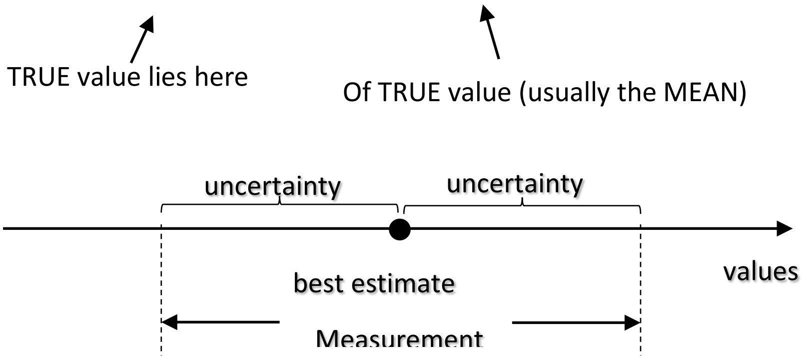


Figure 1-2 Uncertainty shows the area around the average value where the true value of the measurement is likely to be found

Suppose the result of length measurement is (20.1 ± 0.1) cm. This means that the experimenter believes the true value to be closest to 20.1 cm but it could have been anywhere between 20.0 cm and 20.2 cm.

1.2.2 Sources and Types of Error

Measurement errors can arise from three possible origins: the *measuring device*, the *measurement procedure*, and the *measured quantity* itself. Usually the largest of these errors will determine the uncertainty in the data. Errors can be divided into two types: *Systematic and Random errors*

Systematic errors arise from procedures, instruments, bias or ignorance. Systematic errors bias every measurement in the same direction, causing your measurement to consistently be higher or lower than the accepted value. Example: An ammeter with zero error reads higher or lower values of current. Systematic errors can be estimated from understanding the techniques and instrumentation used in an observation.

Figure 1-3 The zero error of an ammeter is an example of systematic error

Random errors are uncontrollable differences between measurements because of equipment, environment or other sources, no matter how well designed and calibrated the tools are. Random errors are *unbiased* small variations that have both positive and negative values. In general, making multiple measurements and averaging can reduce the effect of random errors.

Exercise

Identify the systematic and random errors in the list below:

a. A metre ruler with worn ends
b. A dial instrument with a needle that is not properly zeroed
c. Human reaction time that is always either too late or too early
d. Fluctuations in the readings of an instrument
e. Parallax error (human error) as shown in the diagram below

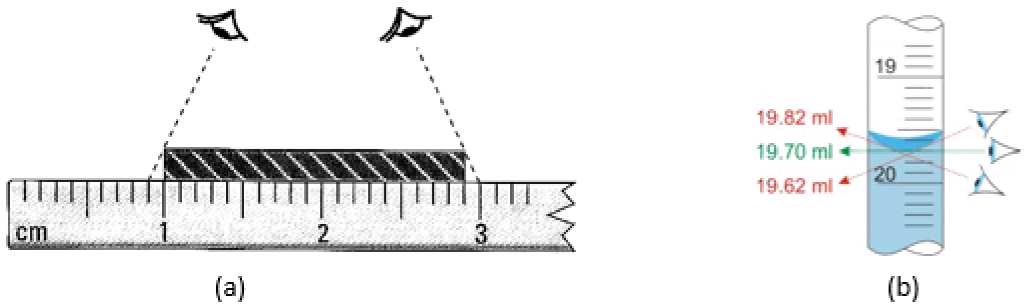


Figure 1-4 Examples of parallax error in (a) measuring length and (b) measuring volume.

1.2.3 Accuracy vs. Precision

In physics, there are two distinct and independent aspects of measurement related to uncertainties:

Accuracy refers to the closeness of a measured value to the 'true' (standard or known) value. It describes *how well we eliminate systematic error*. Example: if you measure the weight of a given substance as 3.2 kg, but the actual or known weight is 10 kg, then your measurement is not accurate. In this case, your measurement is not close to the known value.

Precision refers to the closeness of repeated measurements to each other without referring to the 'true' value. It describes *how well we suppress random errors*. Example: if you weigh an object five times, and get 3.2 kg each time, then your measurement is very precise. The precision of a measuring tool is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.

Precision and accuracy are independent. A measurement can be precise but inaccurate, or accurate but imprecise as illustrated by the several independent trials of shooting at a bullseye target in Figure 1-5 (below).

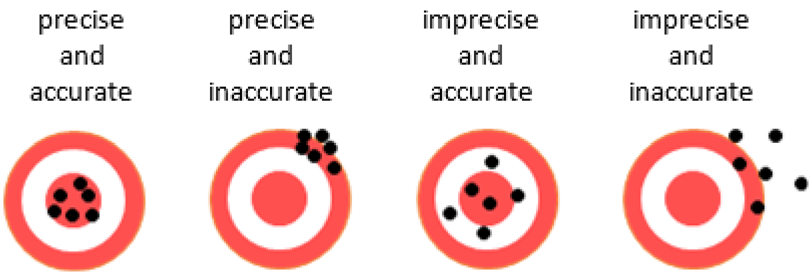


Figure 1-5 Illustration of the difference between accuracy and precision.

Exercise

Given the standard value of g = 9.80665 $\mathrm{m/s^2}$ and the following sets of five measurement values, write "YES" or "NO" in response to the question of accuracy and precision and discuss your responses.

	Set of g values	Accurate?	Precise?
1	g = {9.800, 9.806, 9.807, 9.810, 9.811}		
2	g = {10, 4, 15, 6, 32}		
3	g = {9.80665, 10, 9.8, 9.8067, 9.81}		
4	g = {19.80, 19.806, 19.807, 19.810, 19.799}		

Errors can also be classified as absolute and relative:

1.2.3.1 Absolute error

Absolute error is the difference between the measured value and the accepted value.

$$\textbf{Absolute error} = |\textbf{measured value} - \textbf{accepted value}|$$

1.2.3.2 Relative error

Relative error is a fractional error defined as

$$\textbf{Relative Error} = \frac{\textbf{Absolute error}}{\textbf{Accepted value}}$$

1.2.3.3 Percentage error

Percentage error is relative error expressed as a percentage:

$$\textbf{Percentage error} = \textbf{relative error} \times 100\%$$

Examples

1. Suppose the accepted value of gravity is $g = 9.80665\ m/s^2$. If the measured value is $g = 9.81\ m/s^2$, what is the absolute error?

Solution

$$Absolute\ error = |measured\ value - accepted\ value|$$
$$Absolute\ error = |9.81 - 9.80665|\ m/s^2$$
$$Absolute\ error = 0.00335\ m/s^2$$

2. What is the relative (percentage) error in example 1 above?

Solution

$$Relative\ Error\ = \frac{Absolute\ error}{Accepted\ value} = \frac{0.00335}{9.80665} = 3.4 \times 10^{-4}$$

$$Percentage\ error = relative\ error \times 100\% = 3.4 \times 10^{-4} \times 100\% = 0.03\%$$

3. A digital ammeter gives the value of a current as 456mA. The accuracy or absolute uncertainty of the meter is 1mA. How should the reading be expressed? as (456 ± 1) mA.

Solution

For this reading, 1mA is the absolute error and 456 mA is the estimated value. Therefore, the reading should be expressed as (456 ± 1) mA.

Exercises

1. A measure of the length of two rods using a metre ruler gives:

$$Length\ 1 = 2.000m\ \pm\ 1 \times 10^{-3}\ m$$
$$Length\ 2 = 0.100m\ \pm\ 1 \times 10^{-3} m$$

   a. Find the relative errors in the two measurements.
   b. Find the percentage errors in the two measurements.

2. A student measured the length of a laboratory table as 4.5 m accurate to 0.1 of a meter. Find the absolute, relative and percentage errors in this measurement.
3. The values below are results of a road test for a new car.

Speed	Speedometer correction (km/h)			
Indicated	60	80	100	111
Actual	59	78	96	104

(a) Find the relative (or fractional) error at an actual speed of 59 km/h.

(b) What is the percentage error at 59 km/h?

(c) Find the relative (or fractional) error at an actual speed of 96 km/h.

(d) What is the percentage error at 96 km/h?

(e) Draw a graph of the relative error versus the speed of the car.

(f) what is the relation of the relative errors to the speed of the car?

1.2.4 Quantifying Uncertainties

We will now apply some basic statistics to quantify random errors.

1.2.4.1 The mean

Suppose a quantity x is measured N times. A sample of the measured values is $(x_1, x_2, \dots x_N)$. We want the mean, μ, of the *population* from which such a data set was randomly drawn. We can approximate μ with the sample mean (average) of this particular set of N data points:

$$\mu \cong \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Note that $\bar{x}$ is not the true mean of the population, because we only measured a small subset of the population. But it is our best guess and, statistically, it is an *unbiased predictor* of the true mean μ.

1.2.4.2 The standard deviation

The precision of the value of x is determined by the sample standard deviation, s_x,defined as

$$s_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

The square of the sample standard deviation is called the sample variance, s_x^2. The sample standard deviation is our best estimate of the true statistical standard deviation σ_x of the population from which the measurements were randomly drawn.

1.2.4.3 The standard error (uncertainty)

If we do not care about the standard deviation of a single measurement x but, rather, how well we can rely on the mean value, $\bar{x}$, then we should use the standard error or standard deviation of the mean $s_{\bar{x}}$. This is found by dividing the sample standard deviation by $\sqrt{N}$:

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}}$$

1.2.4.4 Reporting Data

Under normal circumstances, the best estimate of a measured value x predicted from a set of measurements $\{x_i\}$ is reported as $x = \bar{x} \pm s_{\bar{x}}$, where the standard error is now the statistical uncertainty $\delta x = s_{\bar{x}}$. The uncertainties should be given to the same number of decimal places as the measured values. Example: $x = \bar{x} \pm s_{\bar{x}} = (434.2\ \pm\ 1.6)$ nm.

Example: g revisited

Suppose, in a physics lab session, students measured the acceleration due to gravity (g) 40 times. How well is the value of g determined by these measurements?

Values of g measured in cm/s^2									
961	972	979	983	986	965	976	979	966	975
981	985	987	991	983	984	974	981	989	996
968	978	979	984	987	993	990	984	970	977
981	984	992	994	988	985	974	975	971	980

Solution

Mean of g: $\bar{g} = \frac{\sum_{i=1}^{N} g_i}{N} = \frac{39227}{40} = 981 \text{ cm/s}^2$

Standard deviation

First find the deviations of the values of g from their mean value of 981 cm/s^2

-20	-9	-2	2	5	-16	-5	-2	-15	-6
0	4	6	10	2	3	-7	0	8	15
-13	-3	-2	3	6	12	9	3	-11	-4
0	3	11	13	7	4	-7	-6	-10	-1

Then find the sum of squared deviations and divide it by the number of values minus 1. Finally, take the square root to determine the standard deviation:

$$s_g = \sqrt{\frac{\sum_{i=1}^{N}(g_i - \bar{g})^2}{N-1}} = \sqrt{\frac{2711}{39}} = 8.33\ cm/s^2$$

The standard error (uncertainty in the measurement of g)

$$s_{\bar{g}} = \frac{s_g}{\sqrt{N}} = \frac{8.33}{\sqrt{40}} = 1.32 \text{ cm/s}^2$$

Therefore, the students should report the value of g as

$$g = \bar{g} \pm s_{\bar{g}} = (981 \pm 1)\ cm/s^2.$$

Exercise

The temperature of air is measured at different times of a certain day and the following set of readings was recorded.

Rec. No.	1	2	3	4	5	6	7	8	9	10
T (°C)	16	19	18	16	17	19	20	15	17	13

Find (a) the mean temperature of the day, (b) the standard deviation of the temperature data, (c) the standard error and (d) the final result in the form $T = \bar{T} \pm \delta T$.

1.2.5 Error Propagation

Measurement uncertainties propagate through calculations that depend on several uncertain quantities. Suppose that you have two quantities x and y, each with an uncertainty δx and δy, respectively. What is the uncertainty of the quantity $x \pm y$ or xy $(or\ x/y)$? The rules for uncertainty propagation assume that the errors δx and δy are uncorrelated, i.e., they are completely random.

1. Multiplication by an exact number: If $z = cx$, then $\delta z = c\delta x$
2. Addition or subtraction by an exact number: If $z = c + x$, then $\delta z = \delta x$
3. Addition or subtraction: If $z = x \pm y$, then $\delta z = \sqrt{(\delta x)^2 + (\delta y)^2}$
4. Multiplication or division: If $z = xy$ or $z = x/y$, then $\frac{\delta z}{z} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$
5. Power: If $z = x^c$, then $\frac{\delta z}{z} = c\,\frac{\delta x}{x}$

Examples

1. A measurement of the thickness of a pack of cards gives the value $l_1 = (32.3 \pm 0.5)$ mm. Some cards are removed and the thickness is measured again, giving the value $l_2 = (20.4 \pm 0.5)$ mm. what is the thickness of the removed cards together?

Solution

The estimated thickness of the removed cards together is $l = 32.3 - 20.4 = 11.9$. Then by rule 3,

$$\delta l = \sqrt{(\delta l_1)^2 + (\delta l_2)^2} = \sqrt{(0.5)^2 + (0.5)^2} = 0.7 \text{ mm}$$

$$l = (11.9 \pm 0.7\ ) \text{ mm}$$

2. A piece of paper is measured and found to be 5.63 ± 0.15mm wide and 64.2 ± 0.7mm long. What is the area of this piece of paper?

Solution

Data: length = 64.2 ± 0.77mm and width = 5.63 ± 0.15mm

$$Area\ =\ length\ \times\ width$$

$$A =\ (64.2\ \pm\ 0.7mm)\ \times\ (5.63\ \pm\ 0.15mm)$$

First work out the answer by just using the numbers, forgetting the errors

$$Estimated\ Area\ (A) = l \times w =\ 64.2 \times 5.63 =\ 361.446$$

Then, by rule 4:

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta w}{w}\right)^2} = \sqrt{\left(\frac{0.7}{64.2}\right)^2 + \left(\frac{0.15}{5.63}\right)^2} = 0.0288$$

$$\delta A = 361.446 \times 0.0288 = 10.4$$

Report the area as

$A = (361.4 \pm 10.4)\ mm^2$

The final answer should have as many decimal places as the data with *least* number of decimal places.

1.2.6 Significant Figures

Significant figures (sig. figs) are those digits in a measurement that carry meaning and contribute to its precision. Significant figures express the precision of a measuring tool. Here are the rules for identifying significant figures in a measurement:

1. All non-zero figures are significant:

   25.4 has three significant figures.

2. All zeros between non-zeros are significant:

   30.08 has four significant figures.

3. Zeros to the right of a non-zero figure but to the left of the decimal point are not significant (unless specified with a bar):

   109 000 has three significant figures.

4. Zeros to the right of a decimal point but to the left of a non-zero figure are not significant:

   0.050, only the last zero is significant.

5. Zeros to the right of the decimal point and following a non-zero figure are significant:

   304.50 have five significant figures.

Example

Determine the number of sig. figs. For the following numbers

21000	3250000	42210000
0.0012	469	1786
1.0	0.00843	508.6
0.18	0.234	0.6780
67	65.0	5.060

Solution

Number	sig. figs.	Number	sig. figs.	Number	sig. figs.
21000	Two (rule 3)	3250000	Three (rule 3)	42210000	Four (rule 3)
0.0012	Two (rule 4)	469	Three (rule 1)	1786	Four (rule 1)
1.0	Two (rule 5)	0.00843	Three (rule 4)	508.6	Four (rule 2)
0.18	Two (rule 4)	0.234	Three (rule 4)	0.6780	Four (rule 4)
67	Two (rule 1)	65.0	Three (rule 5)	5.060	Four (rule 5)

Exercises

1. Find the number of sig. figs for the following numbers

90000	70000000.0	84.10000
10082	0.0025	3008000
70000000	0.00008914	0.000339

2. Which are significant: Trailing zeros or leading zeros?

When performing calculations, we must be careful about significant figures. When adding, subtracting, multiplying or dividing numbers, the answer should not be more precise than the number with the least number of significant figures.

Examples

1. Find the difference $264.68 - 2.4711$

Solution

Put a question mark at all doubtful places and do the calculation:

$$264.68?? - 2.4711 = 262.21?? = 262.21$$

In this calculation, the least number of sig. figs. is five so the final answer must have five sig. figs

2. Evaluate the product $2.345 \times 3.56 = 8.3482 = 8.35.$

Solution

Multiply the numbers using question marks at all doubtful places.

$$\begin{array}{r} 2.345 \\ \times\ \underline{3.56?} \\ ???? \\ 14070 \\ 11725 \\ \underline{7035} \\ \underline{8.34????} \end{array}$$

The final answer has three sig. figs because the least number of sig. figs. in the operation is three

Exercises

1. The following values are part of a set of experimental data: 618.5 cm and 1450.6mm. Write the sum of these values correct to the right number of significant figures.
2. The following values are part of a set of experimental data: 34.7cm and 19.65mm. How many significant figures would be present in the product and ratio of these two figures?

1.2.7 Order of magnitude

The order of magnitude of a number is the value of the number rounded to the nearest power of ten (no significant figures). It is used if you need to give only an indication of how large or small a number is, and only the power of ten is given. It also indicates that the accuracy of the measurement is limited.

Examples

1. The velocity of light is 3.0×10^8 m/s. The order of magnitude of this velocity is 10^8.
2. The order of magnitude of 142 particles is 10^2. Since 142 in scientific notation is 1.42×10^2.
3. The order of magnitude for 10kV would be given as 10^4.
4. The mean free path of a nitrogen molecule at room temperature and one atmosphere is 59 nm. The order of magnitude is 10^{-8}.
5. The number of molecules in a mole is of the order of 10^{23}.
6. Planck's constant is of the order of 10^{-34}.

Exercises

1. What is the order of magnitude of the gravitational constant?
2. What is the order of magnitude of the distance between the Earth and the Sun?
3. What is the order of magnitude of a charge of 100nC?
4. What is the order of magnitude of the electron charge?
5. What is the order of magnitude of the mass of a proton?

1.3 Vectors: Addition, Components, Magnitude and Direction

Learning Outcome

After completing this section, students are expected to:

- describe the difference between vector and scalar quantities.
- recognise quantities as either scalars or vectors.
- Use geometric methods to add vectors and find their magnitudes and directions in a plane.
- use algebraic method to find resultant of vectors and find components of vectors.
- define a unit vector.
- solve problems about vectors.

1.3.1 Vectors

Vectors are physical quantities such as force, velocity, acceleration and momentum that are expressed in terms of both magnitude (with a unit) and direction. Geometrically, they are represented by arrows in two or three dimensions because arrows have both characteristics of a vector: magnitude and direction.

Note that quantities with a sense of direction such as angles and time are not vectors, because they do not obey the law of parallelogram. A physical quantity is a genuine vector if it adds to another vector according to the law of parallelogram. That is true vectors obey the law of parallelogram. Finite angles are physical quantities with a sense of direction (clockwise or counterclockwise), but they are not vectors because they violate the law of parallelogram.

Physical quantities such as length, volume, mass, density, temperature and time can be expressed in terms of magnitude or size alone (together with a unit). These are called scalar quantities.

Exercise

Categorize each quantity as being either a vector or a scalar.

Quantity	Category
5 m	
30 m/sec, East	
5 km, North	
20 degrees Celsius	

Quantity	Category
256 bytes	
4000 Calories	
5 kg, down	
5 N, down	

Quantity	Category
9.8 m/s^2, up	
10 μC	
45° South of East	
45° clockwise	

1.3.2 Vector notation

There are many ways of writing the symbol of a vector. Vectors are denoted by bold-face letter or a letter with an arrow above it. For example,

Bold face: $\boldsymbol{A}$ Arrow above: $\vec{v}$ Harpoon above: $\vec{F}$ Overbar: $\overline{PQ}$

1.3.3 Geometrical representation of vectors

Vectors are geometrically represented by drawing arrows. The length of the arrow gives the magnitude of the vector and the arrowhead indicates direction of the vector. Figure ***1***-***6***(c) (below) shows that a ruler is used to measure the magnitude of the vector $\overrightarrow{AB}$ as 10.3 units, defining 1 cm as 1 unit. If, for example, the vector is a displacement vector, 10.3 cm on the ruler may represent 13.3 km on the ground; if it is a force vector, 10.3 cm may represent 10.3 N; if it is a velocity vector 10.3 cm may represent 10.3 m/s and so on. A protractor is used to specify the direction of the vector, which is shown to be 29.1^{o} North of East.

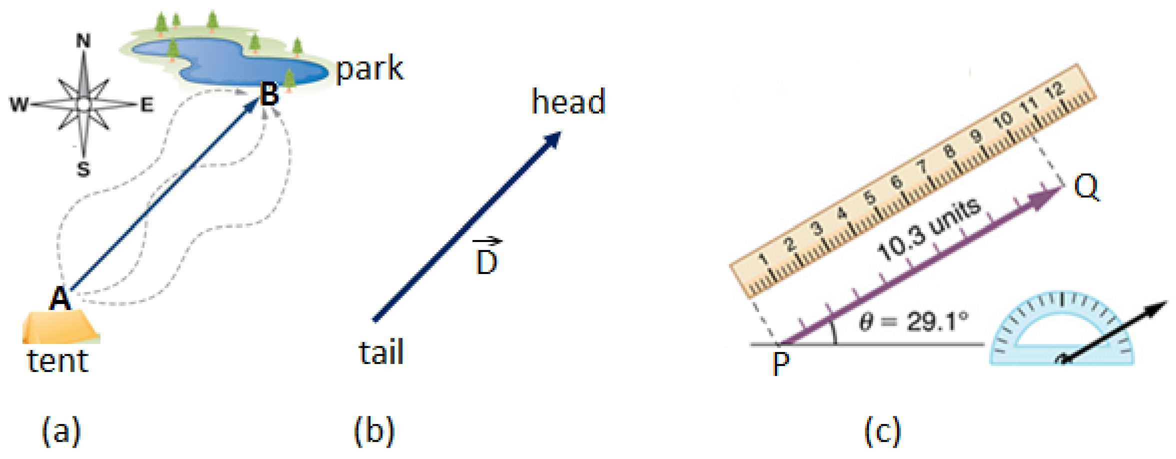


Figure 1-6 A geometrical representation of a vector. (a) the displacement vector from the tent to the park is represented by the arrow AB. (b) this displacement is denoted by $\vec{\boldsymbol{D}}$. (c) The magnitude and direction of a vector are determined using a ruler and a protractor.

Figure 1-6(a)(above) clearly shows the difference between distance (scalar) and displacement (vector) between the tent and the camp. The distance between the tent and the camp depends on the route taken while the displacement from the tent to the camp has a fixed magnitude and a particular direction.

1.3.4 Equality of Two Vectors

Two vectors are equal if they have the same magnitude and direction. In Figure 1-7 (below), vectors $\vec{A}$ and $\vec{B}$ are equal whereas vectors $\vec{C}$ and $\vec{D}$ are not equal eventhough they have the same magnitude. Note that two vectors need not be located at the same point in space to be equal. Moving a vector from one point in space to another doesn't change its magnitude or its direction.

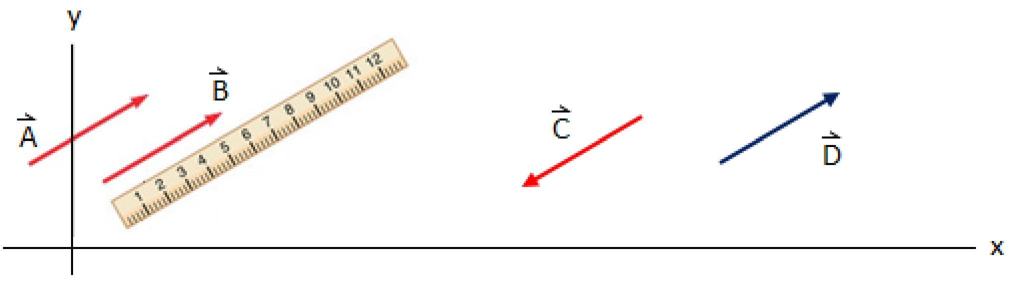


Figure 1-7 The vectors $\vec{\boldsymbol{A}}$, $\vec{\boldsymbol{B}}$ and $\vec{\boldsymbol{D}}$ are equal vectors. Although vector $\vec{\boldsymbol{C}}$ and vector $\vec{\boldsymbol{D}}$ have the same magnitude they are not equal vectors as they have different directions.

Exercises

1. Using a ruler and a protractor determine the magnitude and direction of the vector in Figure 1-6(b).
2. Using a ruler and a protractor check that three of the vectors in Figure 1-7 are equal vectors.

1.3.5 Adding and Subtracting Vectors geometrically

Two vectors can be added geometrically using the tail-to-head method (also called the triangle rule) or the parallelogram rule.

1.3.5.1 Tail-to-head method (triangle rule)

To add vectors, place the tail of one vector at the head of the other vector. The resultant is obtained by joining the tail of the first vector to the head of the second vector. Figure 1-8 (b) and (c) show that vector addition is commutative. The tail-to-head method can be applied to three or more vectors (Figure 1-9).

For any two vectors,

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}, \qquad \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} \neq A + B, \qquad \vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} \neq A - B$$

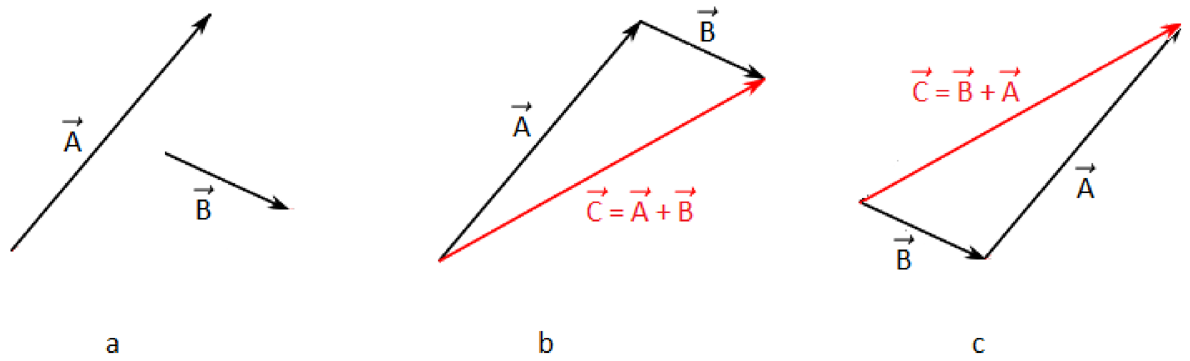


Figure 1-8 Head-to-tail method of vector addition. Geometry shows that $\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}}$.

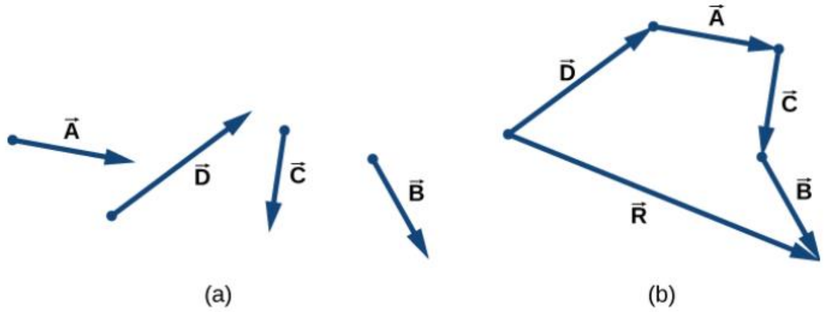


Figure 1-9 Tail-to-head method for drawing the resultant of four vectors

1.3.5.2 Parallelogram Rule

Alternatively, place both vectors with their tails joined. Construct a parallelogram taking the two vectors as the two adjacent sides. The diagonal is the resultant vector (Figure 1-10).

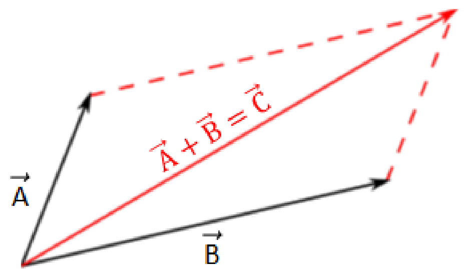


Figure 1-10 Parallelogram rule of vector addition

Subtraction of vectors is the same as adding the negative of the second vector to the first as shown in Figure 1-11.

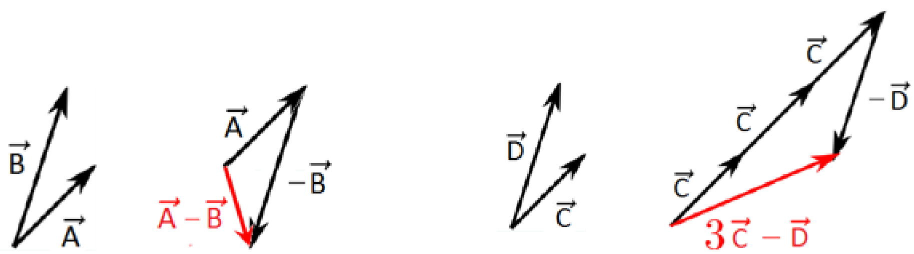
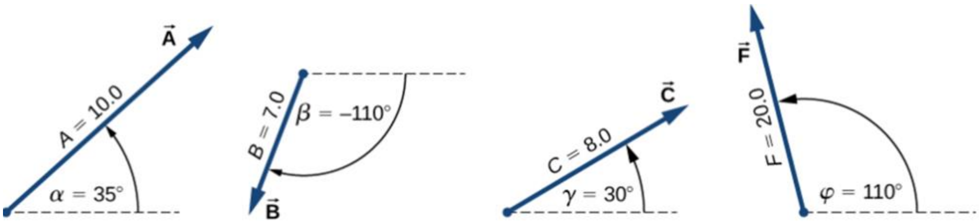


Figure 1-11 Geometric subtraction of vectors, $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$.

If the sum of two vectors is zero, one is said to be the negative of the other. That is, if $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{0}$, then $\boldsymbol{B}=-\boldsymbol{A}$.

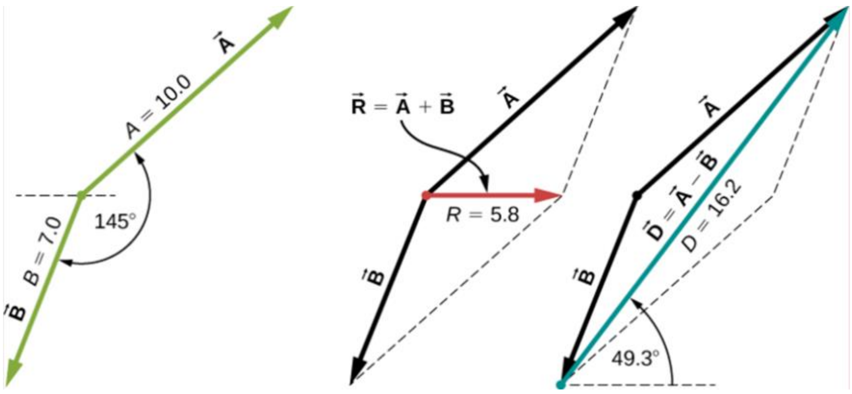
Example

Three displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are specified by their magnitudes in centimeters and by their direction angles with a horizontal line as shown. Choose a convenient scale and use a ruler and a protractor to find the following vector sums: (a) $\vec{R} = \vec{A} + \vec{B}$, (b) $\vec{D} = \vec{A} - \vec{B}$, and (c) $\vec{S} = \vec{A} - 3\vec{B} + \vec{C}$. For parts (a) and (b) we use the parallelogram rule. For (c) we use the tail-to-head method.

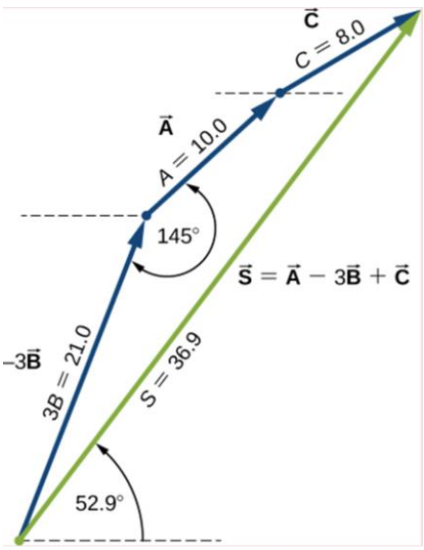


Solution

For parts (a) and (b): R = 5.8 cm and $\theta_R \approx 0°$; D = 16.2 cm and $\theta_D = 49.3°$



For part (C)



Exercises

1. Using a ruler and protractor, find the sum and difference of the following vectors. Draw the vectors to scale, say 1 cm = 2 units of the given vector.
   a. $\vec{A} = 5\text{ N, East}$ and $\vec{B} = 8\text{ N, North of East}$
   b. $\vec{C} = 4\text{ m/s, East}$ and $\vec{D} = 8\text{ m/s, South}$
   c. $\vec{E} = 4\text{ m, North}$, $\vec{F} = 4\text{ m, East}$ and $\vec{G} = 8\text{ m South}$
2. Using the three displacement vectors $\vec{B}$, $\vec{C}$ and $\vec{F}$ in the example above, choose a convenient scale, and use a ruler and a protractor to find vectors (a) $\vec{S} = \vec{C} + \vec{F}$, (b) $\vec{G} = \vec{F} - \vec{C}$ and (c) $\vec{J} = \vec{A} + 2\vec{B} - \vec{F}$. Use the tail-to-head method.
3. Discuss why the following equality does not hold: $\vec{A} + \vec{B} = A + B$, where A and B without arrows represent magnitudes only.

1.3.6 Components of a vector

One way of finding the components of a vector uses the rectangular coordinate system as shown in Figure 1-12. When we know the scalar components A_x and A_y of a vector $\vec{A}$, we can find its magnitude A and its direction angle θ_A. The direction angle—or direction, for short—is the angle the vector forms with the positive direction on the x-axis. The angle θ_A is measured in the *counterclockwise* direction from the +x-axis to the vector.

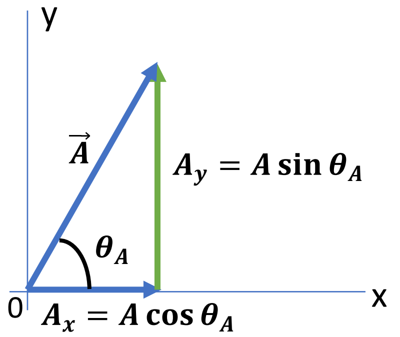


Figure 1-12 Components of a vector.

The vector $\vec{A}$ can be expressed as the sum of two vectors, $\vec{A}_x$ and $\vec{A}_y$, which stand perpendicular to each other in the rectangular (aka Cartesian) coordinate system. That is,

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$\vec{A}_x$ and $\vec{A}_y$ are *vector* components along the x-axis and y-axis respectively. Applying simple trigonometry, we find the *scalar* components of the vector $\vec{A}$ in terms of its magnitude (A) and direction angle (θ_A):

$$A_x = A\cos\theta_A, \qquad A_y = A\sin\theta_A$$

Given the rectangular components of a vector, we can also determine the magnitude and direction of any vector by the (inverse) equations:

$$A = \sqrt{A_x^2 + A_y^2} \qquad \theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Scalar components of a vector $\vec{A}$ may be positive or negative depending on the quadrant in which the vector lies. Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant (III) have both scalar components negative. The calculated angle θ in the first quadrant is identical to the direction angle θ_A. The calculated angle θ in the IV quadrant is identical to the direction angle θ_A. For vectors in quadrants II and III, the direction angle of a vector is given by $\theta_A = \theta + 180°$ counterclockwise from the positive x-axis. In Figure 1-13 all possibilities are shown.

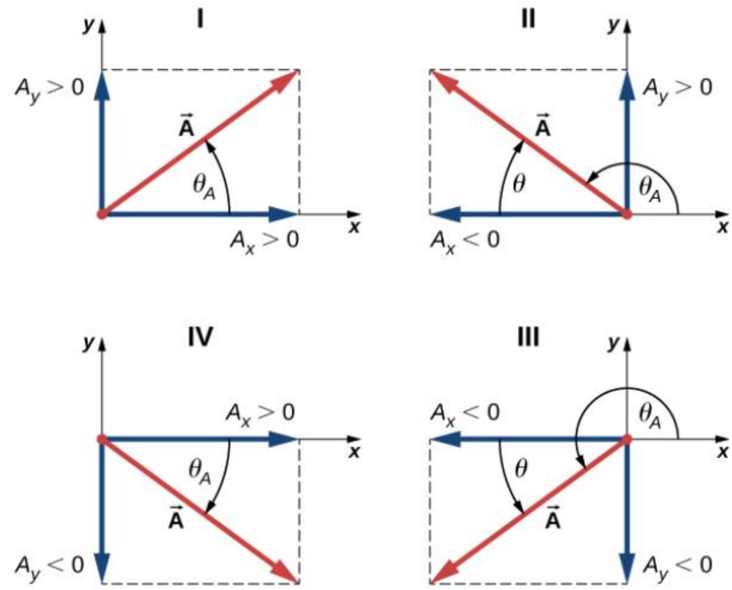


Figure 1-13 Generally, the components of a vector can be positive and negative scalar components. In quadrants I and IV the direction angle of a vector is the same as calculator outputs; in quadrants II and III the direction angle is 180° plus the calculated values.

Example

Find the magnitude and direction of each of the four vectors given below.

Vector	x-component	y-component
A	3 units	4 units
B	-6 units	8 units
C	-9 units	-12 units
D	12 units	-16 units

Solution

Magnitude and direction of vector A

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2 + 4^2} = 5$ units

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$ [correct calculator answer]

Vector A is in the first quadrant at an angle of 53.13° counterclockwise from +x-axis.

Magnitude and direction of vector B

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-6)^2 + 8^2} = 10 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{8}{-6}\right) = -53.13° \qquad \text{[incorrect calculator answer]}$$

The inverse tangent function of a calculator returns values between -90° and +90°, so the answer will be acceptable only for the first and fourth quadrants. To get the right angles for vectors in the second or third quadrant, add 180° to the calculator result.

So, the correct angle for vector B is $\theta = -53.13 + 180 = 126.87°$ counterclockwise from +x-axis (second quadrant)

Magnitude and direction of vector C

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-9)^2 + (-12)^2} = 15 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-12}{-9}\right) = 53.13° \quad \text{[incorrect calculator answer]}$$

The correct angle for vector C is $\theta = 53.13 + 180 = 233.13°$ counterclockwise from +x-axis (third quadrant)

Magnitude and direction of vector D

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{12^2 + (-16)^2} = 20 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{-16}{12}\right) = -53.13° \quad \text{[correct calculator answer]}$$

Vector D is in the fourth quadrant at an angle of 53.13° clockwise from +x-axis.

Note that the calculator answer is correct only half of the time.

A ball is shot at an angle of 35° with the horizontal. If the initial velocity of the ball has a magnitude of 50 m/s find its horizontal (x) and vertical (y) components.

Solution

Horizontal component: $v_x = v\cos\theta = 50 \times \cos 35° \cong 41\ m/s$

Vertical component: $v_y = v\sin\theta = 50 \times \sin 35° \cong 29\ m/s$

Exercises

1. The magnitude of vector $\vec{A}$ is 35.0 units and points in the direction 325° counter-clockwise from the positive x-axis. Calculate the x- and y-components of this vector.
2. A boy ran 3 blocks west, 5 blocks north. Find the magnitude and direction of his resultant displacement with reference to East.
3. The rectangular components of four vectors are given below. Find the magnitude and direction of each vector.

Vector	x-component	y-component
A	3 units	-4 units
B	5 units	1 unit
C	-2 units	-3 units
D	-12 units	6 units

1.3.7 Adding and Subtracting Vectors Algebraically

Algebraic addition and subtraction of vectors makes use of the components of the vectors. The algebraic sum of two or more vectors is obtained by adding x and y components separately. So, if $\vec{R} = \vec{A} + \vec{B}$, then the components of the resultant vector are:

$$R_x = A_x + B_x$$
$$R_y = A_x + B_y$$

Similarly, if $\vec{D} = \vec{A} - \vec{B}$, then the components of difference vector are

$$D_x = A_x - B_x$$
$$D_y = A_x - B_y$$

Remember: Algebraic addition and subtraction of vectors are carried out component by component.

Example

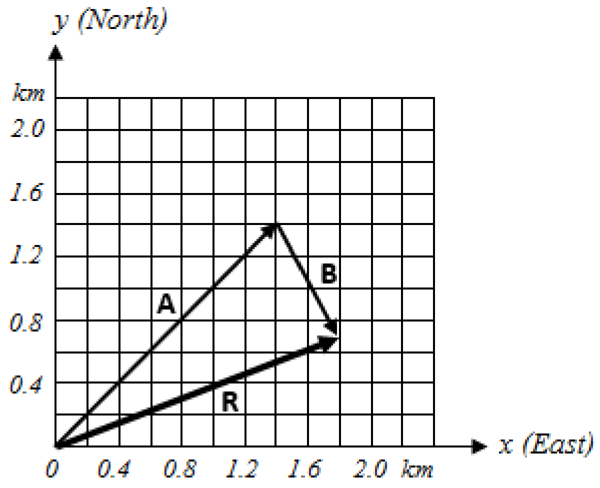
On a certain day, a student goes to school by first walking 2.0 km 45.0° north of east from her home to her ant's home to drop a message. Then she walks 0.8 km in a direction 60.0° south of east of north where her school is located. (a) Determine the components of the student's displacements in the first and second parts of her walk. (b) Determine the components of her total displacement for the trip from home to school.

Solution

Let $\vec{A}$ and $\vec{B}$ be the first and second displacements. Their components are:

$$A_x = A\cos\theta = 2.0\cos 45° = 1.4\ km$$
$$A_y = A\sin\theta = 2.0\sin 45° = 1.4\ km$$

$$B_x = B\cos\theta = 0.8\cos(-60°) = 0.40\ km$$
$$B_y = B\sin\theta = 0.8\sin(-60°) = -0.69\ km$$



The components of the total displacement $\vec{R}$ are:

$$R_x = A_x + B_x = 1.4 + 0.4 = 1.8\ km$$

$$R_y = A_y + B_y = 1.4 - 0.69 = 0.7\ km$$

The diagram shows the student's displacements. Compare the calculated values of the components to the corresponding components on the diagram.

Exercises

1. The magnitude and direction (with respect to the +x axis) of two force vectors are A(50 N, 30°) and B (25 N, -60°). Find
   a. The components of $\vec{A}$ and $\vec{B}$,
   b. The components of the sum $\vec{S} = \vec{A} + \vec{B}$ and difference $\vec{D} = \vec{B} - \vec{A}$.

2. Find the magnitude and direction of the vectors whose xy-components are
   a. (7 m, 11 m)and b. (24 m/s, -15 m/s)

1.3.8 Unit Vectors

A convenient way of analysing vectors is to first describe them in terms of unit vectors.

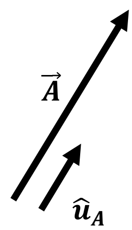
1.3.8.1 The unit vector defined

Definition 1: A unit vector is a vector of magnitude one.

So, if $\hat{u}$ denotes a unit vector, then $|\hat{u}| = 1$.

Definition 2: A unit vector in the direction of any vector $\vec{A}$ is given by

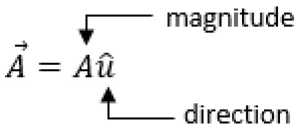
$$\hat{u}_A = \frac{\vec{A}}{A}, \qquad \text{where } A = |\vec{A}|$$



Definition 2 is consistent with Definition 1 because

$$|\hat{u}_A| = \left|\frac{\vec{A}}{A}\right| = \frac{A}{A} = 1$$

Using Definition 2, we can write any vector $\vec{A}$ in terms of its magnitude and parallel unit vector:



1.3.8.2 Unit vectors of the rectangular xy-coordinate system

In the rectangular xy coordinate system, unit vectors are defined in the directions of $+x$ and $+y$. The unit vector in the direction of $+x$ is denoted by $\hat{\imath}$ and the unit vector in the direction of $+y$ is denoted by $\hat{\jmath}$. Unit vectors provide a convenient notation in vector algebra.

In a rectangular (Cartesian) xy-coordinate system in a plane (Figure 1-14, below), the vector component $\vec{A}_x$ of a vector $\vec{A}$ can be described by the scalar component (A_x) and the unit vector $\hat{\imath}$:

$$\vec{A}_x = A_x\hat{\imath}$$

Similarly, the vector component in the y direction is described as

$$\vec{A}_y = A_y\hat{\jmath}$$

Where (A_x, A_y) are the scalar components of the vector $\vec{A}$.

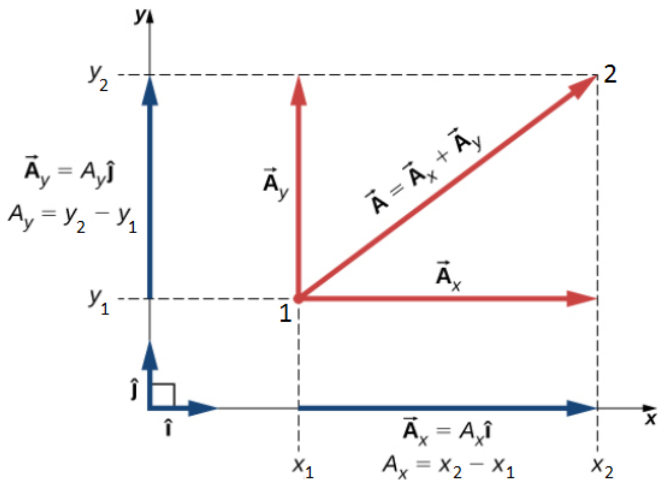


Figure 1-14 Vector components and scalar components of a vector in the Cartesian coordinate system. The vector components $\vec{A}_x$ and $\vec{A}_y$ are multiples of the unit vectors, the multiplying factors being the scalar components A_x and A_y.

Suppose $\vec{A}$ and $\vec{B}$ are any two vectors in the rectangular coordinate system given by

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \qquad \text{and} \qquad \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

The sum or difference, $\vec{A} \pm \vec{B}$, is carried out component-by-component as shown below:

$$\vec{S} = \vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$\vec{D} = \vec{A} - \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) - (B_x\hat{\imath} + B_y\hat{\jmath}) = (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$

Similarly, the scalar multiple of a vector $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ can also be found by multiplying each component of $\vec{A}$ by the given scalar. So, if $\vec{B} = \alpha\,\vec{A}$, then

$$\vec{B} = \alpha\,\vec{A} = \alpha(A_x\hat{\imath} + A_y\hat{\jmath}\,) = (\alpha A_x)\hat{\imath} + (\alpha A_y)\hat{\jmath} = B_x\hat{\imath} + B_y\hat{\jmath},$$

where $B_x = \alpha A_x$ and $B_y = \alpha A_y$

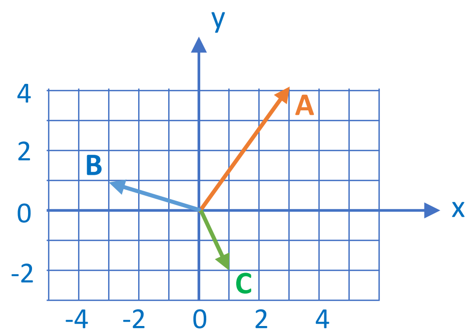
Example

Given three vectors (a) $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$, (b) $\vec{B} = -3\hat{\imath} + \hat{\jmath}$ and (c) $\vec{C} = \hat{\imath} - 2\hat{\jmath}$. Assume all the vectors start at the origin.

1. Show the three vectors in the xy-coordinate system
2. Find $\vec{A} + \vec{C}$ and $\vec{A} - \vec{B} + 3\vec{C}$,
3. Find he vector $\vec{D}$ such that $\vec{D} + \vec{B} - \vec{C} = 0$

Solution

1. The vectors in the xy-coordinate system



2. $\vec{A} + \vec{C} = (3\hat{\imath} + 4\hat{\jmath}) + (\hat{\imath} - 2\hat{\jmath}) = 4\hat{\imath} + 2\hat{\jmath}$

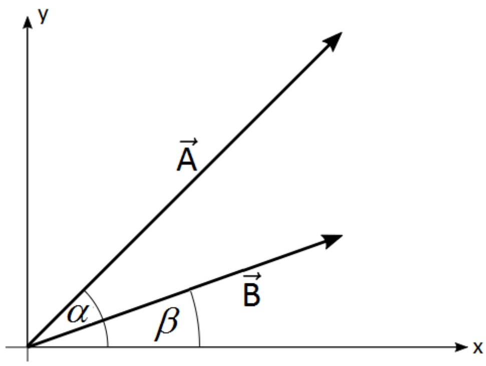
   $\vec{A} - \vec{B} + 3\vec{C} = (3\hat{\imath} + 4\hat{\jmath}) - (-3\hat{\imath} + \hat{\jmath}) + 3(\hat{\imath} - 2\hat{\jmath}) = 9\hat{\imath} - 3\hat{\jmath}$

3. $\vec{D} + \vec{B} - \vec{C} = 0 \Rightarrow \vec{D} = \vec{C} - \vec{B} = (\hat{\imath} - 2\hat{\jmath}) - (-3\hat{\imath} + \hat{\jmath}) = 4\hat{\imath} - 3\hat{\jmath}$

Exercises

1. Given $\vec{A} = 4\hat{\imath} - 10\hat{\jmath}$ and $\vec{B} = 7\hat{\imath} + 5\hat{\jmath}$, find b such that $\vec{A} + b\vec{B}$ is a vector pointing along the x-axis (i.e. has no y component).
2. If $\vec{M} = 2\hat{\imath} - \hat{\jmath}$ and $\vec{N} = 4\hat{\imath} + 3\hat{\jmath}$, find k and l such that $k\vec{M} + l\vec{N} = 2\hat{\imath} - 6\hat{\jmath}$.

3. If $|11\hat{\imath} - k\hat{\jmath}| = |\sqrt{5}(3i + j)|$, find the possible values for k.
4. Find the angle that each of the following vectors makes with the x-axis:
   a. $\hat{\imath} + \hat{\jmath}$
   b. $2.4\hat{\imath} - 2\hat{\jmath}$
   c. $(3.2\hat{\imath} - \hat{\jmath}) + \overrightarrow{4\hat{\imath} + 3\hat{\jmath}}$
   d. $4.2\vec{x} + \vec{y}$, where $\vec{x} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{y} = 6\hat{\imath} - 8\hat{\jmath}$
5. Find the unit vector in the direction of
   a. $\vec{A} = \hat{\imath} + \hat{\jmath}$
   b. $\vec{B} = 2.4\hat{\imath} - 2\hat{\jmath}$
   c. $\vec{x} = 4\hat{\imath} + 3\hat{\jmath}$
   d. $\vec{y} = 6\hat{\imath} - 8\hat{\jmath}$
6. In the diagram below, $\vec{A}$ is a vector of magnitude 35 cm; $\vec{B}$ is a vector of magnitude 13 cm. If $\tan\alpha = 4/3$ and $\tan\beta = 5/12$,
   a. write A and B in terms of $\hat{\imath}$ and $\hat{\jmath}$
   b. Show that $\vec{A} + \vec{B}\ q$makes an angle of $45°$ to the x-axis.



1.4 Chapter Summary

This chapter introduced a quantity as a definite or indefinite amount or size of something and discussed physical and nonphysical quantities. A physical quantity is a quantity that can be measured by defining its units of measurement or using a measuring instrument and is always expressed in terms of a numerical value (magnitude) and a unit. Nonphysical quantities include feelings such as love and exam anxiety which could be quantified by preparing a scale such as a questionnaire.

A unit of measurement (defined and adopted by convention) is a standard by means of which the amount of a physical quantity is expressed. There are seven fundamental SI units and all other units, called derived units, can be expressed as combinations of the fundamental units. These combinations of fundamental units determine the dimensions of a derived quantity. The SI system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.

A measurement error is the difference between the observed (measured) value and the true (accepted) value and it can be expressed as absolute, relative or percentage error. Uncertainty is the quantification of the doubt about measured values which can be expressed in two ways: accuracy and precision. The accuracy of a measured value refers to how close a measurement is to the accepted value. The precision of measured values refers to how close the agreement is between repeated measurements. Significant figures express the precision of a measuring tool. In calculations the final answer should not be more precise than the number with the least number of significant figures. When a set of measured values is given, the measurement uncertainty is determined by the standard error of the mean of the given values.

Vectors are physical quantities characterized magnitude and direction and which obey the law of parallelogram. Geometrically, vectors are represented by arrows. A unit vector is a vector of magnitude 1. Two vectors are equal if and only if they have the same magnitudes and directions.

1.5 Conceptual Questions

1. Identify some advantages of SI units over the British (Imperial) system of units.
2. What is the relationship between the accuracy and uncertainty of a measurement?
3. Estimate the order of magnitude of the length, in meters, of each of the following a dust particle, a fly, a 40-story building and an elephant.
4. What types of natural phenomena could serve as time standards?
5. Why is using a pulse rate a poor method of measuring time?
6. Find the order of magnitude of your age in seconds.
7. Estimate the number of times your heart beats in the average lifetime of an Ethiopian.
8. Estimate the time duration of each of the following events in the units suggested in parentheses: (a) the blink of an eye (seconds), (b) a walk from your home to a nearby shop (minutes), (c) a photon to move across the milky way (years).
9. Estimate the number of air molecules in a volume of 1 cm x 1 cm x 1 cm (assume the mean distance between air molecules is of the order of a nanometer).
10. The left side of an equation has dimensions of energy and the right side has dimensions of pressure times volume. Can the equation be correct? explain your answer.

11. A vector has zero magnitude. Is it necessary to specify its direction? Explain.
12. If two vectors are equal, what can you say about their components?
13. If three vectors sum up to zero, what geometric condition do they satisfy?
14. Suppose two quantities, A and B, have different dimensions. Determine which of the following arithmetic operations could be physically meaningful. (a) $A + B$, (b) $B - A$, (c) $A - B$, (d) A/B, (e) AB, (f) $A = B$.
15. Two different measuring devices are used by students to measure the length of a metal rod. Students using the first device report its length as 0.5 m, while those using the second report 0.502 m. Can both answers be correct (choose one)? (a) Yes, because their values are the same when both are rounded to the same number of significant figures. (b) No, because they report different values.
16. If vector B is added to vector A, under what conditions does the resultant vector have a magnitude equal to A + B? Under what conditions is the resultant vector equal to zero?
17. Under what circumstances would a vector have components that are equal in magnitude?
18. A student writes, “A bird that is diving for prey has a speed of −10 m/s.” What is wrong with the student’s statement? What has the student actually described? Explain.
19. A weather forecast states that the temperature is predicted to be −5°C the following day. Is this temperature a vector or a scalar quantity? Explain.
20. Give an example of a nonzero vector that has a component of zero.
21. If two vectors are equal, what can you say about their components?
22. If vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other, what is the component of $\vec{B}$ along the direction of $\vec{A}$? What is the component of $\vec{A}$ along the direction of $\vec{B}$?
23. If two vectors have the same magnitude, do their components have to be the same?

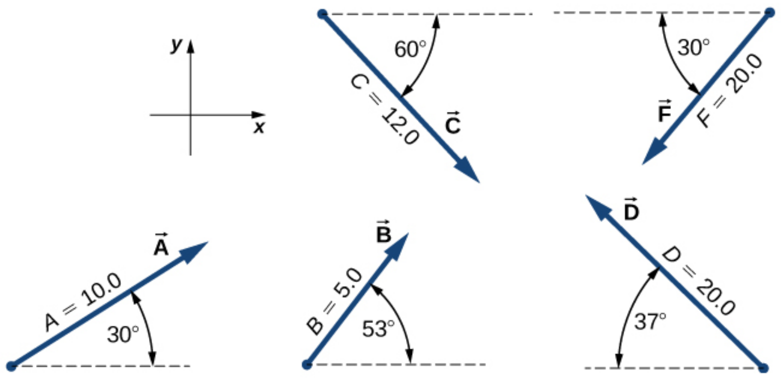
1.6 Problems

Express your answers to problems to the correct number of significant figures and proper units.

1. Show that 1.0 m/s = 3.6 km/h.
2. The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?
3. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?
4. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?
5. A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90km/h?
6. An infant’s pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

7. State how many significant figures are proper in the results of the following calculations:
   (a) $(106.7)(98.2)/(46.210)(1.01)$
   (b) $(18.7)^2$
   (c) $(1.60 \times 10^{-19})(3712)$.
8. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?
9. (a) If your speedometer has an uncertainty of 2.0km/h at a speed of 90km/h, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads 60km/h, what is the range of speeds you could be going?
10. (a) A person's blood pressure is measured to be 120±2mm Hg. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of 80mm Hg?
11. A marathon runner completes a 42.188-km course in 2h, 30 min, and 12s. There is an uncertainty of 25m in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?
12. The sides of a small rectangular box are measured to be 1.80 ± 0.01 cm, 2.05 ± 0.02 cm, and 3.1 ± 0.1 cm long. Calculate its volume and uncertainty in cubic centimeters.
13. The length and width of a rectangular room are measured to be 3.955 ± 0.005m and 3.050 ± 0.005m. Calculate the area of the room and its uncertainty in square meters.
14. A car engine moves a piston with a circular cross section of 7.500 ± 0.002 cm diameter a distance of 3.250 ± 0.001 cm to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.
15. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?
16. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg.)
17. Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?
   (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium.
   (b) Making the same assumption, how many cells are there in a human?
18. The Atwood machine consists of two masses M and m (with $M > m$) attached to the ends of a light string that passes over a light, frictionless pulley. When the masses are released, they accelerate with $a = g(M - m)/(M + m)$. Suppose that M and m are measured as $M = 100 \pm 1$ and $m = 50 \pm 1$, both in grams. Find the uncertainty in the acceleration measurement, δa.

19. Suppose $[V] = L^3$, $[\rho] = ML^{-3}$, $[\rho] = ML^{-3}$, and $[t] = T$. (a) What is the dimension of ρV? (b) What is the dimension of $\Delta V/\Delta t$? (c) What is the dimension of $\rho(\Delta V/\Delta t)$?
20. The arc length formula says the length s of arc subtended by angle θ in a circle of radius r is given by the equation $s = r\,\theta$. What are the dimensions of (a) s, (b) r, and (c) θ?
21. Consider the physical quantities m, s, v, a, and t with dimensions $[m] = M$, $[s] = L$, $[v] = LT^{-1}$, $[a] = LT^{-2}$, and $[t] = T$. Assuming each of the following equations is dimensionally consistent, find the dimension of the quantity on the left-hand side of the equation: (a) $F = ma$; (b) $K = 0.5mv^2$; (c) $p = mv$; (d) $W = mas$; (e) $L = mvr$.
22. In what follows, assume A is area, V is volume, and all other variables are lengths. Determine which formulas are dimensionally consistent. (a) $V = \pi r^2 h$; (b) $A = 2\pi r^2 + 2\pi rh$; (c) $V = 0.5bh$; (d) $V = \pi d^2$; (e) $V = \pi d^3/6$.
23. A student drives 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances she would have to drive straight east and then straight north to arrive at the same point. (b) Show that she still arrives at the same point if the east and north legs are reversed in order. Assume the +x-axis is to the east.
24. A sledge is being pulled by two horses on a flat terrain. The net force on the sledge can be expressed in the Cartesian coordinate system as vector $\vec{F} = (-2980.0\hat{\imath} + 8200.0\hat{\jmath})N$, where $\hat{\imath}$ and $\hat{\jmath}$ denote directions to the east and north, respectively. Find the magnitude and direction of the pull.
25. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the velocity of a fly, the age of Earth, the boiling point of water, the cost of a book, Earth's population, or the acceleration of gravity?
26. For the vectors given in the following figure, use a graphical method to find the following resultants: (a) $\vec{A} + \vec{B}$, (b) $\vec{C} + \vec{B}$, (c) $\vec{D} + \vec{F}$, (d) $\vec{A} - \vec{B}$, (e) $\vec{D} - \vec{F}$, (f) $\vec{A} + 2\vec{F}$, (g) $\vec{A} - 4\vec{D} + 2\vec{F}$.



2 Kinematics in one Dimensions

Learning Outcome

After completing this Chapter, students are expected to:

- Understand motion and position
- Define distance and displacement
- Define speed and velocity
- Identify the average velocity and instantaneous velocity
- Define the average and instantaneous acceleration
- Derive the equations of motion with constant acceleration
- Solve related problems

Introduction

The study of motion and of physical concepts such as force and mass is called *dynamics*. The part of dynamics that describes motion without regard to its causes is called *kinematics*. In this chapter, we study the basic physics of motion where the object (race car, tectonic plate, blood cell, or any other object) moves along a single axis. Such motion is called *one-dimensional motion.*

Though everything around us and elsewhere on earth seems stationary, move with Earth's rotation, Earth orbits around the Sun, the Sun orbits around the centre of the Milky Way galaxy, and that galaxy migrates relative to other galaxies.

If the *motion* is along a straight line only the line may be vertical, horizontal, or slanted, but it must be straight. With this, questions like does the moving object speed up, slow down, stop, or reverse direction and if the motion does change, how is time involved in the change can be attempted. Generally, motion is a continuous change of position; whereas *position* is location of an object relative to some reference point, often the origin (or zero point) of an axis such as the *x*-axis.

In *one-dimensional motion*, moving objects are restricted to motion along a straight line. To describe such motion a + or - sign are all that is needed to specify direction.

The *positive direction* of the axis is in the direction of increasing numbers (coordinates), which is to the right, as in Fig. 2.1. The opposite is the *negative direction*.

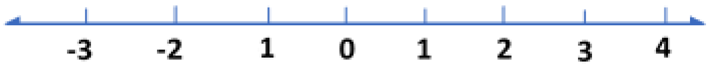


Figure 2-1: Position on an axis that is marked in units of length.

The aforementioned information is, therefore, essential to understand and explain the remaining topics such as distance, displacement, speed, velocity and acceleration.

2.1 Distance and Displacement

It is important to recognize the difference between distance and displacement.

2.1.1 Distance

Distance travelled by a particle is the length of the path a particle takes from its initial position to its final position. Distance is a *scalar* quantity that can be denoted by any English alphabet s, d…, usually and is always indicated by a positive number.

2.1.2 Displacement

Displacement of a particle is the shortest distance between its initial and final positions. It is an example of a *vector* quantity. Many other physical quantities, including *position*, *velocity*, and *acceleration*, also are vectors. *Displacement* is usually denoted by *s*, in which other English alphabets can be also used. Moreover, one can define *displacement* as change in position in some time interval.

As a particle moves from an initial position x_i to a final position x_f, along the +*x* - axis, the displacement becomes $x_f - x_i$. We use the Greek letter delta (Δ) to denote the *change* in a quantity, therefore,

$$\Delta x = x_f - x_i \tag{2-1}$$

Note that Δx is not the product of Δ and *x*; it is a single symbol that means "the change in the quantity *x*."

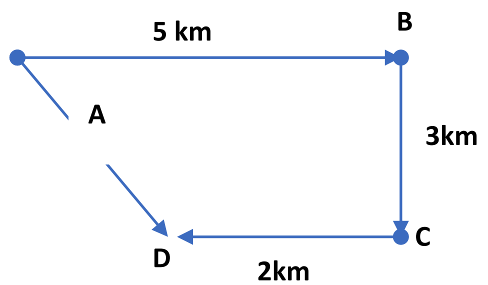
Remark:

Displacement is the change in position of a particle. It is positive if the change in position is in the direction of increasing *x* (the + *x* direction), and negative if it is in the -*x* direction.

Distance is the total path length travelled by a particle and is a scalar physical quantity that can be denoted by any English alphabet s, d…, usually. Distance travelled by a particle cannot be zero where displacement can be.

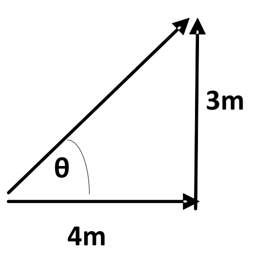
Examples:

1. Consider the case of a particle moving from point A to point D, as shown. The shortest distance is the distance between A and D in the counter clock wise direction which is $\sqrt{(5\ km - 2\ km)^2 + (3\ km)^2} = 3\sqrt{2}\ km$, and hence is the displacement of the particle at 45^0 in the south east direction, whereas the distance traversed is that followed the longer path, $5\ km + 3\ km + 2\ km = 10km$.



Similarly, a car travelled 10 km from known initial position and then back to the same point covers 20 km of total distance whereas its displacement is zero. For only forward or backward straight-line motion distance can be magnitude of a displacement.

2. Find the distance and displacement referring to the following diagram, if an object travels 4m to the positive x-axis and then 3m to the y-axis.



Solution:

Distance,

$$s = 4m + 3m = 7m$$

Magnitude of the Displacement,

$$s = \sqrt{(4m)^2 + (3m)^2} = 5m$$

and its direction will be

$$\theta = \tan^{-1}\frac{3}{4} = \tan^{-1} 0.75 = 37^0$$

Therefore, the displacement is $\vec{s} = 5m, 37^0\ with\ respect\ to\ the\ x - axis.$

2.2 Speed and Velocity

2.2.1 Speed and Average Speed

Speed is the ratio of the distance travelled by any object, irrespective of its direction, to the time it takes to travel that distance. Speed involves both distance and time. Therefore, its unit in SI system is meter per second (*m/s or* ms^{-1}). There are other non-SI units like cm/s, km/h, km/min, ft/s, ...etc.

Ordinarily, the speed of a body does not remain uniform over a certain time interval we are considering. For instance, a bus that carry passengers between Addis Ababa and Ambo, with a half dozen intermediate stops, gains speed when it starts from a station and loses speed when it is approaching a station. Again, it slows down in motion while passing over the bridges etc. It also changes its direction quit often while proceeding along its journey. Thus, the speed of the bus is not uniform but variable.

When the speed of a body varies, then we should use the term average speed since we are determining the average value of the spree over the time interval we are considering. Thus, the average speed of a body between two points is measured by the total distance covered by the body between those points divided by the total time taken by the body to travel that distance.

Mathematically, this is expressed as

$$\boldsymbol{Average\ speed} = \frac{\Delta x}{\Delta t} \quad (2\text{-}2)$$

Where $\Delta \boldsymbol{x}$ is the total distance covered by the body in the corresponding time interval $\Delta \boldsymbol{t}$.

Remark:

Speed is a *scalar physical quantity* that refers to "how fast an object is moving." Speed can be thought of as the rate at which an object covers distance. A fast-moving object has a high speed and covers a relatively large distance in a short amount of time. Contrast this to a slow-moving object that has a low speed; it covers a relatively small amount of distance in the same amount of time. An object with no movement at all has a zero speed. Speed is represented by "*v*" usually, and hence,

$$v = \frac{\Delta x}{\Delta t}$$

2.2.2 Velocity Average Velocity

Velocity is related to speed in the same way that displacement is related to distance. It is the rate of displacement *i.e.,* it is the ratio of the displacement which takes account of direction as well distance to time interval. In other words, velocity is the rate of change of position of a body in a particular direction.

The velocity of a body is also expressed in meters per second (ms^{-1}) or kilometres per sec or kilometres per hour (*km/h*) like speed. Its dimensions are $M^0L^1T^{-1}$.

Consider a particle moving along the x-axis, as shown in Figure 2.1.

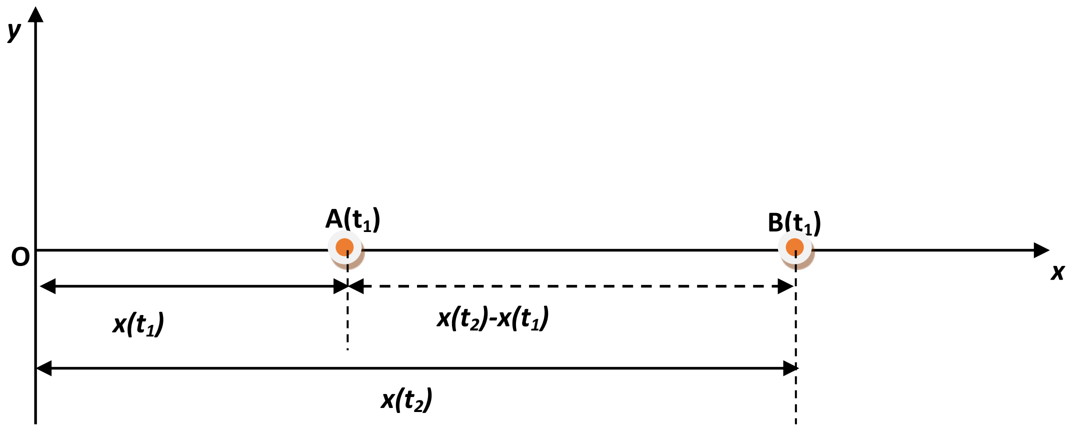


Figure 2-2: Variation of position with time.

The instantaneous positions of the particle are uniquely determined by its distance from the origin O, *i.e*., its *x* coordinate. The distance travelled during the time interval $t_2 - t_1$ is $x(t_2) - x(t_1)$.

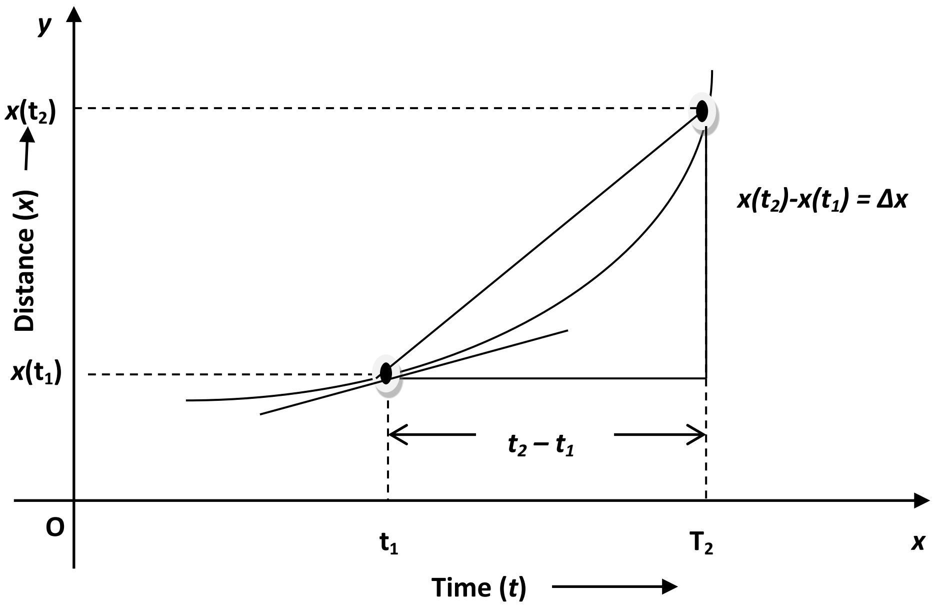


Figure 2-3: Distance – time graph

The ratio of the displacement of the particle to this time interval gives the average velocity between points A and B. *i.e.,* on a graph of x versus t, v_{av} is the slope of the straight line that connects two particular points on the x (t) curve: one is the point that corresponds to $x(t_2)$ and t_2, and the other is the point that corresponds to $x(t_1)$ and t_1.

$$\boldsymbol{Average\ velocity} = \boldsymbol{v_{av}} = \boldsymbol{\overline{v}} = \frac{\boldsymbol{Displacement}}{\boldsymbol{Time}},$$

Or

$$\boldsymbol{v_{av}} = \frac{\boldsymbol{x(t_2)-x(t_1)}}{\boldsymbol{t_2-t_1}} = \frac{\boldsymbol{\Delta x}}{\boldsymbol{\Delta t}} \qquad (2\text{-}3)$$

Remark:

When the velocity of a body is constant, the velocity is very simply defined as the displacement travelled divided by time taken; but when the velocity changes with time, (i.e., either its speed changes or direction of motion changes or both changes), then a more careful definition is

2.2.3 Instantaneous velocity:

We have now seen two ways to describe how fast something moves: average velocity and average speed, both of which are measured over a time interval Δt. However, the phrase "how fast" more commonly refers to how fast a particle is moving at a given instant—its instantaneous velocity (or simply velocity) v.

The velocity at any instant is obtained from the average velocity by shrinking the time interval Δt closer and closer to 0.

$$v_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

For uniform motion

$$v_{inst} = v_{av}$$

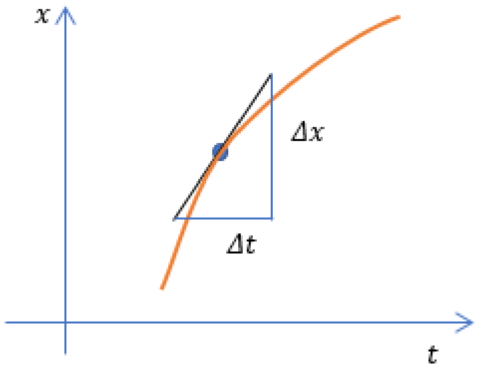


Figure 2-4: x-t curve for non-uniform motion.

Examples:

1. A car travelled a distance of 150 km in 3 hours on straight level road. What is the speed of the car?

Solution:

$$v = \frac{s}{t} = \frac{150km}{3hr} = 5km/hr$$

2. A bus travelling at a speed of 120km/hr east wards continues its journey steadily on a straight level road for 3 hours. What is the actual displacement of the bus after 3 hours?

Solution:

$$\vec{s} = \vec{v}_{av}t = \left(\frac{120km}{hr}east\right)(3hrs) = 360\ km\ east$$

3. A turtle and a rabbit engage in a footrace over a distance of 4.00 km. The rabbit runs 0.500 km and then stops for a 90.0-min nap. Upon awakening, he remembers the race and runs twice as fast. Finishing the course in a total time of 1.75 h, the rabbit wins the race. (a) Calculate the average speed of the rabbit. (b) What was his average speed before he stopped for a nap?

Solution:

Finding the overall average speed in part (a) is just a matter of dividing the total distance by the total time. Part (b) requires two equations and two unknowns, the latter turning out to be the two different average speeds: v_1 before the nap and v_2 after the nap. One equation is given in the statement of the problem (v_2 _ $2v_1$), whereas the other comes from the fact that the rabbit ran for only 15 minutes because he napped for 90 minutes.

(a) Find the rabbit's overall average speed.

$$\textbf{Average speed} = \frac{\textbf{total distance}}{\textbf{\textit{total time}}}$$

$$= \frac{4.0\,km}{1.75\,hr}$$

$$= 2.29\,km/hr$$

(b) Find the rabbit's average speed before his nap.

Sum the running times, and set the sum equal to 0.25 h:

$$t_1 + t_2 = 0.250\,h.$$

Substitute $t_1 = d_1/v_1$ and $t_2 = d_2/v_2$:

$$\frac{d_1}{v_1} + \frac{d_2}{v_2} = 0.250h \qquad (1)$$

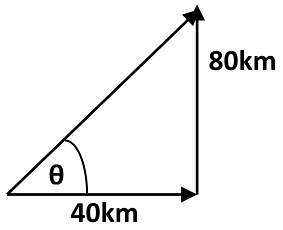
Substitute $v_2 = 2v_1$ and the values of d_1 and d_2 into Equation (1)

$$\frac{0.500\,km}{v_1} + \frac{3.50\,km}{2v_1} = 0.250\,h \qquad (2)$$

Solve Equation (2) for v_1:

$$v_1 = 9.00\,km/h$$

4. A car travelled 40km east in 1hr and then travelled 80km north in 2hrs. Calculate
   (a) its average speed, and
   (b) its average velocity



Solution:

(a) average speed,

$$\boldsymbol{v_{av}} = \frac{\boldsymbol{total\ distance}}{\boldsymbol{total\ time}}$$
$$= \frac{\boldsymbol{40km + 80km}}{\boldsymbol{1hr + 2hr}} = \boldsymbol{40km/hr}$$

(b) Average velocity,

$$\vec{\boldsymbol{v}}_{\boldsymbol{av}} = \frac{\boldsymbol{total\ displacement}}{\boldsymbol{total\ time}} = \vec{\boldsymbol{v}}_{\boldsymbol{east}} + \vec{\boldsymbol{v}}_{\boldsymbol{north}}$$

Magnitude of the average velocity,

$$\boldsymbol{v_{av}} = \sqrt{\boldsymbol{v^2}_{\boldsymbol{east}} + \boldsymbol{v^2}_{\boldsymbol{north}}}$$

$$\boldsymbol{v_{av}} = \sqrt{\left(\frac{\boldsymbol{40km}}{\boldsymbol{hr}}\right)^2 + \left(\frac{\left[\frac{80}{2}\right]\boldsymbol{km}}{\boldsymbol{hr}}\right)^2} = \frac{\boldsymbol{40\sqrt{2}km}}{\boldsymbol{hr}} = \boldsymbol{56.56\ km/hr}$$

And its direction,

$$\boldsymbol{\theta} = \tan^{-1}\frac{\boldsymbol{80km}}{\boldsymbol{40km}} = \tan^{-1}\boldsymbol{2} = \boldsymbol{64^0}$$

Therefore,

$$\vec{\boldsymbol{v}}_{\boldsymbol{av}} = \boldsymbol{56.56\ \ km/hr}\,, \boldsymbol{64}° \text{ with respect to the east direction.}$$

5. The position vector of a body moving along the x-axis is given by $x = 10\,cm/s^2\ t^2$. Compute its instantaneous velocity at time $t = 2s$.

Solution:

$$\vec{v}_{ins}(t) = \lim_{\Delta t \to 0} \frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{10\,cm/s^2\,(t+\Delta t)^2 - 10\,cm/s^2\;t^2}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{10\,cm/s^2\,t^2 + 10\,cm/s^2\,(2t\Delta t) + 10\,cm/s^2\;(\Delta t)^2 - 10\,cm/s^2\,t^2}{\Delta t}$$

$$= \lim_{\Delta t \to 0}\left[\frac{10cm/s^2(2t\Delta t)}{\Delta t} + \frac{10cm/s^2\;(\Delta t)^2}{\Delta t}\right] = (20t)\;cm/s^2;$$

Hence at $t = 2sec, v_{ins} = 40cm/s$

2.3 Acceleration

When velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the magnitude of the velocity of a car increases when you step on the gas and decreases when you apply the brakes. Let us see how to quantify acceleration.

Suppose an object that can be modelled as a particle moving along the *x*- axis has an initial velocity v_{xi} at time t_i and a final velocity v_{xf} at time t_{xf},

The average acceleration $\bar{a}_x = a_{av}$of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that changes occur:

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2\text{-}4)$$

As with velocity, when the motion being analysed is one-dimensional, we can use positive and negative signs to indicate the direction of the acceleration. Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T^2. The SI unit of acceleration is meters per second squared (m/s^2). There are other non SI unit, such as (cm/s^2, ft/s^2).

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the *instantaneous acceleration* as the limit of the average acceleration as Δt approaches zero.

$$\vec{a}_{ins}(t) = lim_{\Delta t \to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t} \qquad (2\text{-}5)$$

Example:

1. A particle is in motion and is accelerating. The functional form of the velocity is $v(t) = 20t - 5t^2\ m/s$. Find the instantaneous acceleration at t = 1, 2, 3, and 5 s.

Solution:

$$\vec{a}_{ins}(t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

$$\vec{a}_{ins}(t) = \lim_{\Delta t \to 0} \frac{20(t+\Delta t) - 5(t+\Delta t)^2 - (20t - 5t^2)}{\Delta t}$$

$$\vec{a}_{ins}(t) = \lim_{\Delta t \to 0}(20 - 5\Delta t - 10t) = 20 - 10t$$

i) $\vec{a}_{ins}(t=1) = 20 - 10t = 10 m/s^2$

ii) $\vec{a}_{ins}(t=2) = 20 - 10t = 0 m/s^2$

iii) $\vec{a}_{ins}(t=3) = 20 - 10t = -10 m/s^2$

iv) $\vec{a}_{ins}(t=5) = 20 - 10t = -30 m/s^2$

Remark:

When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down (which we call *deceleration*). Average acceleration and instantaneous acceleration are equal during uniformly accelerated motion. Acceleration of a body is 2 m/s^2 means that the velocity increases by 2 m/s every one second.

2.3.1 Motion with constant acceleration

The simplest kind of accelerated motion is straight-line motion in which the acceleration is constant. This means that the velocity changes at the same rate throughout the motion. As can be seen from the velocity-time graph, the velocity is increasing by equal amounts in equal intervals of time. The slope of a chord between any two points on the line is the same as the slope of a tangent at any point, and the average and instantaneous accelerations are equal.

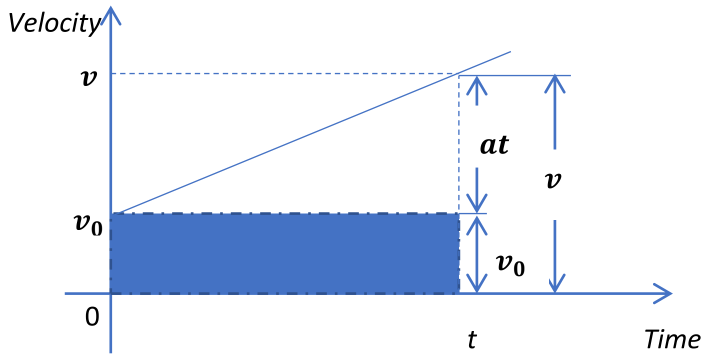


Figure 2-5: velocity – time graph for rectilinear motion

Hence the average acceleration a_{av} can be replaced by the constant acceleration a, and we have

$$\boldsymbol{a} = \frac{\boldsymbol{v_f - v_i}}{\boldsymbol{t_f - t_i}} \tag{2-6}$$

Let $t_i = 0$ and let t_f be any arbitrary later time t. Let v_0 represent the velocity when $t = 0$ (called the initial velocity), and let v be the velocity at any later time t (final velocity). Then the preceding equation becomes

$$a = \frac{v - v_0}{t - 0}$$

Or

$$\boldsymbol{v = v_0 + at} \tag{2-7}$$

To find the displacement of a particle moving with constant acceleration, we make use of the fact that when the acceleration is constant and the velocity-time graph is a straight line, see Figure 2.3, the average velocity in any time interval equals one-half the sum of the velocities at the beginning and the end of the interval. Hence, the average velocity between zero and t is

$$\overline{\boldsymbol{v}} = \frac{\boldsymbol{v_0 + v}}{\boldsymbol{2}} = \boldsymbol{v_{av}} \tag{2-8}$$

(This is not true in general, when the acceleration is not constant and the velocity-time graph is curved, as in figure 4 below.)

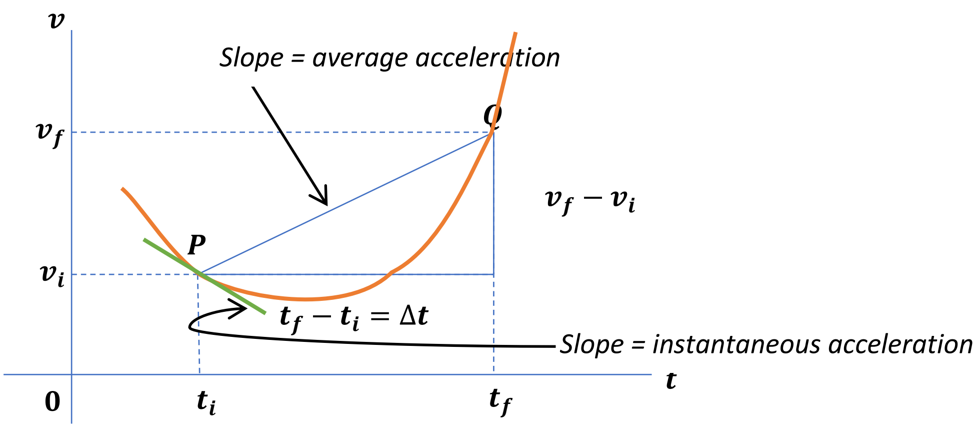


Figure 2-6: velocity-time graph. The average acceleration between t_i and t_f equals the slope of the chord PQ.

By definition, the average velocity is

$$v_{av} = \frac{x_f - v_i}{t_f - t_i}$$

Let $t_i = 0$ and let t_f be any arbitrary time t. Let x_0 represent the position when $t = 0$ (the initial position) and let x be the position at time t. Then the preceding equation becomes

$x - x_0 = v_{av} t.$

Substituting for v_{av} we obtain

$x - x_0 = \left(\frac{v_0 + v}{2}\right) t.$

If we eliminate v,

$x - x_0 = \left(\frac{v_0 + v_0 + at}{2}\right) t$

Or

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \qquad (2\text{-}9)$$

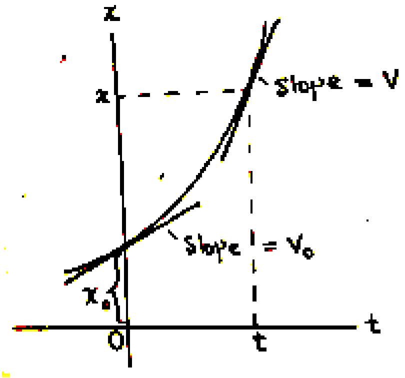


Figure 2-7: coordinate – time graph for motion with constant acceleration.

This equation gives a *parabolic* coordinate- time curve as in Figure 2-7. Eliminating the time t between equations (2.7) and (2.9) we get:

$$\boldsymbol{v^2 - v_0{}^2 = 2a(x - x_0)} \quad (2\text{-}10)$$

Example:

1. An object moves along the x – axis with constant acceleration $a = 5\ m.s^{-2}$. At time $t = 0$ it is at $x = 6m$ and has velocity $v = 3m/s$.

(a) Find the position and velocity at time *t = 2s.*

(b) Where is the body when its velocity is *6m/s*?

Solution:

Using Equation (2.9), the position can be calculated:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$x = 6m + \left(\frac{3m}{s}\right)(2s) + \frac{1}{2}(5\ m.s^{-2})(2s)^2 = 22m$$

We can also obtain the velocity from Expression (2.7):

$$v = v_0 + at$$
$$v = 3\frac{m}{s} + (5\ m.s^{-2})(2s) = 13\ m/s.$$

To know where the body is when the velocity is 6m/s means to find the coordinate (or x). Hence, we can use Equation (2.10)

$$v^2 - {v_0}^2 = 2a(x - x_0)$$
$$(6\ \mathrm{m/s})^2 - (3\ \mathrm{m/s})^2 = 2(5\ m.s^{-2})(x - x_0)$$

Or

$$x - x_0 = \frac{27\, m^2/s^2}{10\ m/s^2} = 2.7m$$

Since $x_0 = 6m$,

$$x = 2.7m + 6m = 8.7m$$

We can also calculate the time and find the position:

$$t = \frac{v - v_0}{a} = \frac{6\,\mathrm{m}/s - 3\,\mathrm{m}/s}{5\, m/s^2} = 0.6\ s$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 8.7m$$

Remark: If the velocity is constant the acceleration is zero, and hence,

$$v = constant$$
$$x = x_0 + vt$$

2. A certain automobile manufacturer claims that its super-deluxe sport car will accelerate uniformly from rest to a speed of 87 mi/h in 8s.

(a) Determine the acceleration of the car.

Solution:

$$a = \frac{v - v_0}{t} = \frac{38.9\mathrm{m/s} - 0}{8\ s} = +4.86\ m/s^2$$

Where $1mi = 1.60934km;\ 1\ mi/h = 0.447m/s; 87.0\ \ mi/h\ = 38.9m/s$

(b) Find the distance the car travels in the first 8 s.

Solution:

$$x = v_{av}t = \frac{1}{2}(v + v_0)$$

$$= \frac{1}{2}(38.9\ \mathrm{m/s} + 0)(8\ s) = 156m$$

(c) What is the velocity of the car 10s after it begins its motion if it continuous with the same acceleration?

Solution

$$v = v_0 + at = 0 + 4.86\frac{m}{s^2} \times 10s = 48.6m/s$$

(d) Find the distance travelled during the 9th second.

Solution:

$$\Delta S_9 = S_9 - S_8 = v_0(t_9 - t_8) + \frac{1}{2}a({t_9}^2 - {t_8}^2)$$
$$= 0 + \frac{1}{2}(4.86)(9^2 - 8^2)$$
$$= (0.5)(4.86)(17)$$
$$= 41.31m$$

2.3.2 Free fall motion

The most common example of motion with (nearly) constant acceleration is that of a body falling toward the earth. In the absence of air resistance it is found that all bodies, regardless of their size or weight, fall with the same acceleration at same point on the earth's surface; and if the distance covered is small compared to the radius of the earth, the acceleration remains constant throughout the fall. The effect of air resistance and the decrease in acceleration with altitude will be neglected. This idealized motion is spoken of as "free fall," although the term includes rising as well as falling motion.

The acceleration of a freely falling body is called *acceleration due to gravity*, or the *acceleration of gravity*, and is denoted by the letter g.

Remark:

At or near the earth's surface the magnitude of g is approximately 9.8 m/s^2, or $980c\,m/s^2$, or 32 ft/s^2. More precise values, and small variations with latitude and elevation. On the surface of the moon, it is due to the attractive force exerted on a body by the moon and not by the earth. On the moon $g = 1.67m/s^2$, and near the surface of the sun, $g = 274m/s^2$. The gravitational force accelerates an object toward the earth and decelerates an object moving upward near the surface of the earth, and direction of g is always (positive) towards earth's centre.

For freely falling bodies the motion is vertical along *y*- axis so that a is replaced by g and *x* is replaced by *y* in the equations of motion for rectilinear motion.

1. $\boldsymbol{v_y = v_{0y} + gt}$ final velocity at any time $\boldsymbol{t}$
2. $\boldsymbol{y = v_{0y}t + \frac{1}{2}gt^2}$ vertical position at any time $\boldsymbol{t}$
3. $\boldsymbol{{v_y}^2 = {v_{0y}}^2 + 2gy}$ (2-11)
4. For freely falling object $\boldsymbol{v_0 = 0}$.
5. When an object reaches a maximum height $\boldsymbol{v = 0}$.

Example:

1. A body is released from rest and falls freely. Compute its position and velocity after 1 and 2s. Take the origin at the elevation of the starting point, the y-axis vertical, and the upward direction as positive.

Solution:

The initial coordinate $y_0$and the initial velocity v_0 are both zero. The acceleration is downward, in the negative y-axis, so $g = -9.8\ m/s^2$. (It is convenient to set $g = +9.8\ m/s^2$ for downward motion and $g = -9.8\ m/s^2$ for upward motion to directly use the equations of rectilinear motion without change following direction of the acceleration due to gravity which is always directed down ward.)

When $t = 1s$,

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$y = 0m + (0\ m/s)(1s) + \frac{1}{2}(9.8\ m/s^2)(1s)^2 = 4.9m$$

Hence, the body is at 4.9 m below the origin. The velocity, then, becomes

$$v_y = v_{0y} + gt$$

$$v_y = 0\frac{m}{s} + \left(\frac{9.8m}{s^2}\right)(1s) = 9.8\ m/s$$

The velocity is negative, of magnitude9.8 m/s.

$ii.$ When *t=2s*,

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$y = 0m + (0\ m/s)(2s) + \frac{1}{2}(9.8\ m/s^2)(2s)^2 = 9.6m$$

The velocity can be calculated as:

$$v_y = v_{0y} + gt$$

$$v_y = 0\ m/s + (9.8\ m/s^2)(2s) = 19.6\ m/s$$

The velocity is negative, of magnitude 19.6 m/s.

2. A stone is thrown from the top of a building with an initial velocity of 20 m/s straight upward. The building is 50 m high, and the stone just misses the edge of the roof on its way down. Determine (a) the time needed for the stone to reach its maximum height, (b) the maximum height, (c) the time needed for the stone to return to the level of thrower, (d) the velocity of the stone at this instant, and (e) the velocity and the position of the stone at $t = 5s$.

Solution:

(a) Since the motion is upward, we take $+9.8\ m/s^2$ and the final velocity, $v_y = 0$ at maximum height. To find the time necessary to reach the maximum height, we use the equation

$v_y = v_{0y} + gt$ where $v_{0y} = 20\ m/s$.

$$t = \frac{v_y - v_{0y}}{g}$$

$$t = \frac{0 - 20\ m/s}{-9.8\ m/s^2} = 2.041\ s$$

(b) The maximum height h is calculated as:

$$y = v_{0y}t + \frac{1}{2}gt^2$$

$$y = (20\ m/s)(2.041\ s) + \frac{1}{2}(-9.8\ m/s^2)(2.041\ s)^2 = 20.41m$$

as measured from the position of throw.

(c) The time required to return to the level of the thrower is twice of the time calculated in part (a) which is *4.082s*, or calculated with the assumption that displacement is zero.

$$y = v_{0y}t + \frac{1}{2}gt^2$$

$$0 = (20\ m/s)\ t + \frac{1}{2}(-9.8\ m/s^2)t^2$$

$hence, t = 0, and\ \ t = 4.082\ s. t = 0$ corresponds to the time the stone starts its motion and $t = 4.082\ s$ is the required time.

(d) The velocity of the stone at this instant means the velocity at which the body reached the level of the throw point back. Hence,

$$v_y = v_{0y} + gt$$
$$v_y = 20\ m/s + (-9.8\ m/s^2)(4.082\ s)$$
$$= -20\ m/s$$

Note that the magnitude of this velocity is the same as the magnitude of the initial velocity.

(e) After 5 seconds the position can be obtained using the expression

$$y = v_{0y}t + \frac{1}{2}gt^2$$

$$y = \left(20\frac{m}{s}\right)(5\ s) + \frac{1}{2}\left(-\frac{9.8m}{s^2}\right)(5\ s)^2 = -22.5m$$

This means that the body is 22.5m below the point of throw; or *50m +(- 22.5m) = + 27.5 m* from the base of the building.

The velocity, now becomes

$$v_y = v_{0y} + gt$$
$$= 20\ m/s + (-9.8\ m/s^2)(5\ s)$$
$$= -29\ m/s$$

Or, *29 m/s* down ward.

(f) Find the velocity of the stone just before it hits the ground.

Use vertical distance -50m, initial velocity 20 m/s and g = -9.8 m/s^{2}since the motion started at above 50m below the reference point. Hence,

$$
\begin{aligned}
{v_y}^2 &= {v_{0y}}^2 + 2gy \\
&= (20\, m/s)^2 + 2(-9.8\, m/s^2)(-50\, m) \\
&= 1380\ m^2/s^2 \\
v_y &= -37.15\, m/s
\end{aligned}
$$

or by first calculating the total time of flight,

$$
\begin{aligned}
y &= v_{0y}t + \frac{1}{2}gt^2 \\
-50 &= 20t + (0.5)(-9.8)t^2
\end{aligned}
$$

Solving the quadratic equation for *t* yields *t = 5.831s*.

Finally,

$$
\begin{aligned}
v_y &= v_{0y} + gt \\
&= 20 + (-9.8)(5.831) \\
&= 20 - 57.1438 \\
&= -37.14\, m/s
\end{aligned}
$$

2.4 Chapter Summary

Displacement

The displacement of an object moving along the *x* - axis is defined as the change in position of the object,

$$\Delta x = x_f - x_i$$

where x_i is the initial position of the object and x_f is its final position.

Velocity

The average speed of an object is given by

$$\text{Average speed} = \frac{path\ lenth}{elapsed\ time}$$

The average velocity $\vec{\boldsymbol{v}}_{\boldsymbol{av}}$ during a time interval $\Delta \boldsymbol{t}$ is the displacement $\Delta\vec{\boldsymbol{x}}$ divided by $\Delta \boldsymbol{t}$.

$$\vec{\boldsymbol{v}}_{\boldsymbol{av}} = \frac{\Delta\vec{\boldsymbol{x}}}{\Delta \boldsymbol{t}} = \frac{\vec{\boldsymbol{x}}_{\boldsymbol{f}} - \vec{\boldsymbol{x}}_{\boldsymbol{i}}}{\boldsymbol{t}_{\boldsymbol{f}} - \boldsymbol{t}_{\boldsymbol{i}}}$$

The average velocity is equal to the slope of the straight line joining the initial and final points on a graph of the position of the object versus time. The slope of the line tangent to the position versus time curve at some point is equal to the instantaneous velocity at that time. The instantaneous speed of an object is defined as the magnitude of the instantaneous velocity.

Acceleration

The average acceleration *a* of an object undergoing a change in velocity $\Delta\vec{v}$ during a time interval Δt is

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

The instantaneous acceleration of an object at a certain time equals the slope of a velocity versus time graph at that instant.

The most useful equations that describe the motion of an object moving with constant acceleration along the *x* – axis are as follows:

$$v = v_0 + at$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = {v_0}^2 + 2a\Delta x$$

All problems can be solved with the first two equations alone, the last being convenient when time doesn’t explicitly enter the problem. After the constants are properly identified, most problems reduce to one or two equations in as many unknowns.

2.5 Conceptual Questions

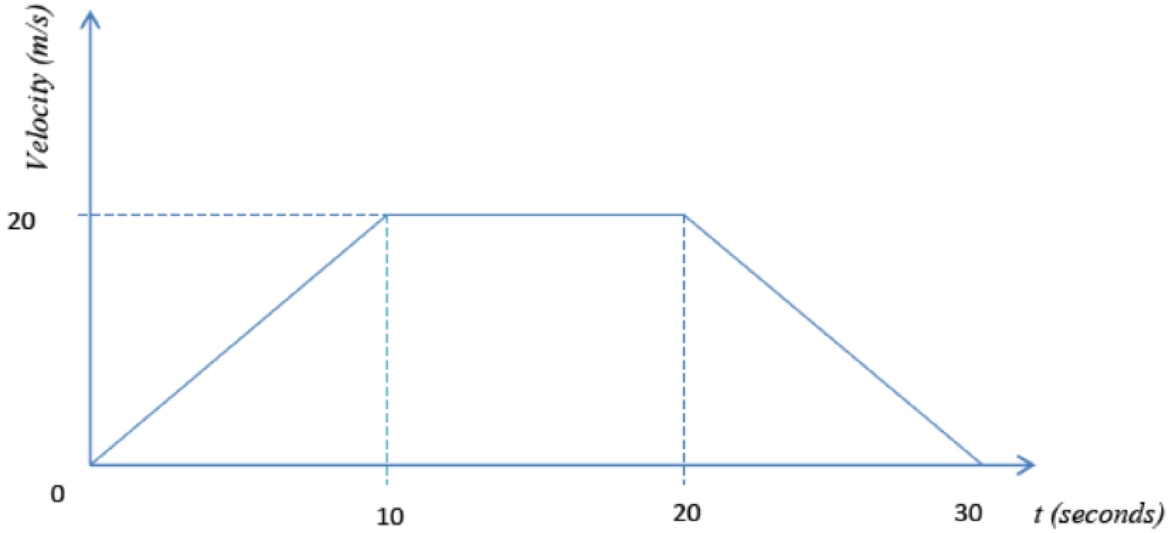
1. If the velocity of a particle is nonzero, can the particle’s acceleration be zero? Explain.
2. If the velocity of a particle is zero, can the particle’s acceleration be nonzero? Explain.
3. If a car is traveling eastward, can its acceleration be westward? Explain.
4. A ball is thrown vertically upward.
   (a) What are its velocity and acceleration when it reaches its maximum altitude?
   (b) What is the acceleration of the ball just before it hits the ground?

2.6 Problems

1. A cyclist goes south at $15km/hr$ for $10km$ and then west at $20\ \mathrm{km/hr}$ for 10 km. Find his average speed and his average velocity.
2. A car travels east at $40\ km/hr$ for $30\ min$, and then north at $30\ km/hr$ for $30\ min$.
    a. What distance did it travel?
    b. What was its displacement?
    c. Calculate its average speed.
    d. Calculate its average velocity.
3. A body accelerates uniformly from $16\ \mathrm{m/s}$ to $20\ m/s$ while covering a distance of $72m$. calculate
    a. Its uniform acceleration.
    b. The time taken.
    c. The distance covered in the last second of its journey.
4. An object starts from rest with constant acceleration of $8m/s^2$ along a straight line. Find
    a. The speed at the end of 5 seconds.
    b. The average speed for the 5 seconds interval.
    c. The distance travelled in 5 seconds.
    d. The distance travelled during the 5th second.
5. A body falls freely from rest for 6 $seconds$. Find
    a. The height covered.
    b. The final speed of the body.
    c. The distance covered in the last 2 $seconds$.
6. A ball is dropped from rest at a height of 50m above the ground.
    a. How long does it take to reach the ground?
    b. What is its speed just before it hits the ground?
7. A car moved east ward 120km/hr in 3 hrs and then north by 60km/hr in 2hrs. calculate:
    a. Its average speed.
    b. Its average velocity.
8. A student walked at 12km/hrs for 2hrs and run at a speed of 16km/hr for 5hrs on a straight road. Calculate his average speed.
9. A stone is flung down from the top of a cliff with a velocity of 6.3m/s and reach the bottom in 3 seconds. How high is the cliff, and with what velocity does the stone hit the ground?
10. A body accelerates uniformly from 12m/s to 16m/s while covering a distance of 70m. calculate:
    a. Its uniform acceleration
    b. The time taken

c. The distance covered in last second of its journey.

11. The following figure represents the velocity of a body during the first 30 seconds of its motion. Find distance and acceleration of the body from 20 seconds to 30 seconds.



12. A body is dropped from rest from the top of a very high building. Taking the acceleration due to gravity to be $10\ m/s^2$, draw
    a. The speed-time graph.
    b. The displacement-time graph of the body for the first 6 seconds of its fall, assuming that the body reaches a constant velocity after 4 seconds.

3 Kinematics in Two Dimensions

Learning Outcome

After completing this Chapter, students are expected to:

- Define and formulate displacement, velocity and acceleration of arbitrary particle in two dimensions.
- Calculate instantaneous velocity and acceleration in two dimensions.
- Familiar with motions in two dimensions.
- Discuss motion with constant acceleration
- Familiar with Projectile motions.
- Explain Uniform Circular motion.
- Attempt to solve related problems in this chapter.

Introduction

The physical world is full of moving objects to describe the motion of real objects you usually need to make simplifying assumptions. Perhaps the most important simplification in applied mathematics is ignoring the size and shape of an object we are consider objects as particles and this then called the particle model. The physical quantities required for the kinematics description of the motion of a particle are its *position*, *velocity* and *acceleration*. The form which the description of these vector quantities takes depends on the coordinates in terms of which and the coordinate system with respect to which we choose to describe the motion of the particle.

In this chapter we explore the kinematics of a particle moving in two dimensions. Knowing the basics of two-dimensional motion will allow us to examine—in future chapters—a wide variety of motions, ranging from the motion of satellites in orbit to the motion of electrons in a uniform electric field. We begin by studying in greater detail the vector nature of position, velocity, and acceleration. As in the case of one-dimensional motion, we derive the kinematics equations for two-dimensional motion from the fundamental definitions of these three quantities. We then treat projectile motion and uniform circular motion as special cases of motion in two dimensions.

3.1 Displacement, Velocity and acceleration in two dimensions

Learning outcome

After completing this section, students are expected to:

- Drive motion of equation for an arbitrary particle in two dimensions.

- Define displacement, velocity and acceleration in two dimensions.
- Define instantaneous velocity and acceleration
- Solve problems related with this section

3.1.1 Position and displacement

We begin by describing the position of a particle by its position vector $\bar{\boldsymbol{r}}$, drawn from the origin of some coordinate system to the particle located in the *xy* plane, as in Figure 3.1. The position vector $\bar{\boldsymbol{r}}$ locates an object relative to the origin of a reference frame "o" shown in Fig 3.1 and mathematically given in Eq.3.1 in two-dimensions.

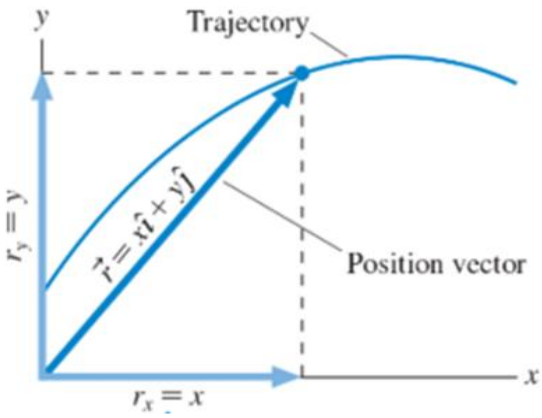


Figure 3.1: Position vector in xy plane

$$\vec{\boldsymbol{r}} = \boldsymbol{x}\hat{\boldsymbol{\imath}} + \boldsymbol{y}\hat{\boldsymbol{\jmath}} \qquad 3.1$$

Where x and y are object co- ordinates. At time t_i the particle is at point P, described by position vector r_i. At some later time t_f it is at point Q, described by position vector r_f. The path from P to Q is not necessarily a straight line. As the particle moves from P to Q in the time interval $\Delta t = t_f - t_i$, its position vector changes from $\bar{\boldsymbol{r}}_{\boldsymbol{i}}$ to $\bar{\boldsymbol{r}}_{\boldsymbol{f}}$. As we learned in Chapter 2, displacement is a vector, and the displacement of the particle is the difference between its final position and its initial position. We now define the displacement vector $\Delta\bar{\boldsymbol{r}}$ for the particle of Figure 3.2 as being the difference between its final position vector and its initial position vector:

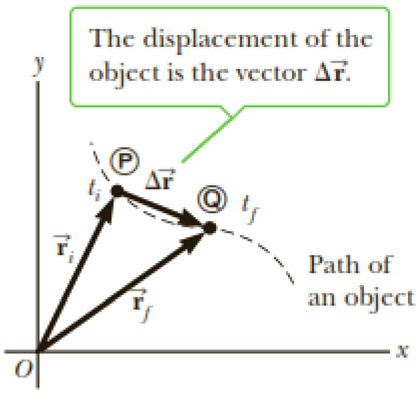


Figure 3.2: An object moving along some curved path between points P and Q. The displacement vector $\Delta\bar{r}$ is the difference in the position vectors $\bar{r}_i$ and $\bar{r}_f$.

From the vector diagram in Figure 3.2, the final position vector is the sum of the initial position vector and the displacement $\bar{r}_f = \bar{r}_i + \Delta\bar{r}$. From this relationship, we obtain the displacement of an object. An object's displacement is defined as the change in its position vector, or

$$\Delta\bar{r} = \bar{r}_f - \bar{r}_i \qquad 3.2$$

The displacement vector $\Delta\bar{r}$ has components along Δx and Δy (See Fig 3.3) for an object that moves from location (x_i, y_i) to (x_f, y_f) in a time interval Δt. The x-component of the object`s displacement is $\Delta x = x_f - x_i$ and the y-component of the object`s displacement is $\Delta y = y_f - y_i$.

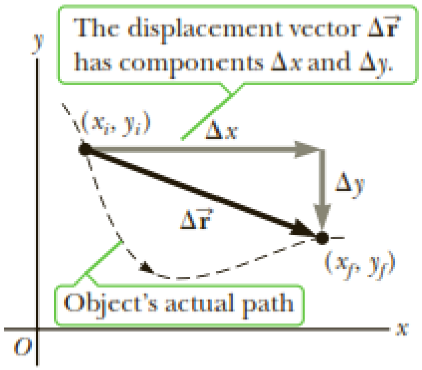


Figure 3.3: The displacement vector components along the x and y axes.

Therefore, Eq. 3.2 can be rewrite as:

$$\Delta\bar{r} = \Delta x\boldsymbol{i} + \Delta y\boldsymbol{j} \qquad 3.3$$

The direction of $\Delta\bar{r}$ is indicated in Figure 3.2. As we see from the figure, the magnitude of $\Delta\bar{r}$ is *less* than the distance travelled along the curved path followed by the particle.

3.1.2 Velocity

It is often useful to quantify motion by looking at the ratio of a displacement divided by the time interval during which that displacement occurs, which gives the rate of change of position. In two-dimensional (or three-dimensional) kinematics, everything is the same as in one-dimensional kinematics except that we must now use full vector notation rather than positive and negative signs to indicate the direction of motion. We define the average velocity of a particle during the time interval Δ*t* as the displacement of the particle divided by the time interval:

$$\vec{V}_{av} = \frac{\Delta \bar{r}}{\Delta t} \qquad 3.4$$

The *x* and *y* components of the average velocity are given by

$$V_{av,x} = \frac{\Delta x}{\Delta t} \quad \text{and} \quad V_{av,y} = \frac{\Delta y}{\Delta t} \qquad 3.5$$

and express the rate at which an object's position is changing along the *x* and *y* axes, respectively. Note that the magnitude of the average velocity is just the distance between the endpoints divided by the elapsed time and the direction of the average velocity is the same as the direction of $\Delta\bar{r}$. Consider again the motion of a particle between two points in the *xy* plane, as shown in Figure 3.4.

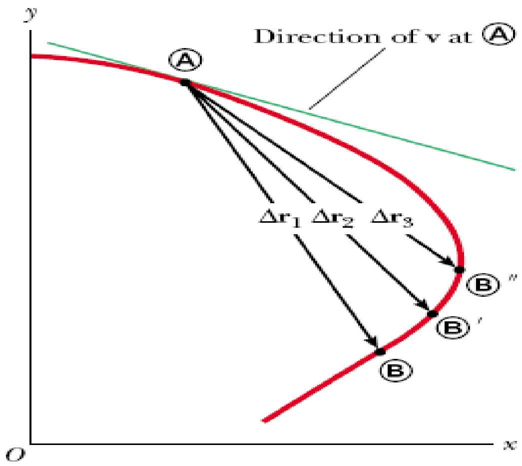


Figure 3.4: Displacement of the particle in xy plane

As the time interval over which we observe the motion becomes smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at A. The instantaneous velocity $\overline{V}$ is defined as the limit of the average velocity $\frac{\Delta \bar{r}}{\Delta t}$ as Δt approaches zero:

$$\overline{V} = \lim_{\Delta t \to 0} \Delta \overline{V} = \lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t} \qquad 3.6$$

The direction of the instantaneous velocity vector is along a line that is tangent to the object's path and in the direction of its motion.

3.1.3 Acceleration

As a particle moves from one point to another along some path, its instantaneous velocity vector changes from V_i at time t_i to V_f at time t_f. Knowing the velocity at these points allows us to determine the average acceleration of the particle. An object's average acceleration during a time interval Δt is the change in its velocity $\Delta \overline{V}$ divided by Δt, or

$$\bar{\boldsymbol{a}}_{av} = \frac{\Delta\bar{V}}{\Delta t} \qquad 3.7$$

Average acceleration has components given by

$$\boldsymbol{a}_{av,x} = \frac{\Delta V_x}{\Delta t} \quad \text{and} \quad \boldsymbol{a}_{av,y} = \frac{\Delta V_y}{\Delta t} \qquad 3.8$$

When the average acceleration of a particle changes during different time intervals, it is useful to define its instantaneous acceleration. The instantaneous acceleration $\vec{a}$ is defined as the limiting value of the ratio $\frac{\Delta\bar{V}}{\Delta t}$ as Δt approaches zero:

$$\vec{\boldsymbol{a}} = \lim_{\Delta T\to 0}\frac{\Delta\vec{V}}{\Delta t} \qquad 3.9$$

Remarks:

- It is important to recognize that various changes can occur when a particle accelerates. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Finally, both the magnitude and the direction of the velocity vector may change simultaneously.
- When an object's speed is increasing, the object's acceleration always has a component in the direction of its velocity. When an object's speed is decreasing the object's acceleration always has a component opposite its velocity.

Examples

1. Which of the following objects can't be accelerating?
    a. An object moving with a constant speed;
    b. an object moving with a constant velocity;
    c. an object moving along a curve.

Answer: *b*

2. Explain whether the following particles do or do not have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.

Exercises

1. If the instantaneous velocity does not change, will the average velocities for different intervals differ?
2. Can the speed of an object be negative? If so, give an example. If not, explain why not.
3. Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are (a) all three controls (b) the gas pedal and the brake (c) only the brake (d) only the gas pedal.
4. A hiker walks 2.00 km north and then 3.00 km east, all in 2.50 hours. Calculate the magnitude and direction of the hiker's (a) displacement (in km) and (b) average velocity (in km/h) during those 2.50 hours. (c) What was her average speed during the same time interval?
5. A car is traveling east at 25.0 m/s when it turns due north and accelerates to 35.0 m/s, all during a time of 6.00 s. Calculate the magnitude of the car's average acceleration.
6. A rabbit is moving in the positive x direction at 2.00 m/s when it spots a predator and accelerates to a velocity of 12.0 m/s along the negative y-axis, all in 1.50 s. Determine (a) the x component and (b) the y component of the rabbit's acceleration.

3.2 Projectile motion

The ball or body is in motion through the air, the only forces acting on it being its weight and the resistance to its motion due to the air. A motion like this is called a projectile motion and is very common especially in sports, for example, the motion of a basketball. The jumps of insects such as locusts, fleas and grass hoppers are projectile motions. It has a number of applications, for example, police accident investigators want to determine car speed from the position of glass and other objects at the scene of an accident. In this section we are going discuss motion that is slightly more complicated.

Learning outcome

After completing this section, students are expected to:

- Recognize that projectile motion is common.
- Explain how to obtain a simple mathematical model of projectile motion.
- Know how to use the model to investigate real life projectile problems.

3.2.1 Projectile Motion with Constant Acceleration

Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration *g* is constant over the range of motion and is directed downward, and (2) the effect of air resistance is negligible.

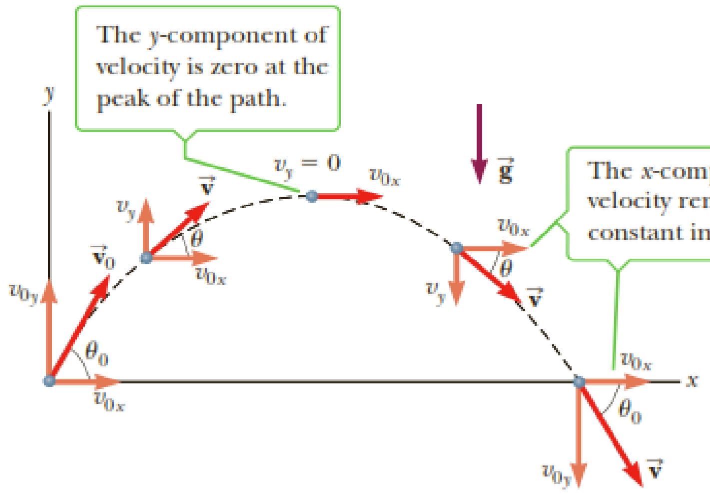


Figure 3.5: The parabolic trajectory of a particle that leaves the origin with a velocity of $\overline{\boldsymbol{V}}_{\mathbf{0}}$

With these assumptions, we find that the path of a projectile, which we call its *trajectory,* is *always* a parabola (see Fig. 3.5). From the figure one can also understand how the x and y components of the velocities are varied through the motion. The positive *x*- direction is horizontal and to the right, and the *y*-direction is vertical and positive upward. The most important experimental fact about projectile motion in two dimensions is that **the horizontal and vertical motions are completely independent of each other**. This means that motion in one direction has no effect on motion in the other direction.

In general, the equations of constant acceleration developed in chapter- 2 follow separately for both the *x*-direction and the *y*-direction. An important difference is that the initial velocity now has two components, not just one as in that topic. We assume that at $\boldsymbol{t = 0}$, the projectile leaves the origin with an initial velocity $\overline{\boldsymbol{V}}_{\boldsymbol{o}}$ at launching angle $\boldsymbol{\theta}$. (See Fig.3.6).

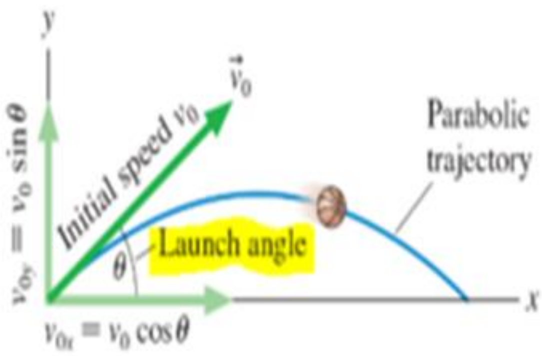


Figure 3.6: A projectile launched with an initial velocity $\overline{\boldsymbol{V}}_{\boldsymbol{o}}$

From the definitions of the cosine and sine functions and Figure 3.6 we have

$$\overline{V}_{ox} = \overline{V}_o cos\theta \quad \text{and} \quad \overline{V}_{oy} = \overline{V}_o sin\theta \qquad 3.10$$

where $\overline{\boldsymbol{V}}_{\boldsymbol{ox}}$is the initial velocity (at $\boldsymbol{t} = \boldsymbol{0}$) in the *x*-direction and $\overline{\boldsymbol{V}}_{\boldsymbol{oy}}$ is the initial velocity in the *y*-direction. For motion with constant acceleration in one dimension carry over to the two-dimensional case; there is one set of three equations for each direction, with the initial velocities modified as just discussed. In the *x*-direction, with $\overline{\boldsymbol{a}}_{\boldsymbol{x}}$ constant, we have

$$V_{fx} = V_{0x} + a_x t\,; \quad \Delta x = V_{0x}\, t + \frac{1}{2} a_x t^2 \;; \quad V_{fx}^2 = V_{0x}^2 t + 2a_x\, \Delta x \qquad 3.11$$

where $\overline{V}_{ox} = \overline{V}_o cos\theta$. In the *y*-direction, we have

$$V_{fy} = V_{0y} + a_y t\,; \quad \Delta y = V_{0y}\, t + \frac{1}{2} a_y t^2 \;; \quad V_{fy}^2 = V_{0y}^2 t + 2a_x\, \Delta y \qquad 3.12$$

where $\overline{V}_{oy} = \overline{V}_o sin\theta$ and $\overline{a}_y$ is constant. The object’s speed can be calculated from the components of the velocity using the Pythagorean theorem

$$V = \sqrt{V_x^2 + V_y^2} \qquad 3.13$$

The angle that the velocity vector makes with the *x*-axis is given by

$$\theta = tan^{-1}\left(\frac{V_y}{V_x}\right) \qquad 3.14$$

The kinematic equations are easily adapted and simplified for projectiles close to the surface of the Earth. In this case, assuming air friction is negligible, the acceleration in the *x*-direction is 0 (because air resistance is neglected). This means that $\bar{a}_x = 0$, and the projectile's velocity component along the x-direction remains constant. If the initial value of the velocity component in the *x*-direction is $\bar{V}_{ox} = \bar{V}_o cos\theta$, then this is also the value of $\bar{V}_x$ at any later time, so

$$\bar{V}_x = \bar{V}_{ox} = \bar{V}_o cos\theta = constant \qquad 3.15$$

whereas the horizontal displacement is simply

$$\Delta x = \bar{V}_{0x} t = (V_0 cos\theta)t \qquad 3.16$$

For the motion in the *y*-direction, we make the substitution $\bar{a}_y = -g$ and $\bar{V}_{oy} = \bar{V}_o sin\theta$

in Equations 3.12, giving

$$V_y = V_0 sin\theta - gt$$
$$\Delta y = (V_o sin\theta)t - \frac{1}{2}gt^2$$
$$V_y^2 = (V_0 sin\theta)^2 - 2g\Delta y \qquad 3.17$$

The important facts of projectile motion can be summarized as follows:

1. Provided air resistance is negligible, the horizontal component of the velocity $\bar{V}_x$. remains constant because there is no horizontal component of acceleration.
2. The vertical component of the acceleration is equal to the free-fall acceleration $-g$.
3. The vertical component of the velocity V_y and the displacement in the *y* - direction are identical to those of a freely falling body.
4. Projectile motion can be described as a superposition of two independent motions in the *x*- and *y*-directions.

Let us derive the trajectory or the path of the motion to do so let us consider the horizontal and vertical displacement of the motion which is mathematically given in the following manner respectively.

$\Delta x = (V_0 cos\theta)t$ and $\Delta y = (V_o sin\theta)t - \frac{1}{2}gt^2$ 3.18

It is possible to get the following relation from horizontal displacement

Examples

1. From the results in Eqs. (3.18) and (3.20) drive (a) The maximum height to which a projectile rise above the horizontal plan of the projection (b) The maximum horizontal displacement (Range) (c) The total time taken by the object to return to the same level.

Solution:

(a) To calculate it, we make use of the fact that the velocity $V_y(t)$ is zero at maximum height and let us assume that the time taken by the projectile to reach the maximum height is t_1. It is clear that the y-component of velocity is given as:

$$V_y = V_0 sin\theta - gt \quad \Rightarrow \quad 0 = V_0 sin\theta - gt$$

Thus, $t_1 = \frac{V_0 sin\theta}{g}$ (i)

If we substitute Eq (i) in to the y-component of Equation 3.20, we obtain the maximum vertical displacement.

$$H = V_0 sin\theta \left(\frac{V_0 sin\theta}{g}\right) - \frac{1}{2}\left(\frac{V_0 sin\theta}{g}\right)^2 \qquad \text{(ii)}$$

$$H = \frac{V_0^2 sin^2\theta}{2g} \qquad \text{(iii)}$$

(b) The total time is given by $T = 2t_1 \Rightarrow T = \frac{2V_0 sin\theta}{g}$

(c) If we substitute the total time in to the horizontal displacement of Eq. 3.18, we can get the horizontal displacement.

$$R = \frac{2V_0 cos\theta\ (sin\theta)}{g} = \frac{V_0^2 sin2\theta}{g} \qquad \text{(iv)}$$

N.B: There are two angles of projection for the same horizontal range θ and $90° - \theta$.

2. A jet of water flows from a hosepipe with speed 40m/s at an angle of 60^0 to the horizontal. Given that the particles of water travel as projectiles, find the equation of the path of the jet.

Solution:

To find the equation of the path substitute $V_0 = 40$ m/s $\theta = 60^0$ in to Eqs. 3.19 and 3.20.

$$t = \frac{\Delta x}{V_0 cos\theta} = \frac{\Delta x}{(40\ m/s)\cos 60°} = \frac{\Delta x}{20} \qquad \text{(i)}$$

Substitute Eq (i) into Eq 20, then we will get

$$\Delta y = \frac{\Delta x}{20} tan60 - \frac{g\Delta x^2}{2(40\ m/s)^2 cos^2 60}$$

$$\Delta y = 1.73\ \Delta x - 0.015\ \Delta x^2 \qquad \text{(ii)}$$

Eq (ii) can be rewritten as: $y\ (x) = 1.73\ x - 0.015\ x^2$

$$t = \frac{\Delta x}{V_0 cos\theta} \qquad 3.19$$

Now substitute the time constant in the vertical displacement of Eq.3.18 and we can get the trajectory or the path of the motion.

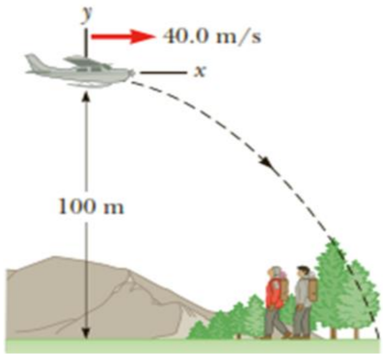
$$\Delta y = \Delta x\ tan\theta - \frac{g\Delta x^2}{2V_0^2 cos^2\theta} \qquad 3.20$$

Exercise

1. Make a list of the quantities you think determine the motion of a projectile such as a basketball which do you think are the most important?
2. When a bullet is fired horizontally, it takes the same amount of time to reach the ground as bullet dropped from rest from the same height. Yes or No
3. Suppose you are running at constant velocity and you wish to throw a ball such that you will catch it as it comes back down. In what direction should you throw the ball relative to you? (a) straight up (b) at an angle to the ground that depends on your running speed (c) in the forward direction.
4. As a projectile moves in its parabolic path, where are the velocity and acceleration vectors perpendicular to each other? (a) Everywhere along the projectile's path, (b) at the peak of its path, (c) nowhere along its path (d) not enough information is given.

Example

An Alaskan rescue plane drops a package of emergency rations to stranded hikers, as shown in the following figure. The plane is traveling horizontally at 40 m/s at a height of 100 m above the ground. Neglect air resistance. (a) Where does the package strike the ground relative to the point at which it was released? (b) What are the horizontal and vertical components of the velocity of the package just before it hits the ground? (c) What is the angle of the impact?



Solution:

(a) First we have to get the time "t" where the package strike the ground by considering the vertical displacement (see Eq.3.18). Here $V_{0y} = 0$ and $y_f = -100\ m$.

$$\Delta y = (V_o sin\theta)t - \frac{1}{2}gt^2$$
$$-100\ m = -(4.9\ m/s^2)t^2$$
$$t = 4.52\ sec$$

To calculate the horizontal displacement, let us substitute the time found here in to the following mathematical relation:

$$\Delta x = V_{0x}t = 40\ {}^{m}/_{s}\,(4.52\ s) = 181\ m$$

(b) Here we have to Find the *x* and y-components of the velocity at the time of impact

$$V_x = V_0\,Cos\theta = (40\ {}^{m}/_{s})cos0 = 40\ {}^{m}/_{s}$$
$$V_y = V_0 sin\theta - gt = -9.80\ {}^{m}/_{s^2}\ (4.52\ s) = -44.3\,{}^{m}/_{s}$$

(c) To get the angle of impact (θ) let us use the following mathematical relation i.e.,

V_y -44.3

Exercises

1. A projectile is thrown with an initial velocity *x i + y j*. The range of the projectile is twice the maximum height of the projectile calculate the ratio of *y/x*.
2. A tennis player makes a return at a speed of 15m/s and at a height of 3m to land in the court at a horizontal distance of 12m from her. What are the possible angles of projection of the ball?
3. A grasshopper jumps a horizontal distance of 1.5 m from rest, with an initial velocity at a 35.0° angle with respect to the horizontal. Find (a) the initial speed of the grasshopper and (b) the maximum height reached.
4. A ball is thrown upward from the top of a building at an angle of 25.0° above the horizontal and with an initial speed of 28.0 m/s. The point of release is 55.0 m above the ground. (a) How long does it take for the ball to hit the ground? (b) Find the ball's speed at impact. (c) Find the horizontal range of the ball. Neglect air resistance.

3.3 Kinematics of circular motion

We have studied the kinematics and dynamics of motion using Cartesian co-ordinates. For example, we have studied motion in one dimension, collisions in one dimension and some problems in two dimensions. When a mass moves in a circle we can use Cartesian co-ordinates to describe its behavior, but it is a lot easier if we use angular co-ordinates. Although circular motion sounds kind of trivial, it isn't. To a good approximation the motion of the earth around the sun is on a circle and the analysis of MRI signals from the body depends on understanding a kind of circular motion.

Learning outcome

After completing this section, students are expected to:

- Define uniform circular motion.
- Explain how radial and tangential accelerations are produced.
- Formulate kinematics of uniform circular motion.
- Understand and apply a problem-solving procedure to solve problems related to uniform circular motion.

3.3.1 Uniform Circular Motion

Figure 3.7a shows a car moving in a circular path with *constant speed v*. Such motion is called uniform circular motion, and occurs in many situations. It is often surprising to students to find that even though an object moves at a constant speed in a circular path, it still has acceleration because there is continues change in the *direction* of the velocity (see Fig.3.7b and 3.7c). The velocity vector is always tangent to the path of the object and perpendicular to the radius of the circular path. We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the centre of the circle. An acceleration of this nature is called a centripetal acceleration (*centripetal* means *centre-seeking*), and its magnitude is:

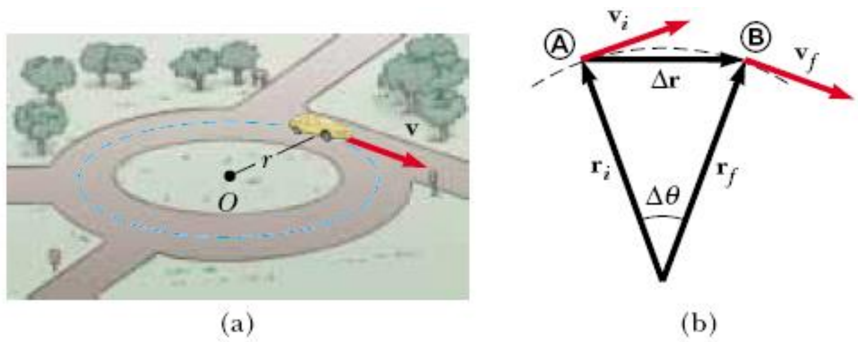


Figure 3.7: A car moving in a circular path with constant speed

Remember that kinematics is described by the position, $\bar{\boldsymbol{x}}$, the velocity $\bar{\boldsymbol{V}}$ and the acceleration $\bar{\boldsymbol{a}}$. What are the corresponding kinematical quantities for a mass moving in a circle? To do so, let us first define the time taken by the particle to complete one complete circle. The time interval it takes the particle to go around the circle of radius *r* once, completing one revolution (abbreviated rev.) is called period and represented by symbol *T.* For a particle moving with constant speed, speed is simply distance/time. The particle moves once around a circle of radius *r* and travels the circumference of 2πr is represented by:

$$V = \frac{2\pi r}{T} \qquad 3.21$$

We can describe the position of a particular in circular motion by its distance *r* from the center of the circle and its angle θ from the positive x-axis (See Fig.3.8).

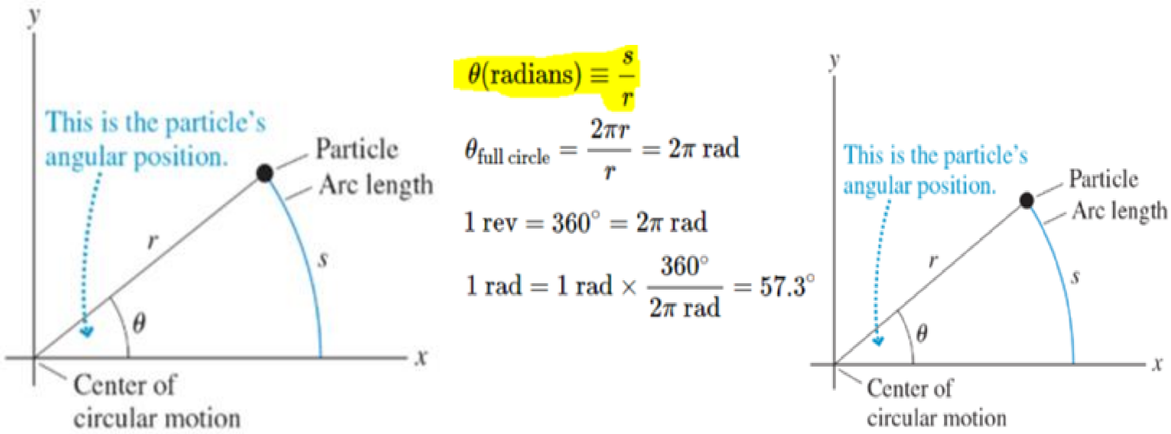


Figure 3.8: Position of particle and relation between its position *r* and angle θ

To define the average angular velocity $\boldsymbol{\omega_{av}}$, let us conceder the change in angular position from an initial angular position θ_i at time t_i to a final angular position θ_f at a later time t_f (See Fig.3.9).

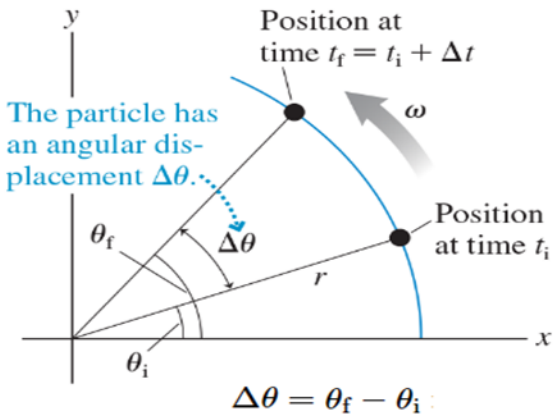


Figure 3.9: A particle moving with angular velocity ω.

Thus, the average angular velocity is mathematically defined as

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_1} \qquad 3.22$$

in a manner which is completely analogous to the definition of velocity in terms of position.

Note: The rate at which a particle's angular position is changing as it moves around a circle. Represented by symbol ω which is a lowercase Greek omega. The units rad/s, rev/s, and rev/min are all common units. In the case of uniform circular motion, the magnitude of centripetal acceleration is constant and this acceleration comes due to changing *direction* rather than *changing speed*. Acceleration vector points toward the center of the circle. (See Fig.3.10).

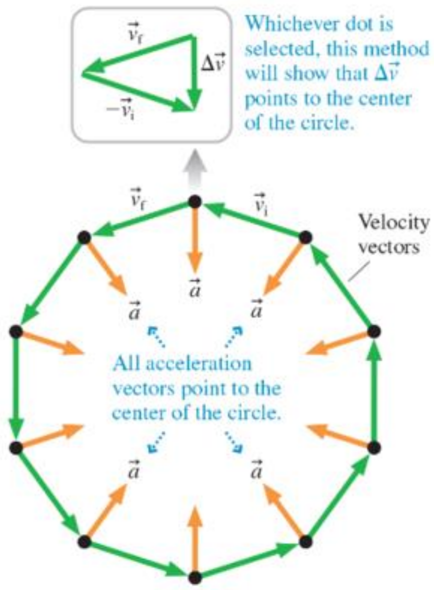


Figure 3.10: Representation of uniform circular motion

Note: Centripetal acceleration is constant during uniform circular motion, but the direction of acceleration is constantly changing. Thus, the constant-acceleration kinematics equations do not apply to circular motion.

The instantaneous value of $\boldsymbol{\omega}$ is the limit in which $\Delta t \to 0$. Similarly, the angular acceleration is defined as,

$$\boldsymbol{\alpha_{av}} = \frac{\Delta\boldsymbol{\omega}}{\Delta \boldsymbol{t}} = \frac{\boldsymbol{\omega_f} - \boldsymbol{\omega_0}}{\Delta \boldsymbol{t}} \qquad 3.23$$

Furthermore, all of the constant acceleration formulae and understanding of graphs of position versus time are completely analogous. For constant angular acceleration we then have,

$$\omega_f = \omega_0 + \alpha t \qquad 3.24$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad 3.25$$

and

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta \qquad 3.26$$

The bottom line is that you can take all of the equations you know for linear kinematics in one dimension and make the replacement $x \to \theta$, $\overline{\boldsymbol{V}} \to \boldsymbol{\omega}$ and $\overline{\boldsymbol{a}} \to \boldsymbol{\alpha}$ and you have the correct equations for angular kinematics on a circle. Note that in angular problems counterclockwise is positive. It is also possible to get the relations between linear and angular kinematics. The key thing to note is that the length of the arc around the circle is related to its angle through (See Fig.3.8):

$S = r\Delta\theta$ (arc-length)

Of course, if $\Delta\theta = 2\pi$, then we go all the way around the circle and so have covered a circumference, so $s = 2\pi r$. Now that we know the relationship between arc-length and angle, it is easy to find the relationship between angular velocity and linear velocity using,

$$\bar{v} = \frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega \qquad 3.27$$

A similar argument shows that the linear and angular accelerations are related by,

$$\overline{\boldsymbol{a}} = \boldsymbol{r\alpha} \qquad 3.28$$

Note:

Circular motion does not produce an outward force and it does not persist without a force.

Examples

1. The earth takes one year to go around the sun. What is its angular velocity ω? Given that the earth sun distance is $1.5\ x\ 10^{11}\ m$, what is the linear velocity of the earth with respect to the sun?

Solution:

The angular velocity is given by,

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{360 \times 24 \times 3600s} = 2x10^{-7}\, rad/s$$

The linear velocity of the earth with respect to the sun is then,

$$\bar{v} = r\omega = (1.5 \times 10^{11}m)\ (2x10^{-7}\, rad/s) = 3 \times 10^{4}\, m/s$$

2. A particle moves in a circle of radius 20 cm at a speed that increases uniformly. If the speed changes from 5 m/s to 6 m/s in 2 s, find the angular acceleration.

Solution:

It is given that speed of the particle increases uniformly which means the rate of change of speed is constant (with position or time). Since *magnitude of tangential acceleration* is nothing but the rate of change of linear speed, it is also constant here which again implies that the average and instantaneous values of the same are equal.

$$\bar{a}_t = \frac{\Delta V}{\Delta t} = \frac{6\, m/s - 5\, m/s}{2\ s} = 0.5\ \ m/s^2$$

3.3.2 Tangential and Radial Acceleration

Let us consider the motion of a particle along a smooth curved path where the velocity changes both in direction and in magnitude, as described in Figure 3.11. In this situation, the velocity vector is always tangent to the path; however, the acceleration vector $\bar{\boldsymbol{a}}$ is at some angle to the path. At each of three points A, B, and C in Figure 3.11.

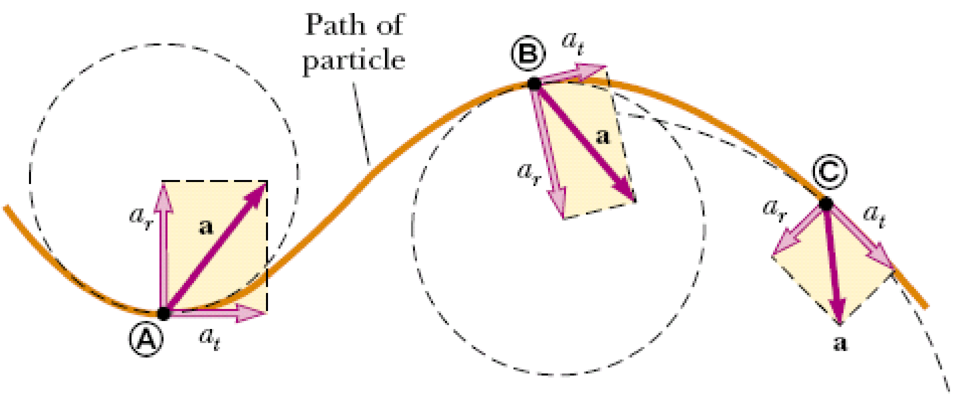


Figure 3.11: Motion of a particle along a smooth curve

The *total* acceleration vector $\bar{\boldsymbol{a}}$ can be written as the vector sum of the radial ($\bar{\boldsymbol{a}}_{\boldsymbol{r}}$) and tangential ($\bar{\boldsymbol{a}}_{\boldsymbol{t}}$) component vectors:

$$\bar{\boldsymbol{a}} = \bar{\boldsymbol{a}}_{\boldsymbol{t}} + \bar{\boldsymbol{a}}_{\boldsymbol{r}} \qquad 3.29$$

The tangential acceleration component causes the change in the speed of the particle. This component is parallel to the tangential velocity and is given by:

$$\bar{a}_t = \frac{\Delta v_t}{\Delta t} \qquad 3.30$$

The radial acceleration component arises from the change in direction of the velocity vector and is given by

$$\bar{\boldsymbol{a}}_{\boldsymbol{r}} = -\bar{\boldsymbol{a}}_{\boldsymbol{c}} = \frac{\boldsymbol{V}^2}{\boldsymbol{r}} \qquad 3.31$$

Where *r* is the radius of curvature of the path at the point in question. The negative sign indicates that the direction of the centripetal acceleration is toward the centre of the circle representing the radius of curvature, which is opposite the direction of the radial unit vector *r*, which always points away from the centre of the circle. Because $\bar{\boldsymbol{a}}_{\boldsymbol{r}}$ and $\bar{\boldsymbol{a}}_{\boldsymbol{t}}$ are perpendicular component vectors of $\bar{\boldsymbol{a}}$, it follows that the magnitude of $\bar{\boldsymbol{a}}$ is given by:

$$\bar{a} = \sqrt{a_r^2 + a_t^2} \qquad 3.32$$

At given speed, $\bar{\boldsymbol{a}}_{\boldsymbol{r}}$ is large when the radius of curvature is small (as at points A and B in Fig. 2.7) and small when *r* is large (such as at point C). The direction of $\bar{\boldsymbol{a}}_{\boldsymbol{t}}$ is either in the same direction as *V* (if *V* is increasing) or opposite *V* (if *V* is decreasing).

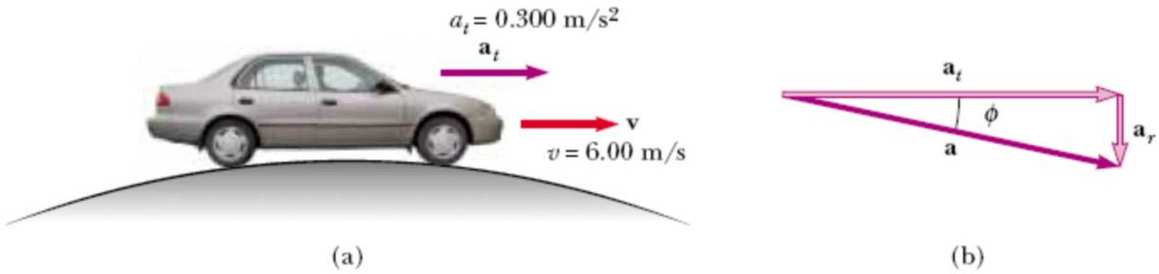
Example

1. A car exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What is the direction of the total acceleration vector for the car at this instant?

Solution:

Conceptualize the situation using the figure below. Because the car is moving along a curved path, we can *categorize* this as a problem involving a particle experiencing both tangential and radial acceleration. Now we recognize that this is a relatively simple plug-in problem. The radial acceleration is given by Eqs.3.31and 3.32. With *V* = 6.00 m/s and *r* = 500 m, we find that:

$$\bar{a}_r = -\bar{a}_c = \frac{V^2}{r} = \frac{(6m/s)^2}{500\ m} = -0.072\ m/s^2$$



The radial acceleration is directed straight downward while the tangential acceleration vector has magnitude 0.300 m/s^2 and is horizontal. Because $\bar{\boldsymbol{a}} = \bar{\boldsymbol{a}}_t + \bar{\boldsymbol{a}}_r$, the magnitude of $\bar{\boldsymbol{a}}$ is

$$\bar{a} = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.72)^2 - (0.30)^2} = 0.309\ m/s^2$$

If ϕ is the angle between a and the horizontal, then

Exercises:

1. A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors parallel? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.
2. A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors perpendicular everywhere along the path? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

3. A fan moves with angular velocity of 10π rad/s. Now it is switched off and an angular retardation of 2π rad/s² is produced. Find the number of rotations made by the fan before it stops.
4. For an object in uniform circular motion rank the changes listed below regarding the effect each would produce on the magnitude of the centripetal acceleration of the objects? Assume all other parameters stay constant except that noted in the description of the change.

Change A: The speed of the object doubles.

Change B: The radius of the motion triples.

Change C: The mass of the object triples.

Change D: The radius of the motion becomes half as big.

Change E: The speed of the object becomes half as big.

5. The angular velocity (ω) of a particle depends on its angular position (θ, measured with respect to a certain line of reference) by the rule ω = 2√θ. Find the angular acceleration α as a function of θ.

3.4 Summary

General Relations

The average velocity of an object is the object's displacement during a time interval dived by the time interval.

$$\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

Speed is the magnitude of the velocity, $|\vec{V}|$

The average acceleration of an object given by $\bar{\boldsymbol{a}}_{\boldsymbol{av}} = \Delta\bar{\boldsymbol{V}}/\Delta t$

Instantaneous acceleration of an object given by $\vec{a} = \lim_{\Delta t\to 0} \Delta\vec{V}/\Delta t$

Projectile motion

The position of a projectile launched with initial speed $\mathbf{V}_0$ at an angle θ with the horizontal is given by the vector equation:

$$\vec{r}\,(t) = \vec{r}_o + \vec{V}_0\, t + \frac{1}{2}\vec{g}t^2$$

In terms of horizontal (x) and vertical (y) components

$$x(t) = x_0 + (V_0 \cos\theta)t \quad \text{and} \quad y = y_0 + (V_0 \sin\theta)t - \frac{1}{2}gt^2$$

The maximum height the projectile reaches above the point of release is $H = \frac{V_0^2 sin^2\theta}{2g}$

The time of flight of the projectile is $T = \frac{2V_0 sin\theta}{g}$

The range of the projectile is $\frac{V_0^2 sin2\theta}{g}$

Uniform Circular

- The centripetal acceleration is given as $a_c = \frac{V^2}{r}$
- The period of the motion is given as $T = \frac{2\pi r}{V}$
- The *total* acceleration vector $\overline{\boldsymbol{a}}$ can be written as $\overline{\boldsymbol{a}} = \overline{\boldsymbol{a}}_{\boldsymbol{t}} + \overline{\boldsymbol{a}}_{\boldsymbol{r}}$
- The radial components of acceleration express as $\overline{\boldsymbol{a}}_{\boldsymbol{r}} = -\overline{\boldsymbol{a}}_{\boldsymbol{c}} = \frac{\boldsymbol{V}^2}{\boldsymbol{r}}$
- The tangential components of acceleration express as $a_t = \frac{\Delta v_t}{\Delta t}$
- The magnitude of $\overline{\boldsymbol{a}}$ is given by $a = \sqrt{a_r^2 + a_t^2}$

3.5 Conceptual Questions

1. Neglecting air resistance, is the magnitude of the velocity vector at impact greater than, less than, or equal to the magnitude of the initial velocity vector? Why?
2. True/False: Because the x-component of the displacement doesn't depend explicitly on g, the horizontal distance travelled doesn't depend on the acceleration of gravity.
3. A person standing at the edge of a cliff throws one ball straight up and another ball straight down, each at the same initial speed. Neglecting air resistance, which ball hits the ground below the cliff with the greater speed:
   a. ball initially thrown upward;
   b. ball initially thrown downward;
   c. neither; they both hit at the same speed.
4. Suppose you are carrying a ball and running at constant velocity on level ground. You wish to throw the ball and catch it as it comes back down. Neglecting air resistance, should you (a) throw the ball at an angle of about 45° above the horizontal and maintain the same speed, (b) throw the ball straight up in the air and slow down to catch it, or (c) throw the ball straight up in the air and maintain the same speed?
5. A ball is projected horizontally from the top of a building. One second later, another ball is projected horizontally from the same point with the same velocity. (a) At what point in the motion will the balls be closest to each other? (b) Will the first ball always be traveling faster than the second? (c) What will be the time difference between them when the balls hit the ground? (d) Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?

6. A projectile is launched at some angle to the horizontal with some initial speed $\boldsymbol{V_0}$, air resistance is negligible. (a) Is the projectile a freely falling body? (b) What is its acceleration in the vertical direction? (c) What is its acceleration in the horizontal direction?
7. Two projectiles are thrown with the same initial speed, one at an angle θ with respect to the level ground and the other at angle 900 - θ. Both projectiles strike the ground at the same distance from the projection point. Are both projectiles in the air for the same length of time?
8. A ball is thrown upward in the air by a passenger on a train that is moving with constant velocity. (a) Describe the path of the ball as seen by the passenger. Describe the path as seen by a stationary observer outside the train. (b) How would these observations change if the train were accelerating along the track?
9. As a projectile moves in its parabolic path, where are the velocity and acceleration vectors perpendicular to each other? (a) Everywhere along the projectile's path, (b) at the peak of its path, (c) nowhere along its path, or (d) not enough information is given.
10. In uniform circular motion what will be the nature of acceleration, if the velocity remain constant?

3.6 Problems

1. How is it possible for a particle moving at constant speed to be accelerating? can a particle with constant velocity be accelerating at the same time?
2. The co- ordinates of an object moving in the xy plane vary with time according to the equations x= -5m sin t and y= 4m- 5m cost, where t is in second.
   a) determine the components of the velocity and acceleration at t = 0
   b) write expressions for the position, velocity and acceleration vector at any time t >0.
3. A projectile is fired up an incline (incline angle ϕ) with an initial speed V0 at an angle θ0 with respect to the horizontal (θ0 > ϕ), as shown in Figure 3.12.
   a) show that the projectile travels a distance d up the incline, where

$$d = \frac{2{V_0}^2 \cos\theta_0 \sin(\theta_0 - \varphi)}{g \cos^2\varphi}$$

   b) For what value of θ0 is d a maximum, and what is the maximum value?

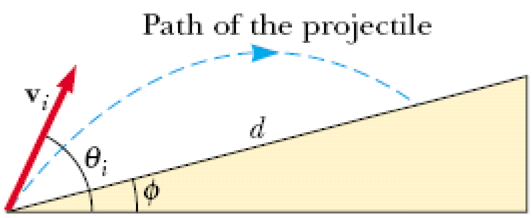


Figure 3.12

4. A golf ball leaves the ground at an angle θ and hits a tree while moving horizontally at height h above the ground. If the tree is a horizontal distance of b from the point of projection, show that
   a) tan θ = 2h/b

b) what is the initial velocity of the ball in terms of b and h?

5. A rock is thrown upward from the level ground in such a way that the maximum height of its flight is equal to its horizontal range R. (a) At what angle u is the rock thrown? (b) In terms of the original range R, what is the range R the rock can attain if it is launched at the same speed but at the optimal angle for maximum range? (c) Would your answer to part (a) be different if the rock is thrown with the same speed on a different planet? Explain.
6. A dive bomber has a velocity of 280 m/s at an angle θ below the horizontal. When the altitude of the aircraft is 2.15 km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ.
7. From the window of a building, a ball is tossed from a height $\boldsymbol{y_0}$ above the ground with an initial velocity of 8.00 m/s and angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) If the base of the building is taken to be the origin of the coordinates, with upward the positive y - direction, what are the initial coordinates of the ball? (b) With the positive x-direction chosen to be out the window, find the x- and y components of the initial velocity. (c) Find the equations for the x - and y - components of the position as functions of time. (d) How far horizontally from the base of the building does the ball strike the ground? (e) Find the height from which the ball was thrown. (f) How long does it take the ball to reach a point 10.0 m below the level of launching?
8. A boy can throw a ball a maximum horizontal distance of R on a level field. How far can he throw the same ball vertically upward? Assume that his muscles give the ball the same speed in each case.
9. An airplane in a holding pattern flies at constant altitude along a circular path of radius 3.50 km. If the airplane rounds half the circle in $1.5 \times 10^2 s$, determine the magnitude of its (a) displacement and (b) average velocity during that time. (c) What is the airplane's average speed during the same time interval?
10. Suppose a rocket-propelled motorcycle is fired from rest horizontally across a canyon 1.00 km wide. (a) What minimum constant acceleration in the x-direction must be provided by the engines so the cycle crosses safely if the opposite side is 0.750 km lower than the starting point? (b) At what speed does the motorcycle land if it maintains this constant horizontal component of acceleration? Neglect air drag, but remember that gravity is still acting in the negative y-direction.
11. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater speed for the stone at the end of the sling? (b) What is the centripetal acceleration of the stone at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?
12. An automobile whose speed is increasing at a rate of 0.60 m/s2 travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.0 m/s, find (a) the tangential acceleration component, (b) the centripetal acceleration component, and (c) the magnitude and direction of the total acceleration.
13. A small steel ball is projected horizontally off the top landing of a long rectangular staircase (see Figure 3.13). The initial speed of the ball is 3m/s. Each step is 0.18m high and 0.3m wide. Which step does the ball strike first?

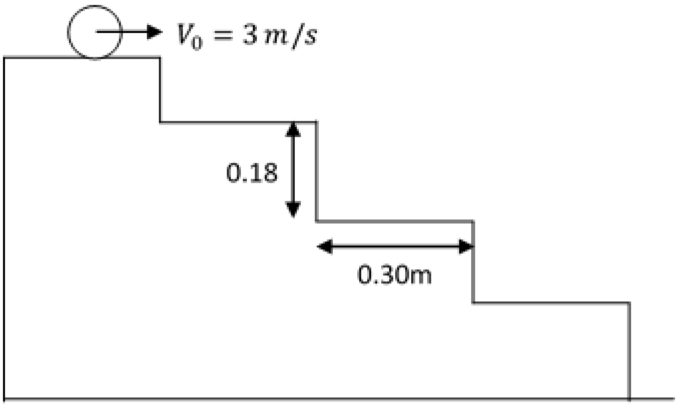


Figure 3.13

14. A ball is thrown upward from the top of a building at an angle of 35.0° above the horizontal and with an initial speed of 25.0 m/s, as in Figure 3.12. The point of release is 45.0 m above the ground. (a) How long does it take for the ball to hit the ground? (b) Find the ball's speed at impact. (c) Find the horizontal range of the ball. Neglect air resistance.
15. Suppose the ball is thrown from the same height of Problem 13, at an angle of 30.0° below the horizontal. If it strikes the ground 57.0 m away, find (a) the time of flight, (b) the initial speed, and (c) the speed and the angle of the velocity vector with respect to the horizontal at impact
16. A pendulum with a cord of length r = 1.00 m swings in a vertical plane (Fig. 3.14). When the pendulum is in the two horizontal positions $\theta = 90.0°$ and $\theta = 270°$, its speed is 5.00 m/s. (a) Find the magnitude of the radial and tangential acceleration for these positions. (b) Draw vector diagrams to determine the direction of the total acceleration for these two positions. (c) Calculate the magnitude and direction of the total acceleration.

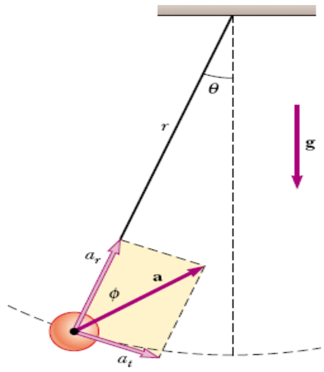


Figure 3.14

4 Dynamics

Learning Outcomes:

After completing this Chapter, students are expected to:

- define the term force,
- understand force as a measure of interactions between objects,
- know the types of interactions; distinguish between contact and field forces,
- understand the four basic forces that underlie the processes in nature,
- define mass and inertia,
- state and understand Newton's three laws of motion,
- define normal and gravitational forces,
- apply Newton's laws of motion to solve problems involving a variety of forces,
- integrate concepts from kinematics to solve problems using Newton's laws of motion,
- define friction and discuss the factors that determine it value,
- describe the types of friction,
- calculate the magnitude of static and kinetic friction,
- define uniform circular motion, and establish the expression for centripetal force,
- explain the centrifugal force and understand it as a fictitious force, and
- Calculate coefficient of friction on a car tire as well as ideal speed and angle of a car on a turn.

Introduction:

In the preceding chapters, we have discussed the concept of motion but not what caused the motion. Indeed, what causes objects to move are forces. The part of mechanics which studies both motion and forces that causes the motion is called dynamics. Chapter 4 introduces Newton's three laws of motion and the dynamics of uniform circular motion. The three laws are simple and sensible. The first law states that a force must be applied to an object in order to change its velocity. Changing an object's velocity means accelerating it, which implies a relationship between force and acceleration. This relationship, the second law, states that the net force on an object equals the object's mass times its acceleration. Finally, the third law says that whenever we push on something, it pushes back with equal force in the opposite direction. In particular, you will learn about the concept of force as a measure of interaction, the fundamental types of interactions, the three Newton's laws of motion, motion with friction as well as the dynamics of uniform circular motion.

4.1 The Concept of Force as a Measure of Interaction

After completing this section, students are expected to:

- define the term force,
- understand force as a measure of interaction between objects, and
- explain the vector nature of force.

4.1.1 The Concept of Force:

A force is commonly imagined as a push or a pull on some object. When you push or pull an object away or towards you, you exert a force on it. Similarly, you exert a force on a ball when you throw or kick it. In these examples, the word *force* refers to an interaction with an object by means of muscular activity and some change in the object's velocity. Whenever there is an interaction between two objects, there is a force acting on each other. When the interaction ceases, the two objects no longer experience a force. Forces exist only as a result of an interaction. In brief, the effects of forces are:

- to accelerate or stop an object,
- to change the direction of a moving object, and
- to change the shape of an object.

4.1.2 The Vector Nature of Force:

It is possible to use the deformation of a spring to measure force. Suppose a vertical force is applied to a spring scale that has a fixed upper end as shown in Fig. 4.1a. The spring elongates when the force is applied, and a pointer on the scale reads the extension of the spring. If the spring is calibrated such that a force $\vec{F}_1$ (with a magnitude of 1.0 unit) as the force that produces a pointer reading of 1.0 cm. Next, if we apply a different downward force $\vec{F}_2$ whose magnitude is twice that of the force $\vec{F}_1$ as seen in Fig. 4.1b, the pointer moves to 2.0 cm. Figure 4.1c shows that the combined effect of the two collinear forces (i.e., forces acting in the same direction) is the sum of the effects of the individual forces, i.e., $\vec{F} = \vec{F}_1 + \vec{F}_2 = 3.0 units$.

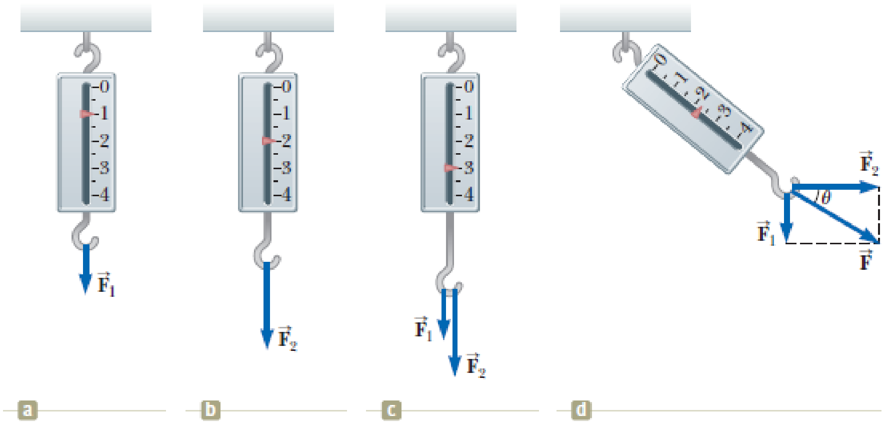


Figure 4.1: Illustration of the vector nature of force.

Now suppose the two forces are applied simultaneously with $\vec{F}_1$ downward and $\vec{F}_2$ horizontal as illustrated in Fig. 4.1d. In this case, the pointer reads 2.24 cm. The single force $\vec{F}$ that would produce this same reading is the sum of the two vectors $\vec{F}_1$ and $\vec{F}_2$ as described in Fig. 4.1d. That is,

$F = \sqrt{F_1^2 + F_2^2} = 2.24units$, and its direction is $\theta = tan^{-1}(-F_1/F_2) = tan^{-1}(-0.50) = -22.6^0$.

4.2 Types of Interactions

Learning Outcomes:

After completing this section, students are expected to:

- define the terms contact and field forces,
- distinguish between contact and field forces, and
- understand the four basic forces that underlie the processes in nature.

4.2.1 Contact and Field Forces:

Force can be classified as either contact forces or field forces.

1. **Contact forces** are forces that involve physical contact between two objects. A contact force must touch or be in contact with an object to cause a change. Examples of contact forces are:

i. When a coiled spring is pulled, as in Fig. 4.2a, the spring stretches.

ii. When a stationary cart is pulled, as in Fig. 4.2b, the cart moves.

iii. When a ball is kicked, as in Fig. 4.2c, it is both deformed and set in motion.

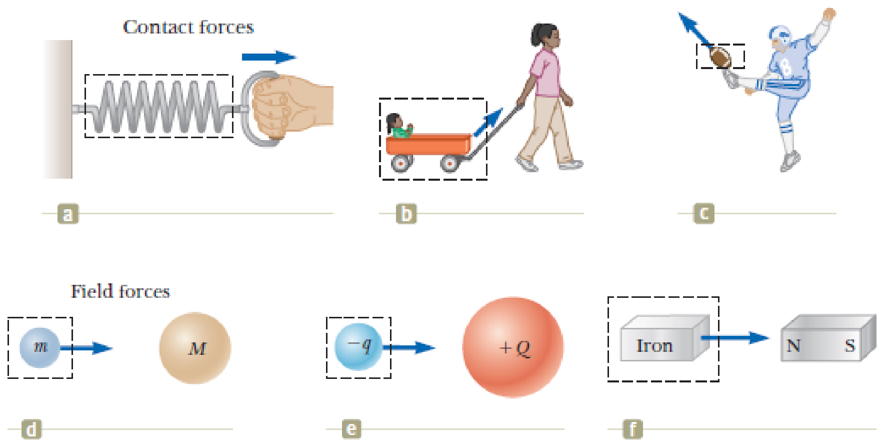


Figure 4.2: Examples of contact and field forces. In each case, a force is exerted on the object within the dashed boxed area by some 'agent' external to it.

2. **Field forces** are forces that do not involve physical contact between two objects. A field force is sometimes referred to as "action at a distance" force. The concept of field force may be explained as follows: "*An object of mass M, such as the Sun, creates an invisible influence that stretches throughout space. A second object of mass m, such as Earth, interacts with the*

field of the Sun, not directly with the Sun itself." So, the force of gravitational attraction between two objects, illustrated in Figure 4.2d, is an example of a field force. The force of gravity keeps objects bound to Earth and also gives rise to what we call the *weight* of those objects.

Other examples of field forces are (i) the electric force that one electric charge exerts on another (Fig. 4.2e), such as the charges of an electron and proton that form a hydrogen atom; and (ii) The force a bar magnet exerts on a piece of iron (Fig. 4.2f).

The distinction between contact forces and field forces is not as sharp as discussed above. When examined at the atomic level, all the forces that are classified as contact forces turn out to be caused by electric (field) forces of the type illustrated in Fig. 4.2e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces.

4.2.2 Fundamental Forces - Forces of Nature:

There are four fundamental forces in nature and all are field forces. These are gravitational force, electromagnetic force, strong nuclear force and weak nuclear force.

1. The gravitational force

It is the force between any two objects in the universe. It is an attractive force by virtue of their masses. The gravitational force is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. Gravitational force is the weakest force among the fundamental forces of nature but has the greatest large-scale impact on the universe. Unlike the other forces, gravity works universally on all matter and energy, and is universally attractive.

2. The electromagnetic force

It is the force between charged particles such as the force between two electrons, or the force between two current carrying wires. It is attractive for unlike charges and repulsive for like charges. The electromagnetic force obeys inverse square law. It is very strong compared to the gravitational force. It is the combination of electrostatic and magnetic forces.

3. The strong nuclear force

It is the strongest of all the basic forces of nature. It, however, has the shortest range, of the order of $10^{-15}\ m$. This force holds the protons and neutrons together in the nucleus of an atom.

4. The weak nuclear force

The weak force is a force that arises in most radioactive decay processes and plays an important role, for instance, in the nuclear reactions that generate the Sun's energy output. This force is not as weak as the gravitational force.

4.3 Newton's Laws of Motion

Isaac Newton proposed the laws of motion to offer a systematic method of calculating an object's motion due to forces exerted on it. This section discusses the three Newton's laws of motion.

Learning Outcomes:

After completing this section, students are expected to:

- state Newton's laws of motion,
- state the first condition of equilibrium,
- know about normal force and gravitational force,
- distinguish between inertial and gravitational masses,
- know about some of the applications of the laws, and
- apply Newton's laws to solve related problems.

4.3.1 Newton's First Law of Motion

Consider a book lying on a table. Obviously, the book remains at rest if left alone. Now imagine pushing the book with a horizontal force great enough to overcome the force of friction between the book and the table, setting the book in motion. Because the magnitude of the applied force exceeds the magnitude of the friction force, the book accelerates. When the applied force is withdrawn, friction soon slows the book to a stop.

Next, imagine pushing the book across a smooth, waxed floor. The book again comes to rest once the force is no longer applied, but not as quickly as before. Finally, if the book is moving on a horizontal frictionless surface, it continues to move in a straight line with constant velocity until it hits a wall or some other obstruction.

Before about 1600, scientists felt that the natural state of matter was the state of rest. Galileo, however, devised thought experiments, such as an object moving on a frictionless surface, and concluded that *"**it's not the nature of an object to stop once set in motion, but rather to continue in its original state of motion**"*. This observation was later formalized as ***Newton's first law of motion***, which states that:

An object moves with a velocity that is constant in magnitude and direction unless a nonzero net force acts on it.

The net force on an object is defined as the vector sum of all external forces exerted on the object. External forces come from the object's environment. If an object's velocity isn't changing in either magnitude or direction, then its acceleration and the net force acting on it must both be zero.

Remarks: Internal forces originate within the object itself and can't change the object's velocity. As a result, internal forces are not included in Newton's first law. It is not really possible to "pull yourself up by your own bootstraps."

4.3.1.1 Mass and Inertia:

Imagine hitting a golf ball off a tee with a driver. If you are a good golfer, the ball will sail over two hundred yards down the fairway. Now imagine teeing up a bowling ball and striking it with the same club. Your club would probably break, you might sprain your wrist, and the bowling ball, at best, would fall off the tee, take half a roll, and come to rest.

From this thought experiment, we conclude that although both balls resist changes in their state of motion, the bowling ball offers much more effective resistance. The tendency of an object to continue in its original state of motion is called ***inertia***.

Although inertia is the tendency of an object to continue its motion in the absence of a force, **mass** is a measure of the object's resistance to changes in its motion due to a force. This kind of mass is often called ***inertial mass*** because it's associated with inertia. The greater the mass of a body, the less it accelerates under the action of a given applied force. The SI unit of mass is the kilogram. Mass is a scalar quantity that obeys the rules of ordinary arithmetic.

Example: Newton's First law and Inertia:

a) A book lying on the table will remain at rest, until it is moved by some external agencies.
b) A person standing in a bus falls backward when the bus suddenly starts moving. This is because, the person who is initially at rest continues to be at rest even after the bus has started moving.
c) A passenger sitting in a moving car falls forward, when the car stops suddenly.
d) When a bus moving along a straight line takes a turn to the right, the passengers are thrown towards left. This is due to inertia which makes the passengers travel along the same straight line, even though the bus has turned towards the right.

4.3.1.2 Applications of the first law:

The following two examples illustrate Newton's first law in practice.

(a) Seat Belts

Inertia can be used to explain the operation of one type of seat belt mechanism. The purpose of the seat belt is to hold the passenger firmly in place relative to the car, to prevent serious injury in the event of an accident. Figure 4.3 illustrates how one type of shoulder harness operates. Under normal conditions, the ratchet turns freely to allow the harness to wind on or unwind from the pulley as the passenger moves. In an accident, the car undergoes a large acceleration and rapidly comes to rest. Because of its inertia, the large block under the seat continues to slide forward along the tracks. The pin connection between the block and the rod causes the rod to pivot about its center and engage the ratchet wheel. At this point, the ratchet wheel locks in place and the harness no longer unwinds.

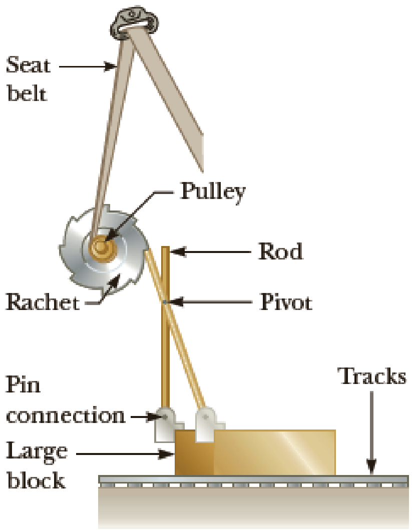


Figure 4.3: A mechanical arrangement for an automobile seat belt.

(b) Rockets

A spaceship is launched into space. The force of the exploding gases pushes the rocket through the air into space. Once it is in space, the engines are switched off and it will keep on moving at a constant velocity. If the astronauts want to change the direction of the spaceship they need to fire an engine. This will then apply a force on the rocket and it will change its direction.

4.3.2 Newton's Second Law of Motion

Newton's first law explains what happens to an object that has no net force acting on it: The object either remains at rest or continues moving in a straight line with constant speed. Newton's second law answers the question of what happens to an object that does have a net force acting on it.

Imagine pushing a block of ice across a frictionless horizontal surface. When you exert some horizontal force on the block, it moves with an acceleration of, say, $2m/s^2$. If you apply a force twice as large, the acceleration doubles to $4m/s^2$. Pushing three times as hard triples the acceleration, and so on. From such observations, we conclude that the acceleration of an object is directly proportional to the net force acting on it.

Mass also affects acceleration. Suppose you stack identical blocks of ice on top of each other while pushing the stack with constant force. If the force applied to one block produces an acceleration of $2m/s^2$, then the acceleration drops to half that value, $1m/s^2$, when two blocks are pushed, to one-third the initial value when three blocks are pushed, and so on. We conclude that *"**the acceleration of an object is inversely proportional to its mass**"*. These observations are summarized in Newton's second law as:

The acceleration $\vec{a}$ of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

The constant of proportionality is equal to one, so in mathematical terms the preceding statement can be written

$$\vec{a} = \frac{\sum \vec{F}}{m},$$

where $\vec{a}$ is the acceleration of the object, m is its mass, and $\sum \vec{F}$ is the vector sum of all forces acting on it. Multiplying through by m, we have

$$\sum \vec{F} = m\vec{a} \qquad (4.1)$$

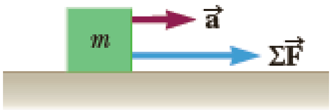


Figure 4.4: Illustration of Newton's second law of motion.

Figure 4.4 illustrates the relationship between the mass, acceleration, and the net force. The second law is a vector equation, equivalent to the following three component equations:

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y, \qquad \text{and} \qquad \sum F_z = ma_z. \qquad (4.2)$$

Remarks: When there is no net force on an object, its acceleration is zero, which means the velocity is constant. In other words, the acceleration on the object is zero, and hence the system is said to be in translational equilibrium. That is,

$$\sum \vec{F} = 0. \qquad (4.3)$$

According to Eq. (4.3), the body is either at rest or moving with constant velocity. Eq. (4.3.3) is commonly referred as *the first condition of equilibrium*.

Unit of Force

The SI unit of force is the Newton. When 1 Newton of force acts on an object that has a mass of $1kg$, it produces an acceleration of $1m/s^2$ in the object. From this definition and Newton's second law, we see that the Newton can be expressed in terms of the fundamental units of mass, length, and time as

$$1N = 1kg \cdot m/s^2. \quad (4.4)$$

An ***external force*** is a force that acts on an object from 'agents' outside the system of interest. For example, in the Figure shown below the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again, looking at the Figure, the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law, while the internal forces actually cancel each other. You must define the boundaries of the system before you can determine which forces are external. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws.

Examples:

1. An airboat with mass $m = 3.50 \times 10^2 kg$, including the passenger, has an engine that produces a net horizontal force of $7.70 \times 10^2 N$, after accounting for forces of resistance (see Fig. 4.5).

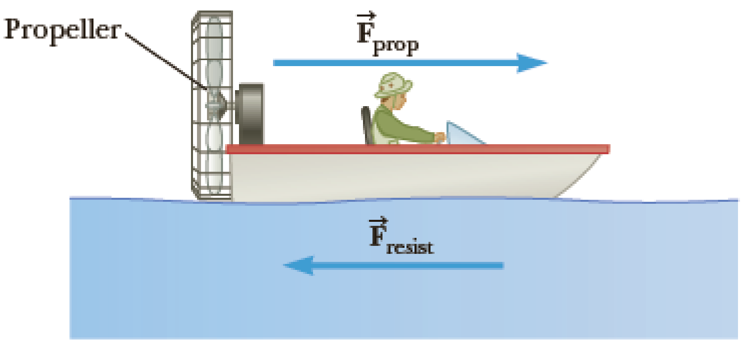


Figure 4.5: A passenger moving on an airboat.

a. Find the acceleration of the airboat.
b. Starting from rest, how long does it take the airboat to reach a speed of $12.0m/s$?
c. After reaching that speed, the pilot turns off the engine and drifts to a stop over a distance of $50.0m$. Find the resistance force, assuming it is constant.

Solution:

a. The acceleration of the airboat is obtained using Newton's second law. i.e.,

$$F_{net} = ma$$

$$\Rightarrow \quad a = \frac{F_{net}}{m} = \frac{7.70\times10^2\ N}{3.50\times10^2\ kg} = 2.2m/s^2$$

b. To find the time, we use the following equation:

$$v = v_0 + at$$

$$\Rightarrow \quad t = \frac{v - v_0}{a} = \frac{12m/s - 0}{2.2m/s^2} = 5.45s$$

c. After the engine is switched off, the only forces acting on the airboat is the resistance forces. Hence, first we calculate the acceleration due to resistance forces. That is,

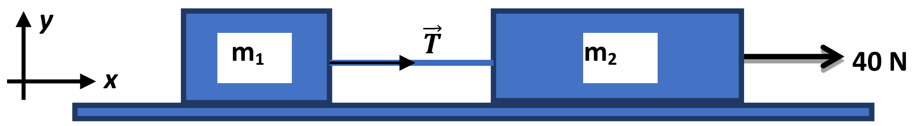
$$v^2 = v_0^2 + 2a\Delta x$$

$$\Rightarrow \quad a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0-(12m/s)^2}{2\times50.0m} = -1.44m/s^2$$

Note that the acceleration is negative - indicating that the airboat is slowing down, i.e., decelerating. The corresponding resistance force is obtained using Newton's second law as:

$$F_{resist} = ma = (3.50\times10^2 kg)(-1.44m/s^2) = -504N.$$

2. Two masses of $m_1 = 2kg$ and $m_2 = 8kg$ are connected by a mass less string. They are supported on a frictionless horizontal surface. A horizontal force of $F = 40N$ is applied to the mass, m_2, as shown. Calculate the tension in the string between the two masses.



Solution:

In the y-direction, the net force in each mass is zero, i.e., the weights and the normal forces are equal but opposite, and hence

$\sum F_y = 0.$

On the other hand, the horizontal forces acting on the masses are:

Mass, m_1: $\sum F_x = T = m_1 a_x$ (a)

Mass, m_2: $\sum F_x = F - T = m_2 a_x$ (b)

Substituting Eq. (a) into (b), we get

$$\sum F_x = F - m_1 a_x = m_2 a_x,$$

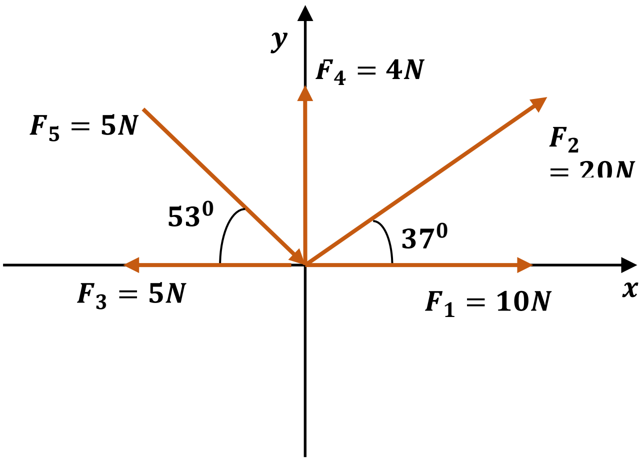
so that the acceleration of the masses is

$$a_x = \frac{F_x}{m_1+m_2} = \frac{40N}{2kg+8kg} = 4m/s^2.$$

Then, substituting the value of the calculated acceleration into Eq. (a), we find the tension, T , in the string to be:

$$T = m_1 a_x = (2kg)(4m/s^2) = 8N.$$

3. Find the resultant of the concurrent forces shown below.



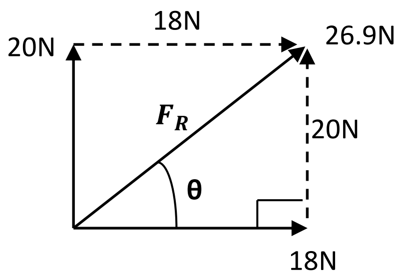
Solution:

Forces	Components	
	x-components	y-components
F_1	$F_{1x} = 10\cos 0^0\ N = 10N$	$F_{1y} = 10\sin 0^0\ N = 0$
F_2	$F_{2x} = 20\cos 37^0\ N = 16N$	$F_{2y} = 20\sin 37^0\ N = 12N$
F_3	$F_{3x} = 5\cos 180^0\ N = -5N$	$F_{3y} = 5\sin 180^0\ N = 0$
F_4	$F_{4x} = 4\cos 90^0\ N = 0$	$F_{4y} = 4\sin 90^0\ N = 4N$
F_5	$F_{5x} = 5\cos(180-53)^0\ N = -3N$	$F_{5y} = 5\sin(180-53)^0\ N = 4N$
Sum	$R_x = \sum_{i=1}^{5} F_{ix} = 18N$	$R_y = \sum_{i=1}^{5} F_{iy} = 20N$

One can see from the table above that, total *x*-component of the force is $R_x = 18N$ and the *y*-component, $R_y = 20N$. Since these two are perpendicular, the magnitude of the resultant force is given by

$$F_R = \sqrt{R_x^2 + R_y^2} = \sqrt{(18N)^2 + (20N)^2} = 26.9N,$$

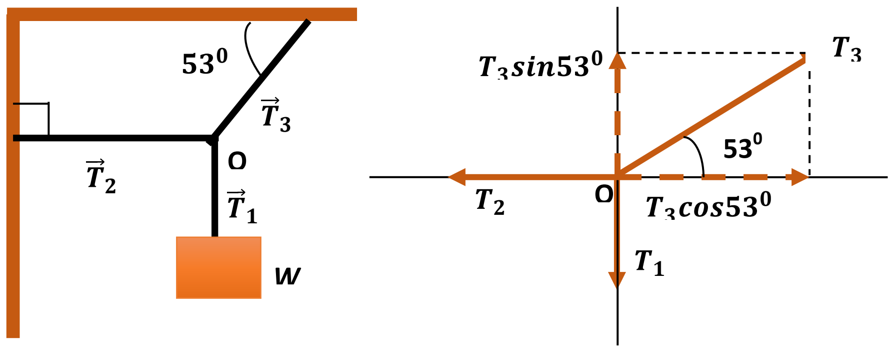
and its direction is



$$\tan\theta = \frac{R_y}{R_x} = \frac{20\,N}{18\,N} = 1.11$$

$$\Rightarrow \quad \theta = \tan^{-1}(1.11) = 48.0^0$$

4. A block of weight W hangs from a cord, which is knotted at point O to two other cords, one fastened to the ceiling, the other to the wall, as shown. We wish to find the tension in these three cords, assuming the weights of the cords to be negligible.



Solution:

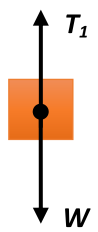
First, we resolve the forces into horizontal and vertical components, as shown in the free-body diagram. From the diagram, the forces sum up to zero at point O, so that the block is at equilibrium.

From Newton's first law and first condition of equilibrium, we have

$$\sum F_x = T_3cos53^0 - T_2 = 0$$

and

$$\sum F_y = T_3sin53^0 - T_1 = 0$$



From the figure, $T_1 + (-W) = 0$

Hence, $T_1 = W$

From the above equations, $T_3 = \frac{T_1}{sin53^0} = \frac{W}{sin53^0} = 1.155W$

This equation can be used in the first equation:

$$T_2 = T_3 cos53^0 = (1.155W)cos53^0 = 0.577W$$

Thus, all the three tensions can be expressed as multiples of the weight of the block, which is assumed to be known. Therefore, if W = 200N,

$T_1 = 200N$,
$T_2 = 0.577 \times 200N = 115.4N$,
$T_3 = 1.155 \times 200N = 231N$.

4.3.2.1 The Gravitational Force and Weight:

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the gravitational force, $\vec{F}_g$. This force is directed toward the center of the Earth, and its magnitude is called the weight of the object. We know that a freely falling object experiences an acceleration $\vec{g}$ acting toward the center of the Earth. Applying Newton's second law $\sum \vec{F} = m\vec{a}$ to a freely falling object of mass m, with $\vec{a} = \vec{g}$ and $\sum \vec{F} = \vec{F}_g$, gives

$$\vec{F}_g = m\vec{g} .$$

Therefore, the weight of an object, being defined as the magnitude of $\vec{F}_g$, is equal to mg:

$$F_g = mg . \qquad (4.5)$$

Because it depends on g , weight varies with geographic location. Because g decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level. For example, suppose a student has a mass of 70.0 kg. The student's weight in a location where $g = 9.80m/s^2$ is 686 N. At the top of a mountain, however, where $g = 9.77m/s^2$, the student's weight is only 684 N.

Equation (4.5) quantifies the gravitational force on the object, but notice that this equation does not require the object to be moving. Even for a stationary object or for an object on which several forces act, Eq. (4.5) can be used to calculate the magnitude of the gravitational force. The mass *m* in Eq. (4.5) determines the strength of the gravitational attraction between the object and the Earth and is called the ***gravitational mass***. When mass m is used as a measure of the resistance to changes in motion in response to an external force, it is known as the ***inertial mass***. Even though gravitational mass is different in behavior from inertial mass, the values of the gravitational mass and inertial mass are the same.

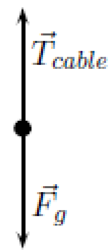
4.3.2.2 Applications of the second law

A) Lifts

Let us consider a 500 kg lift, with no passengers, hanging on a cable. The purpose of the cable is to pull the lift upwards so that it can reach the next floor or lower the lift so that it can move downwards to the floor below. We will look at five possible cases during the motion of the lift and apply our knowledge of Newton's second law of motion to the situation. The 5 cases are: (Let the upwards direction be the positive direction.)

Case 1: The 500 kg lift is stationary at the second floor of a tall building.

The lift is not accelerating. There must be a tension $\vec{T}$ from the cable acting on the lift and there must be a force due to gravity, $\vec{F_g}$. There are no other forces present and we can draw the free body diagram as shown to the right. We apply Newton's second law to the vertical direction (mass of the lift = m):



$$\vec{F}_R = m\vec{a}$$
$$T - F_g = m(0)$$
$$T = F_g$$

The forces are equal in magnitude and opposite in direction.

Case 2: The lift moves upwards at an acceleration of 1 m/s2.

If the lift is accelerating, it means that there is a resultant force in the direction of the motion. This means that the force acting upwards is now greater than the force due to gravity $\vec{F_g}$ (down). To find the magnitude of $\vec{T}$ applied by the cable we apply Newton's second law to the vertical direction:

$$\vec{F}_R = m\vec{a}$$
$$T - F_g = m(1m/s^2)$$
$$T = F_g + m(1m/s^2)$$

The answer makes sense as we need a bigger force upwards to cancel the effect of gravity as well as have a positive resultant force.

Case 3: The lift moves at a constant velocity.

When the lift moves at a constant velocity, the acceleration is zero,

$$\vec{F}_R = m\vec{a}$$
$$T - F_g = m(0)$$
$$T = F_g$$

The forces are equal in magnitude and opposite in direction. It is common mistake to think that because the lift is moving there is a net force acting on it. It is only if it is accelerating that there is a net force acting.

Case 4: The lift slows down at a rate of 2 m/s2

The lift was moving upwards so this means that it is decelerating or accelerating in the direction opposite to the direction of motion. This means that the acceleration is in the negative direction.

$$\vec{F}_R = m\vec{a}$$
$$T - F_g = m(-2m/s^2)$$
$$T = F_g - m(2m/s^2)$$

As the lift is now slowing down there is a resultant force downwards. This means that the force acting downwards is greater than the force acting upwards.

Case 5: The cable snaps

When the cable snaps, the force that used to be acting upwards is no longer present. The only force that is present would be the force of gravity. The lift will fall freely and its acceleration.

4.3.2.3 Apparent weight

Your weight is the magnitude of the gravitational force acting on your body. When you stand in a lift that is stationery and then starts to accelerate upwards you feel you are pressed into the floor while the lift accelerates. You feel like you are heavier and your weight is more. When you are in a stationery lift that starts to accelerate downwards you feel lighter on your feet. You feel like your weight is less.

Weight is measured through normal forces. When the lift accelerates upwards you feel a greater normal force acting on you as the force required to accelerate you upwards in addition to balancing out the gravitational force.

When the lift accelerates downwards you feel a smaller normal force acting on you. This is because a net force downwards is required to accelerate you downwards. This phenomenon is called apparent weight because your weight didn't actually change.

B) Rockets

As with lifts, rockets also are examples of objects in vertical motion. The force of gravity pulls the rocket down while the thrust of the engine pushes the rocket upwards. The force that the engine exerts must overcome the force of gravity so that the rocket can accelerate upwards. The worked example below looks at the application of Newton's second law in launching a rocket.

Example:

A rocket (of mass 5000 kg) is launched vertically upwards into the sky at an acceleration of 20 m/s^2. If the magnitude of the force due to gravity on the rocket is 49,000 N, calculate the magnitude and direction of the thrust of the rocket's engines ($\vec{F}_T$).

Solution:

Applying Newton's second law, we get

$$\vec{F}_R = m\vec{a}$$
$$F_T - F_g = ma$$
$$F_T = 49{,}000N + (5000kg)(20m/s^2)$$
$$F_T = 149{,}000N$$

The force due to the thrust is 149,000 N upwards.

4.3.3 Newton's Third Law of Motion

Newton's third law of motion deals with the interaction between pairs of objects. For example, when we sit on a chair, our body exerts a downward force on the chair and the chair exerts an upward force on our body. There are two forces resulting from this interaction: a force on the chair and a force on our body. These two forces are called action and reaction forces. Newton's third law explains the relation between these action forces. It states that:

The action force is equal in magnitude to the reaction force and opposite in direction.

That is, whenever one body exerts a certain force on a second body, the second body exerts an equal and opposite force on the first.

Consider two bodies 1 and 2 exerting forces on each other. Let the force exerted on the body 1 by the body 2 be $\vec{F}_{12}$ and the force exerted on the body 2 by the body 1 be $\vec{F}_{21}$. Then according to third law, $\vec{F}_{12} = -\vec{F}_{21}$.

One of these forces, say $\vec{F}_{12}$ may be called as *the action*, whereas the other force $\vec{F}_{21}$ may be called as *the reaction* or vice versa. It is to be noted that always the action and reaction do not act on the same body; they always act on different bodies. The action and reaction never cancel each other and the forces always exist in pair.

The effect of third law of motion can be observed in day-to-day activities. Examples are:

i. When a man jumps from a boat to the shore, the boat moves away from him. The force he exerts on the boat (action) is responsible for its motion and his motion to the shore is due to the force of reaction exerted by the boat on him.

ii. A swimmer pushes the water in the backward direction with a certain force (action) and the water pushes the swimmer in the forward direction with an equal and opposite force (reaction).

iii. We will not be able to walk if there were no reaction force. In order to walk, we push our foot against the ground. The Earth in turn exerts an equal and opposite force. This force is inclined to the surface of the Earth. The vertical component of this force balances our weight and the horizontal component enables us to walk forward.

iv. A bird flies by with the help of its wings. The wings of a bird push air downwards (action). In turn, the air reacts by pushing the bird upwards (reaction).

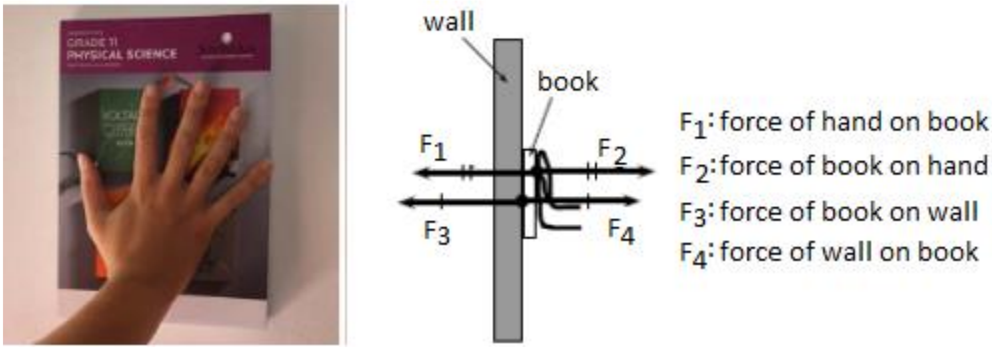


Figure 4.7: A book held pushed to the wall.

v. If you hold a book up against a wall you are exerting a force on the book (to keep it there) and the book is exerting a force back at you (to keep you from falling through the book), as shown in Fig. 4.7. These two forces (the force of the hand on the book (F1) and the force of the book on the hand (F2)) are an action-reaction pair of forces.

There is another action-reaction pair of forces present in this situation. The book is pushing against the wall (action force) and the wall is pushing back at the book (reaction). The force of the book on the wall (F_3) and the force of the wall on the book (F_4) are shown in the diagram.

Examples:

1. A man of mass M = 75.0 kg and woman of mass m = 55.0 kg stand facing each other on an ice rink, both wearing ice skates. The woman pushes the man with a horizontal force of F = 85.0 N in the positive x-direction. Assume the ice is frictionless.
    a. What is the man's acceleration?
    b. What is the reaction force acting on the woman?
    c. Calculate the woman's acceleration.

Solution:

a. Applying Newton's second law for the man, we obtain

$$F = Ma_m$$
$$a_m = \frac{F}{M} = \frac{85.0N}{75.0kg} = 1.13m/s^2.$$

b. Now apply Newton's third law of motion, to find the reaction force R acting on the woman which has the same magnitude and opposite direction. That is,

$$R = -F = -85.0N.$$

c. Finally, applying Newton's second law for the woman we obtain

$$a_w = \frac{F}{m} = \frac{-85.0N}{55.0kg} = -1.55m/s^2.$$

Note that the forces are equal and opposite each other, but the accelerations are not equal because the two masses differ from each other.

2. A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in the Figure shown below. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. (a) Calculate the acceleration produced in the system when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N. (b) Calculate the force the professor exerts on the cart.

Solution:

From the Figure (and the free-body diagrams) note the following:

- Since all bodies accelerate as a unit, we define the system to be the professor, cart, and equipment. This is labelled System 1 in the Figure.
- The professor pushes backward with a force $\vec{F}_{foot}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\vec{F}_{floor}$ of 150 N on System 1.
- Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction.
- As noted, the friction $\vec{f}$ opposes the motion and is thus in the opposite direction of $\vec{F}_{floor}$.
- Also, note that we do not include the forces $\vec{F}_{prof}$ or $\vec{F}_{cart}$ in the free-body diagram of System 1, because these are internal forces, and we do not include $\vec{F}_{foot}$ because it acts on the floor, not on the system.

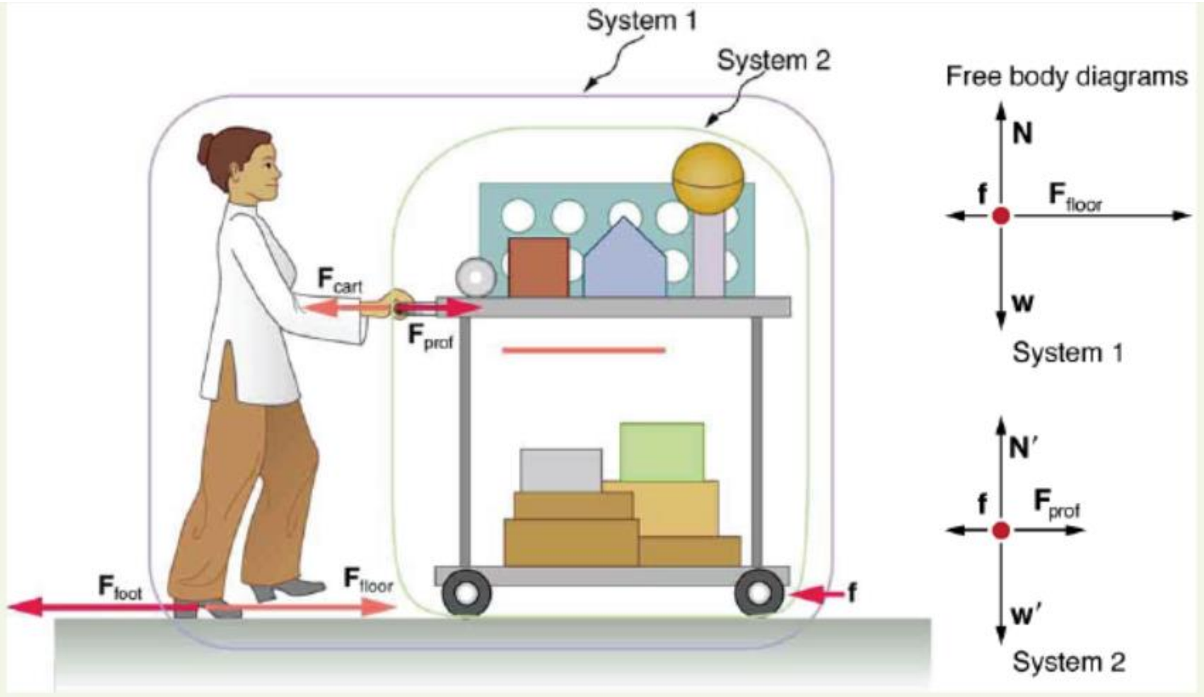


Figure: A professor pushes a cart of demonstration equipment.

(a) From the free-body diagram of System 1, we find that the net external force on System 1 is given by

$$F_{net} = F_{floor} - f = 150N - 24..0N = 126N.$$

The mass of the system (System 1) is

$$m = (65.0 + 12.0 + 7.0)kg = 84kg.$$

Therefore, the acceleration of the system is

$$a = \frac{F_{net}}{m} = \frac{126N}{84kg} = 1.5m/s^2.$$

(b) Now, if we now define the system of interest to be the cart plus equipment (System 2 in the Figure), then the net external force on System 2 is the force the professor exerts on the cart minus friction, f.

The force she exerts on the cart, $\vec{F}_{prof}$, is an external force acting on System 2. $\vec{F}_{prof}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2. Thus, Newton's second law can be used to find $\vec{F}_{prof}$. Starting with

$$F_{net} = ma,$$

and noting that the magnitude of the net external force on System 2 is

$$F_{net} = F_{prof} - f,$$

we solve for F_{prof} . That is,

$$F_{prof} = F_{net} + f.$$

The value of f is given, so we must calculate net F_{net} . That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{net} = ma,$$

where the mass of System 2 is $19.0\,kg$ $(m = 12.0kg + 19.0kg)$ and its acceleration was found to be $a = 1.5m/s^2$ in Part (a). Thus,

$$F_{net} = ma = (19.0kg)(1.5m/s^2) = 28.5N.$$

Thus, the force the Professor exerts on the cart becomes

$$F_{prof} = F_{net} + f,$$

or $$F_{prof} = 28.5N + 24.0N = 52.5N.$$

Remark: It is interesting that this force is significantly less than the 150 N force the professor exerted backward on the floor. Not all of that 150 N force is transmitted to the cart; some of it accelerates the professor.

4.3.3.1 Application of the third law:

Working of a rocket

The propulsion of a rocket is one of the most interesting examples of Newton's third law of motion. The rocket is a system whose mass varies with time. In a rocket, the gases at high temperature and pressure, produced by the combustion of the fuel, are ejected from a nozzle. The reaction of the escaping gases provides the necessary thrust for the launching and flight of the rocket.

4.4 Motion with Friction

Learning Outcomes:

After completing this section, students are expected to:

- define friction and know about the factors that affect friction,
- define static friction and kinetic friction,
- understand what limiting static friction mean,
- mention some applications of friction, and
- solve problems involving motion with friction.

4.4.1 The Normal Force:

If you put a brick on water it will sink because nothing balances the gravitational force. On the other hand, if you put a heavy box on the ground the gravitational force is balanced. We call the force that a surface exerts to balance the forces on an object in contact with that surface as the normal force. It may be defined as:

The normal force is the force exerted by a surface on an object in contact with it.

In generally, we may identify four most common four cases of the normal force acting on an object. These are discussed below:

Case 1: The Normal Force on a Level Surface

Figure 4.8a shows a block at rest on a flat surface. The two forces acting on the block are the normal force, acting upward, and the gravity force, directed downward. The free-body diagram for the block (See Fig. 8b) depicts just the forces acting on the block. Free-body diagrams include only the forces acting directly on the object in question. Forces or reaction forces acting on different bodies are not shown. For example, the reaction force to the gravity force acting on the block is the gravity force exerted by the block on the Earth, which doesn't appear in the block's free-body diagram.

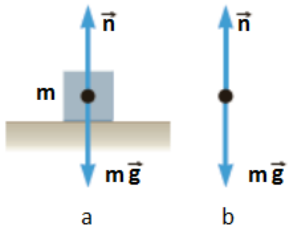


Figure 4.8: (a) A block on a level surface and (b) Free-body diagram.

The y-component (vertical direction) of the second law of motion, with $a_y = 0$, yields:

$$\sum F_y = ma_y,$$
$$n - mg = 0$$

or $$n = mg.$$

It means that the normal force in this case is equal to the weight of the object.

Case 2: The Normal Force on a Level Surface with an Applied Force

Figure 4.9(I-a) shows a block at rest on a flat surface. The three forces acting on the block are the normal force, directed upward; the gravity force, directed downward; and an applied force, directed at a positive angle θ.

The y-component of the second law of motion yields:

$$\sum F_y = ma_y \quad \Rightarrow \quad n - mg + F\sin\theta = 0.$$

That equation can easily be solved for the normal force n:

$$n = mg - F\sin\theta.$$

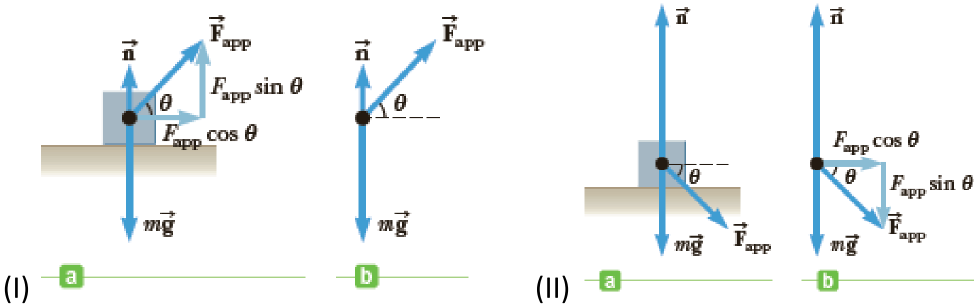


Figure 4.9: An applied force at (I-a) a positive angle and (II-a) a negative angle; and the corresponding free-body diagrams (I-b) and (II-b), respectively.

Notice that if the angle is positive, as in Fig. 4.9(I-a), then the sine of the angle is positive, and the y-component of the applied force supports some of the weight, reducing the normal force. That is, the sum of the normal force and the y-component of the applied force equals the magnitude of the weight. If the angle is negative, however, then the sine of the angle is also negative, making a positive contribution to the normal force, as illustrated in Fig. 4.9(II-b). The normal force must be larger, and in magnitude, equal to the sum of the weight and the y-component of the applied force.

Case 3: The Normal Force on a Level Surface Under Acceleration

Figure 4.10a shows a block on a flat surface that is under acceleration, such as in an elevator. The two forces acting on the block are the normal force, directed upward, and the gravity force, directed downward. An acceleration upward, however, will increase the magnitude of the normal force, because the normal force must not only compensate for gravity, but also provide the acceleration.

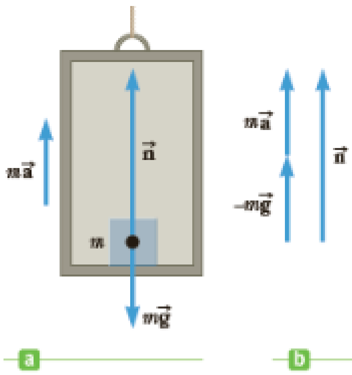


Figure 4.10: (a) A block in an elevator accelerating upward and (b) the free-body diagram.

The y-component of the second law of motion yields:

$$\sum F_y = ma_y \quad \Rightarrow \quad n - mg = ma_y ,$$

or

$$n = mg + ma_y$$

As can be seen in Fig. 4.10b, the magnitude of the normal force vector must equal the sum of the magnitudes of the gravitational force and the inertial quantity, $m\vec{a}$.

Case 4: The Normal Force on a Slope

A common variation on a second law problem is an object resting on a surface tilted at some constant angle. Although optional, in that circumstance a simplification of the problem can be achieved by selecting coordinates that are similarly tilted, with the x'-axis running parallel to the slope and y'-axis perpendicular to the slope, as shown in Fig. 4.11a.

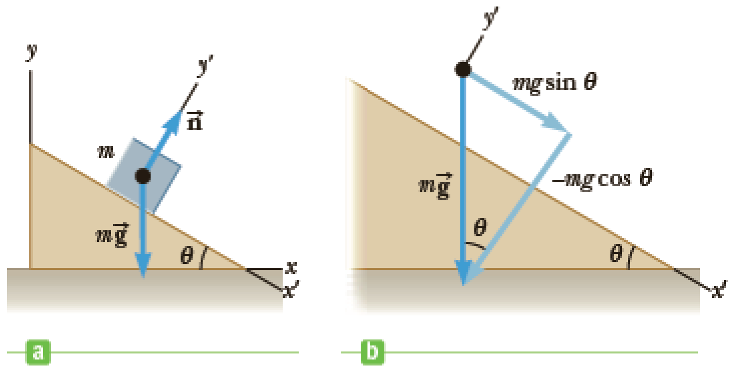


Figure 4.11: (a) A block on a slope, showing the forces acting on it. (b) In tilted coordinates, the gravity force has two components, one perpendicular to the slope and the other parallel to it.

Using Fig. 4.11b, the force due to gravity can then be broken into two components,

$F_{x',grav} = mg\sin\theta$ and $F_{y',grav} = -mg\cos\theta$.

The second law for the y'-direction can be solved for the normal force:

$$\sum F_{y'} = ma_{y'}$$
$$n - mg\cos\theta = 0,$$

or, $$n = mg\cos\theta.$$

The normal force on a slope is equal in magnitude to the component of the gravity force perpendicular to the slope. It is also useful to know that the force on the block directed down the slope due to gravity is given by

$$F_{x',grav} = mg\sin\theta.$$

4.4.2 Friction

Why does a box sliding on a surface eventually come to a stop? The answer is friction. Friction arises where two surfaces are in contact and moving relative to each other.

Friction arises because the surfaces interact with each other. When the surface of one object slides over the surface of another, each body exerts a frictional force on the other. For example, if a book slides across a table, the table exerts a frictional force onto the book and the book exerts a frictional force onto the table. Frictional forces act parallel to surfaces. It may be defined as:

Frictional force is the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with.

The magnitude of the frictional force depends on

- the type of the surface (its roughness) and
- the magnitude of the normal force.

Different surfaces will give rise to different frictional forces, even if the normal force is the same. Friction, f, is proportional to the magnitude of the normal force, $\vec{n}$. That is,

$$f \propto n.$$

For every surface we can determine a constant factor, known as the ***coefficient of friction***. We know that static friction and kinetic friction have different magnitudes so we have different coefficients for the two types of friction:

- μ_s is the coefficient of static friction
- μ_k is the coefficient of kinetic friction

A force is not always large enough to make an object move - for example a small applied force might not be able to move a heavy crate. The frictional force opposing the motion of the crate is equal to the applied force but acting in the opposite direction. This frictional force is called *static friction*. For static friction the force can vary up to some maximum value after which friction has been overcome and the object starts to move. So, we define a maximum value for the static friction by the following equation:

$$f_{s,max} = \mu_s n$$

When the applied force is greater than the maximum static frictional force, the object moves but still experiences friction. This is called kinetic friction. For kinetic friction the value remains the same regardless of the magnitude of the applied force. The magnitude of the kinetic friction is:

$f_k = \mu_k n$

Remember: Static friction is present when the object is not moving and kinetic friction while the object is moving.

Friction is very useful. The following examples illustrates some of the useful applications of friction:

- If there was no friction and you tried to lift a ladder up against a wall, it would simply slide to the ground.
- Rock climbers use friction to maintain their grip on cliffs.
- The brakes of cars would be useless if it wasn't for friction.
- When you rub your hands together fast and pressing hard you will feel that they get warm. This is heat created by the friction.

Examples:

1. A block rests on a horizontal surface. The normal force is 20 N. The coefficient of static friction between the block and the surface is 0.40 and the coefficient of kinetic friction is 0.20.
    a) What is the magnitude of the frictional force exerted on the block while the block is at rest?
    b) What will the magnitude of the frictional force be if a horizontal force of magnitude 5 N is exerted on the block?
    c) What is the minimum force required to start the block moving?
    d) What is the minimum force required to keep the block in motion once it has been started?
    e) If the horizontal force is 10 N, determine the frictional force.

Solution:

a) The magnitude of the frictional force exerted on the block while the block is at rest is zero. It is because that friction arises only when an object is moving or tries to move (by applying a force).
b) The maximum static frictional force is

$f_{s,max} = \mu_s n = 0.40 \times 20\ N = 8\ N$

But the applied force is $F_a = 5N$, which is less than $f_{s,max} = 8\ N$. Therefore, the block remains at rest with a frictional force equal to the applied force. That is,

$f = F_a = 5N$

c) The minimum force required to start the block moving is equal to maximum static frictional force. That is,

$$F_{min} = f_{s,max} = 8\ N$$

d) The minimum force required to keep the block in motion once it has been started is

$$f_k = \mu_k n = (0.20)(20N) = 4N$$

Note that the kinetic friction is less than the maximum static frictional force.

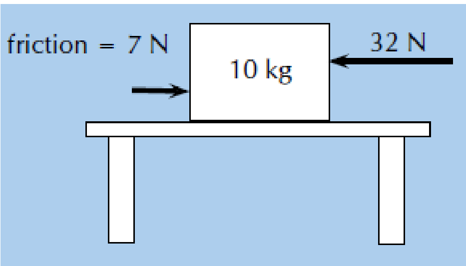
e) Once the block is in motion the frictional force will be equal to f_k, calculated in part (d) (Provided that the applied force is not less f_k). Therefore,

$$f_k = \mu_k n = 4N.$$

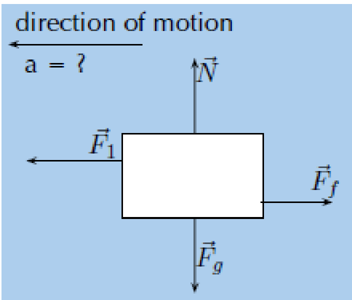
2. A 10 kg box is placed on a table, as shown in the Fig. (a). A horizontal force of magnitude 32 N is applied to the box. A frictional force of magnitude 7 N is present between the surface and the box.

   a) Draw a force diagram indicating all of the forces acting on the box.

   b) Calculate the acceleration of the box.



(a)



(b)

Solution:

a) The force diagram is that shown in the Fig. (b). Note that the forces in the vertical direction must be equal but opposite, that is, $\vec{N} = -\vec{F}_g$ (since the box is moving along the horizontal direction).

b) To calculate the acceleration of the box we will be using Newton's second law. Therefore:

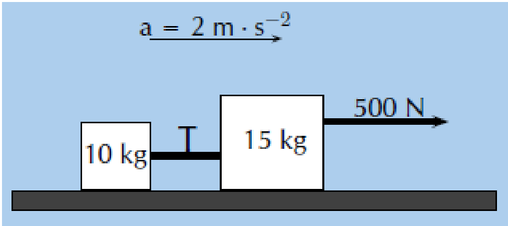
$$\vec{F}_R = m\vec{a}$$
$$\vec{F}_1 + \vec{F}_f = m\vec{a}$$
$$32N - 7N = (10kg)a$$
$$\vec{a} = 2.5\ m/s, \text{ to the left}$$

3. Two crates, m1 = 15 kg and m2 = 10 kg respectively, are connected with a thick rope according to the diagram. A force, to the right, of Fa = 500 N is applied. The boxes

move with an acceleration of a = 2 m/s2 to the right. One third of the total frictional force is acting on the 10 kg block and two thirds on the 15 kg block. Calculate:

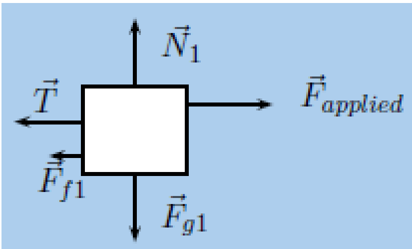
a) the magnitude and direction of the total frictional force present.

b) the magnitude of the tension in the rope at T.



Solution:

(a) First draw the free-body diagram for the two masses.

(a) 

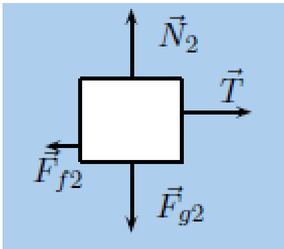
(b) 

Figure: The free-body diagram for the (a) 15 kg mass and (b) 10 kg mass.

The net force on m_1 is $\vec{F}_{R1} = \vec{F}_a + \vec{F}_f + \vec{T}$ ($F_{R1} = F_a - F_{f1} - T$). Applying Newton's second law for the 15 kg mass, we get

$$\begin{aligned}
\vec{F}_{R1} &= m_1\vec{a} \\
F_a - F_{f1} - T &= m_1 a \\
500N - \tfrac{2}{3}F_{fT} - T &= (15kg)(2m/s^2) \\
T &= 470N - \tfrac{2}{3}F_{fT}
\end{aligned} \qquad \text{(a)}$$

Next, applying Newton's second law for the 10 kg mass, we get

$$\begin{aligned}
\vec{F}_{R2} &= m_2\vec{a} \\
T - F_{f1} &= m_2 a \\
T - \tfrac{1}{3}F_{fT} &= (10kg)(2m/s^2) \\
T &= 20N + \tfrac{1}{3}F_{fT}
\end{aligned} \qquad \text{(b)}$$

Then, solving Eqs. (a) and (b) simultaneously, we get the total frictional force, F_{fT} to be:

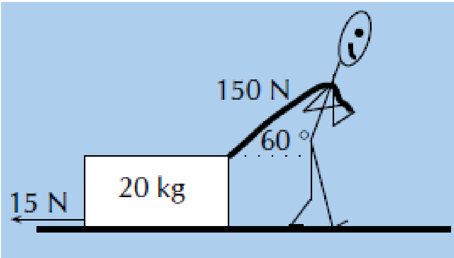
$$F_{fT} = 450N.$$

(b) Finally, substituting the magnitude of F_{fT} into the Eq. (b), we can determine the magnitude of the tension:

$$T = 170N.$$

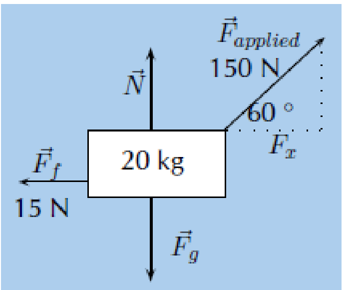
The total force due to friction is 450 N to the left and the magnitude of the force of tension is 170 N.

4. A man is pulling a m = 20 kg box with a rope that makes an angle of θ = 600 with the horizontal. If he applies a force of magnitude Fa = 150 N and a frictional force of magnitude Ff = 15 N is present, calculate the acceleration of the box.

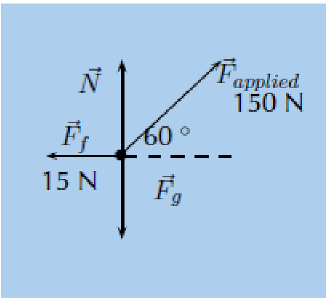


Solution:

The motion is horizontal and therefore we will only consider the forces in a horizontal direction. The free-body diagram is shown below:



or



Let us choose the positive *x*-direction (to the right) to be positive. The applied force is acting at an angle of 60^0 to the horizontal. The horizontal component of the applied force is

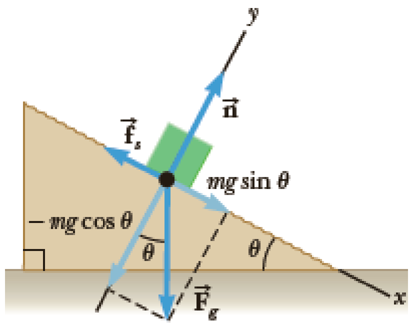
$$F_x = F\cos\theta$$
$$F_x = (150N)\cos 60^0$$
$$F_x = 75N$$

To find the acceleration we apply Newton's second law:

$$\boldsymbol{F_R = ma}$$
$$\boldsymbol{F_x - F_f = (20kg)a}$$
$$\boldsymbol{75N - 15N = (20kg)a}$$
$$\boldsymbol{a = 3m/s^2}$$

Therefore, the acceleration is $a = 3m/s^2$, to the right.

5. As shown in the figure below, a block having a mass of 4.00 kg rests on a slope that makes an angle of 30.00 with the horizontal. If the coefficient of static friction between the block and the surface it rests upon is 0.650, calculate
   (a) the normal force,
   (b) the maximum static friction force, and
   (c) the actual static friction force required to prevent the block from moving.
   (d) Will the block begin to move or remain at rest?



Solution:

(a) The normal force is

$n = mg\cos\theta = (4.00kg)(9.80m/s^2)(\sin 30.0^0)$

$n = 33.9N.$

(b) The maximum force that static friction can exert on the block on this surface is given by

$f_{s,max} = \mu_s n = 0.650 \times 33.9\ N = 22.1\ N$

(c) The Newton's second law applied for the x-direction, down the slope reads

$ma_x = \sum F_x$

Setting $a_x = 0$, and using the expressions for the two forces acting parallel to the x-axis, the gravity force and static friction force, we get

$0 = f_{x,grav} - f_s$

$f_s = f_{x,grav} = mg\cos\theta$

so that $f_s = mg\cos\theta = (4.00kg)(9.80m/s^2)(\sin 30.0^0) = 19.6N.$

(d) In this case, the actual, required static friction force, $f_s = 19.6N$, is less than the maximum possible static friction force, $f_{s,max}$, so the block remains at rest on the slope.

4.5 Dynamics of Circular Motion

Learning Outcomes:

After completing this section, students are expected to:

- define uniform circular motion and centripetal force,
- list and explain some applications of centripetal force, and
- calculate coefficient of friction on a car tire.
- calculate ideal speed and angle of a car on a turn.
- solve other problems related to uniform circular motion.

4.5.1 Centripetal Force:

An object can have a centripetal acceleration only if some external force acts on it. For a ball whirling in a circle at the end of a string, that force is the tension in the string. In the case of a car moving on a flat circular track, the force is friction between the car and track. A satellite in circular orbit around Earth has a centripetal acceleration due to the gravitational force between the satellite and Earth. In brief, the force of tension in the string of a yo-yo whirling in a vertical circle is an example of a centripetal force, as is the force of gravity on a satellite circling the Earth.

Consider a puck of mass *m* that is tied to a string of length *r* and is being whirled at constant speed in a horizontal circular path, as illustrated in Fig. 4.12. Its weight is supported by a frictionless table. Why does the puck move in a circle? Because of its inertia, the tendency of the puck is to move in a straight line; however, the string prevents motion along a straight line by exerting a radial force on the puck - a tension force - that makes it follow the circular path. The tension T is directed along the string toward the center of the circle, as shown in the Figure.

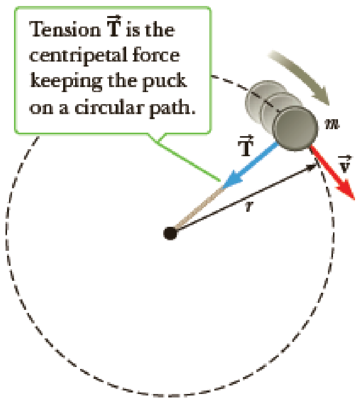


Figure 4.12: A puck rotating in a horizontal plane at a constant speed, v .

In general, converting Newton's second law to polar coordinates yields an equation relating the net centripetal force, F_c, acting on a given object to the centripetal acceleration. The magnitude of the net centripetal force equals the mass times the magnitude of the centripetal acceleration:

$$\boldsymbol{F_c = ma_c = m\frac{v^2}{r}}. \tag{4.6}$$

A net force causing a centripetal acceleration acts toward the center of the circular path and results to a change in the direction of the velocity vector. If that force should vanish, the object would immediately leave its circular path and move along a straight line tangent to the circle at the point where the force vanished.

Any force like gravitational force, frictional force, electric force, magnetic force etc. may act as a centripetal force. Some of the examples of centripetal force are:

i. In the case of a stone tied to the end of a string whirled in a circular path, the centripetal force is provided by the tension in the string.
ii. When a car takes a turn on the road, the frictional force between the tyres and the road provides the centripetal force.
iii. In the case of planets revolving round the Sun or the Moon revolving round the Earth, the centripetal force is provided by the gravitational force of attraction between them.
iv. For an electron revolving round the nucleus in a circular path, the electrostatic force of attraction between the electron and the nucleus provides the necessary centripetal force.

4.5.2 Fictitious Forces

Anyone who has ridden a merry-go-round as a child has experienced what feels like a "center-fleeing" force. Holding onto the railing and moving toward the center feels like a walk up a steep hill.

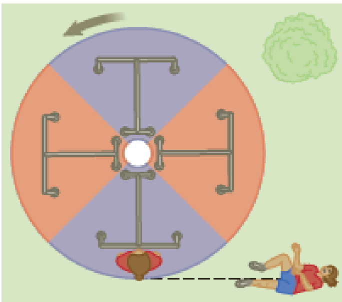


Figure 4.13: A fun-loving student loses her grip and falls along a line tangent to the rim of the merry-go-round.

Actually, this so-called centrifugal force is fictitious. In reality, the rider (See Fig 4.13) is exerting a centripetal force on her body with her hand and arm muscles. In addition, a smaller centripetal force is exerted by the static friction between her feet and the platform. If the rider's grip slipped, she wouldn’t be flung radially away; rather, she would go off on a straight line, tangent to the point in space where she let go of the railing. The rider lands at a point that is farther away from the center, but not by "fleeing the center" along a radial line. Instead, she travels perpendicular to a radial line, traversing an angular displacement while increasing her radial displacement.

Other examples of centrifugal force are:

(i) When a car is turning around a corner, the person sitting inside the car experiences an outward force. It is because of the fact that no centripetal force is supplied by the person. Therefore, to avoid the outward ("center-fleeing") force, the person should exert an inward force.

(ii) The normal force that prevents an object from falling toward the center of the Earth is a centrifugal force.

Examples:

1. A car travels at a constant speed of $13.4m/s$ on a level circular turn of radius $50.0m$, as shown in the bird's-eye view in Fig. 4.14(a). What minimum coefficient of static friction, μ_s, between the tires and roadway will allow the car to make the circular turn without sliding?

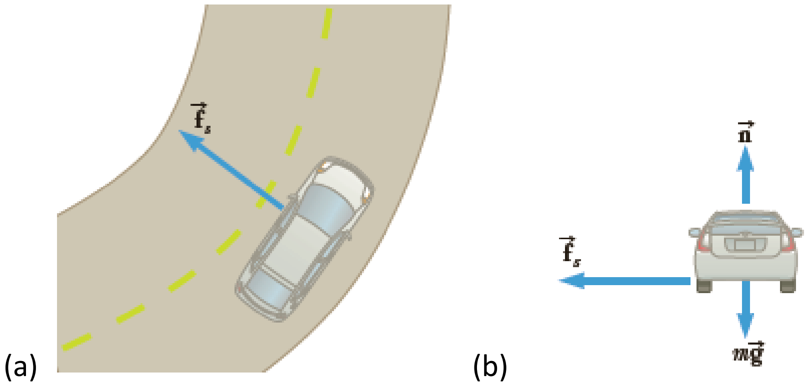


Figure 4.14: (a) A car travelling in a circular path. (b) Force diagram showing, the weight, the normal force, and the static friction force acting on the car.

Solution:

The forces acting on the car are shown in the free-body diagram shown in Fig. 4.14(b). Note that the radial component involves only the maximum static friction force, $f_{s,max}$:

$$f_{s,max} = m\frac{v^2}{r} = \mu_s n, \qquad \text{(a)}$$

Newton's second law applied to the forces acting in the vertical direction results to:

$$n - mg = 0 => n = mg \qquad \text{(b)}$$

Substituting Eq. (b) into (a), we get

$$m\frac{v^2}{r} = \mu_s mg,$$

and hence,

$$\mu_s = \frac{v^2}{rg} = \frac{(1.34m/s)^2}{(50.0m)(9.80m/s^2)} = 0.366.$$

2. Consider a car that rounds a curved road of radius $r = 316m$ and banked at an angle $\theta = 31.0^0$ (See Fig. 4.15). If the car negotiates the curve too slowly, it tends to slip down the incline of the turn, whereas if it is going too fast, it may begin to slide up the incline.

a. Find the necessary centripetal acceleration on this banked curve so the car won't tend to slip down or slide up the incline (Neglect friction).

b. Calculate the speed of the car.

Solution:

(a) Newton's second law applied to the car reads;

$m\vec{a} = \sum \vec{F} = \vec{n} + m\vec{g}$. (a)

Resolving the normal force, $\vec{n}$, into vertical and radial components (see Fig. 4.15(b)), we get the vertical (or y-) component of Eq. (a) to be

$\sum F_y = 0 \Rightarrow n\cos\theta - mg = 0$

so that the normal force becomes

$n = \frac{mg}{\cos\theta}$. (b)

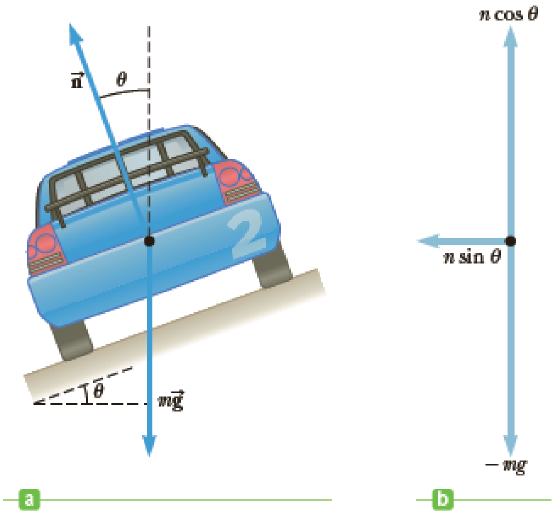


Figure 4.15: A car that rounds a curve banked at an angle θ. (a) Force diagram for the car and (b) components of the forces

Now the centripetal force which is supplied by the radial (or x-) component of the normal force that keeps the car on a circular path becomes

$\sum F_x = F_c = n\sin\theta$. (c)

Substituting Eq. (b) into (c), we obtain

$F_c = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta$. (d)

Then, the centripetal acceleration is obtained from the relation, $a_c = F_c/m$. Thus, using Eq. (d) we find that

$a_c = g\tan\theta = (9.80 m/s^2)(\tan 31.0^0) = 5.80 m/s^2$.

(b) using the relation $a_c = v^2/r$, we obtain the speed of the car to be:

$v = \sqrt{ra_c} = \sqrt{(361m)(5.80m/s^2)} = 43.1m/s.$

4.5.3 Applications of centripetal forces

4.5.3.1 Motion in a vertical circle

Let us consider a body of mass m tied to one end of the string which is fixed at O and it is moving in a vertical circle of radius r about the point O as shown in Fig. 4.16. The motion is circular but is not uniform, since the body speeds up while coming down and slows down while going up.

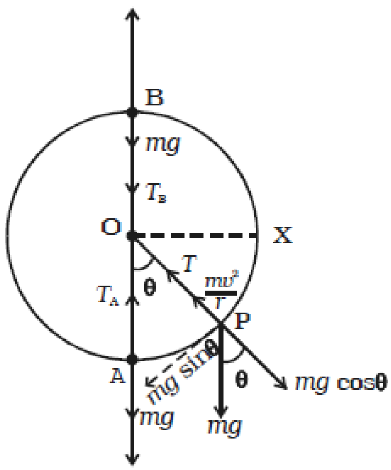


Figure 4.16: Motion of a body in a vertical circle.

Suppose the body is at P at any instant of time t, and the tension T in the string always acts towards O. The weight mg of the body at P is resolved along the string as $mg\cos\theta$ which acts outwards and $mg\sin\theta$, perpendicular to the string. When the body is at P, the following forces acts on it along the string.

i. $mg\cos\theta$ acts along OP (outwards)

ii. tension T acts along PO (inwards)

Thus, the net force on the body at *P* acting along $PO = T - mg\cos\theta$. This must provide the necessary centripetal force mv^2/r. Therefore,

$T - mg\cos\theta = \frac{mv^2}{r}$,

so that the tension in the string becomes

$$T = mg\cos\theta + \frac{mv^2}{r}. \qquad (4.7)$$

At the lowest point A of the path, $\theta = 0^0$, $\cos\theta = 1$ and hence Eq. (4.7) reduces to:

$$T_A = mg + \frac{mv_A^2}{r}. \qquad (4.8)$$

At the highest point of the path, i.e. at *B*, $\theta = 180^0$. Hence, $\cos 180^0 = -1$. Therefore, from Eq. (4.7), we obtain

$$T_B = \frac{mv_B^2}{r} - mg. \qquad (4.9)$$

If $T_B > 0$, then the string remains taut while if $T_B < 0$, the string slackens and it becomes impossible to complete the motion in a vertical circle.

If the velocity v_B is decreased, the tension T_B in the string also decreases, and becomes zero at a certain minimum value of the speed called *critical velocity*. Let v_C be the minimum value of the velocity, then at $v_B = v_C$, $T_B = 0$. Therefore, from Eq. (4.9), we have

$$m\frac{v_C^2}{r} - mg = 0,$$

from which, we get

$$v_C = \sqrt{rg}. \qquad (4.10)$$

If the velocity of the body at the highest point *B* is below this critical velocity, the string becomes slack and the body falls downwards instead of moving along the circular path. In order to ensure that the velocity v_B at the top is not lesser than the critical velocity $\sqrt{rg}$, the minimum velocity v_A at the lowest point should be in such a way that v_B should be $\sqrt{rg}$. That means, the motion in a vertical circle is possible only if $v_B \geq \sqrt{rg}$.

The velocity v_A of the body at the bottom point *A* can be obtained by using law of conservation of energy. When the stone rises from *A* to *B*, i.e., through a height *2r*, its potential energy (P.E.) increases by an amount equal to the decrease in kinetic energy (K.E.). Thus,

$$(P.E.+K.E)_{atA} = (P.E.+K.E)_{atB},$$

i.e., $$0 + \frac{1}{2}mv_A^2 = 2mgr + \frac{1}{2}mv_B^2. \qquad (4.11)$$

But from Eq. (4.10), $v_B^2 = rg$ (since $v_B = v_C$) and hence

$$v_A = \sqrt{5rg}. \qquad (4.12)$$

Substituting v_A from Eq. (4.12) into (4.8), we have

$$T_A = mg + m\left(\frac{5gr}{r}\right) = 6mg. \qquad (4.13)$$

While rotating in a vertical circle, the object tied to the string must have a velocity greater than $\sqrt{5gr}$ or tension greater than $6mg$ at the lowest point, so that its velocity at the top is greater than $\sqrt{gr}$ or tension ≥ 0.

An airplane while looping a vertical circle must have a velocity greater than $\sqrt{5gr}$ at the lowest point, so that its velocity at the top is greater than $\sqrt{gr}$. In that case, a pilot sitting in the airplane will not fall.

4.5.3.2 Motion on a level circular road

When a vehicle goes around a level curved path, it should be acted upon by a centripetal force. While negotiating the curved path, the wheels of the car have a tendency to leave the curved path and regain the straight-line path. Frictional force between the tyres and the road opposes this tendency of the wheels. This static frictional force, therefore, acts towards the centre of the circular path and provides the necessary centripetal force.

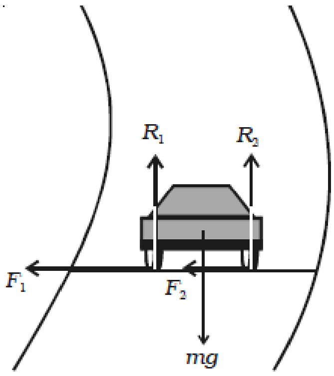


Figure 4.17: Vehicle on a level circular road.

In Fig. 4.17, weight of the vehicle mg acts vertically downwards. R_1, R_2 are the forces of normal reaction of the road on the wheels. As the road is level (horizontal), R_1, R_2 act vertically upwards. Obviously,

$$R_1 + R_2 = mg. \quad (4.14)$$

Let μ_s be the coefficient of static friction between the tyres and the road. Also, let F_1 and F_2 be the forces of friction between the tyres and the road, directed towards the centre of the curved path.

$$F_1 = \mu_s R_1 \text{ and} F_2 = \mu_s R_2. \quad (4.15)$$

If v is velocity of the vehicle while negotiating the curve, the centripetal force required is equal to mv^2/r. As this force is provided only by the friction, we have

$$m\frac{v^2}{r} \leq F_1 + F_2 = \mu_s(R_1 + R_2),$$

or,
$$\frac{mv^2}{r} \leq \mu_s mg,$$

since $R_1 + R_2 = mg$. Solving for the velocity, we get

$$v \leq \sqrt{\mu_s gr}.$$

Hence the maximum velocity with which a car can go around a level curve without skidding is $v = \sqrt{\mu_s gr}$. The value of v depends on radius r of the curve and coefficient of friction μ_s between the tyres and the road.

4.5.3.3 Banking of curved roads and tracks

When a car goes round a level curve, the force of friction between the tyres and the road provides the necessary centripetal force. If the frictional force, which acts as centripetal force and keeps the body moving along the circular road is not enough to provide the necessary centripetal force, the car will skid. In order to avoid skidding, while going round a curved path the outer edge of the road is raised above the level of the inner edge. This is known as banking of curved roads or tracks.

Bending of a cyclist round a curve

A cyclist has to bend slightly towards the centre of the circular track in order to take a safe turn without slipping (See Fig. 4.18).

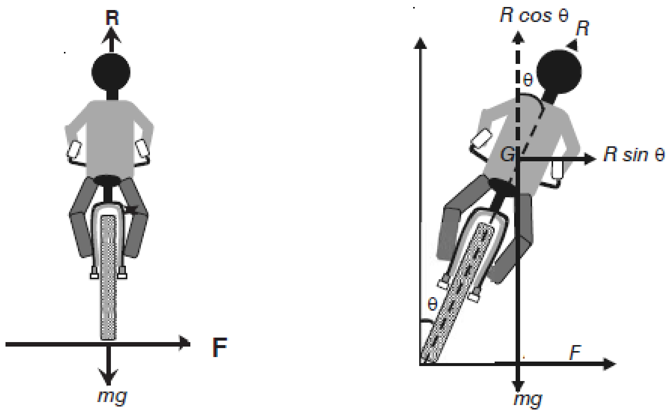


Figure 4.18: Bending of a cyclist in a curved road.

Figure 4.18 shows a cyclist taking a turn towards his right on a circular path of radius r. Let m be the mass of the cyclist along with the bicycle and v, the velocity. When the cyclist negotiates the curve, he bends inwards from the vertical, by an angle θ. Let R be the reaction of the ground on the cyclist. The reaction R may be resolved into two components:

i. the component $R\sin\theta$, acting towards the centre of the curve providing necessary centripetal force for circular motion and

ii. the component $R\cos\theta$, balancing the weight of the cyclist along with the bicycle. That is,

$$R\,sin\,\theta = \frac{mv^2}{r}, \quad (4.16)$$

and $$R\,cos\,\theta = mg. \quad (4.17)$$

Dividing Eq. (4.16) by (4.17),

$$tan\,\theta = \frac{v^2}{rg} \text{ or } \theta = tan^{-1}\left(\frac{v^2}{rg}\right) \quad (4.18)$$

Thus, for less bending of cyclist (i.e., for θ to be small), the velocity *v* should be smaller and radius *r* should be larger.

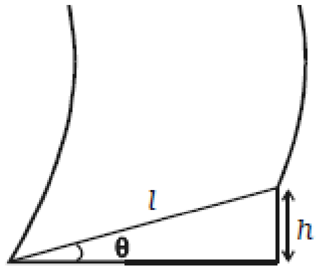


Figure 4.19: Banked road.

For a banked road (Fig. 4.19), let h be the elevation of the outer edge of the road above the inner edge and l be the width of the road then,

$$\sin\theta = \frac{h}{l}. \quad (4.19)$$

For small values of θ, $sin\,\theta \cong tan\,\theta$. Therefore, from Eqs. (4.18) and (4.19)

$$tan\,\theta = \frac{h}{l} = \frac{v^2}{rg}. \quad (4.20)$$

Obviously, a road or track can be banked correctly only for a particular speed of the vehicle. Therefore, the driver must drive with a particular speed at the circular turn. If the speed is higher than the desired value, the vehicle tends to slip outward at the turn but then the frictional force acts inwards and provides the additional centripetal force. Similarly, if the speed of the vehicle is lower than the desired speed it tends to slip inward at the turn but now the frictional force acts outwards and reduces the centripetal force.

Condition for skidding

When the centripetal force is greater than the frictional force, skidding occurs. If µ is the coefficient of friction between the road and tyre, then the limiting static friction (frictional force) is $f = \mu_{s,max}R$ where normal reaction $R = mg$. Thus,

$$f = \mu_{s,max}.$$

Thus, for skidding, the centripetal force must be greater than the frictional force, i.e.,

$$\frac{mv^2}{r} > \mu_{s,max}mg.$$

or $$\frac{v^2}{rg} > \mu_{s,max}. \quad (4.21)$$

But, $v^2/(rg) = tan\,\theta$, and hence

$$tan\,\theta > \mu_{s,max}. \quad (4.22)$$

Equation (4.22) shows that when the tangent of the angle of banking is greater than the coefficient of friction, skidding occurs.

4.6 Summary

Forces/Interactions

A force is commonly imagined as a push or a pull on some object. When you push or pull an object away or towards you, you exert a force on it. Hence, the word *force* refers, in general, to an interaction between two objects. Forces exist only as a result of an interaction. The effect of forces are (a) to accelerate or stop an object, (b) to change the direction of a moving object, and (c) to change the shape of an object.

Fundamental Forces

Force can be classified as either contact forces or field forces. *Contact forces* are forces that involve physical contact between two objects, while field forces are forces that do not involve physical contact between two objects.

There are four known fundamental forces of nature. These are:

(1) the strong nuclear force between subatomic particles;

(2) the electromagnetic forces between electric charges;

(3) the weak nuclear forces, which arise in certain radioactive decay processes; and

(4) the gravitational force between objects.

All the fundamental forces are field forces. Forces such as friction or the force of a bat hitting a ball are called contact forces. However, on a more fundamental level, contact forces have an electromagnetic nature.

Newton's laws of motion

Newton's first law states that an object moves at constant velocity unless acted on by a force.

The tendency for an object to maintain its original state of motion is called inertia. Mass is the physical quantity that measures the resistance of an object to changes in its velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force acting on an object equals the product of its mass m and acceleration, $\vec{a}$:

$$\sum \vec{F} = m\vec{a}.$$

An object in equilibrium has no net external force acting on it, and the second law, in component form, implies that $\sum F_x = 0$ and $\sum F_y = 0$ for such an object. This condition is often known as the first condition of equilibrium.

The weight, $\vec{F}_g$, of an object of mass m is the magnitude of the force of gravity exerted on that object and is given by

$$F_g = mg,$$

where $g = F_g/m$ is the acceleration of gravity.

Newton's third law states that if two objects interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1:

$$\vec{F}_{21} = -\vec{F}_{12}.$$

Any one of the forces is called the "action" while the other is the "reaction".

Friction

Friction is the force that opposes the motion of an object in contact with a surface and it acts parallel to the surface the object is in contact with. The magnitude of the frictional force depends on (i) the type of the surface (its roughness) and (ii) the magnitude of the normal force. The *normal force* is the force exerted by a surface on an object in contact with it.

We may identify two types of frictions: *kinetic and static frictions*.

The magnitude of the kinetic friction force f_k acting on an object moving on a surface is given by

$$f_k = \mu_k n,$$

where μ_k is the coefficient of kinetic friction and n is the magnitude of the normal force.

The magnitude of the static friction force f_s acting on an object at rest satisfies the inequality

$$0 \le f_s \le f_{s,max} = \mu_s n,$$

where n is the normal force and μ_s is the coefficient of the maximum static friction force between the object and the surface. Note that only the maximum static friction force, $f_{s,max}$, involves the use the static friction coefficient, μ_s .

Centripetal force

Centripetal force F_c is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. For an object of mass m moving with constant speed of v on a flat circular track of radius r, the magnitude of centripetal force is given by

$$F_c = ma_c = m\frac{v^2}{r}.$$

4.7 Conceptual Questions

1. State Newton's laws of motion.
2. Which of the following statements are true?
   (a) An object can move even when no force acts on it.
   (b) If an object isn't moving, no external forces act on it.
   (c) If a single force acts on an object, the object accelerates.
   (d) If an object accelerates, at least one force is acting on it.
   (e) If an object isn't accelerating, no external force is acting on it.
   (f) If the net force acting on an object is in the positive x-direction, the object moves only in the positive x-direction.
3. (a) If gold were sold by weight, would you rather buy it in Addis Ababa or in Asseb?

(b) If it were sold by mass, in which of the two locations would you prefer to buy it? Why?

4. If you push on a heavy box that is at rest, you must exert some force to start its motion. Once the box is sliding, why does a smaller force maintain its motion?
5. A ball is held in a person's hand.
   (a) Identify all the external forces acting on the ball and the reaction to each.
   (b) If the ball is dropped, what force is exerted on it while it is falling? Identify the reaction force in this case. (Neglect air resistance.)
6. If only one force acts on an object, can it be in equilibrium? Explain.
7. Identify the action-reaction pairs in the following situations:
   (a) a man takes a step,
   (b) a snowball hits a girl in the back,
   (c) a baseball player catches a ball, and
   (d) a gust of wind strikes a window.
8. Objects moving along a circular path have a centripetal acceleration provided by a net force directed towards the center. Identify the force(s) providing the centripetal acceleration in each of these cases:
   (a) a planet in circular orbit around its sun;
   (b) a car going around an unbanked, circular turn;
   (c) a rock tied to a string and swung in a vertical circle, as it passes through its highest point;
   (d) a dry sock in a clothes dryer as it spins in a horizontal circle.
9. A car of mass m follows a truck of mass $2m$ around a circular turn. Both vehicles move at speed v.
   (a) What is the ratio of the truck's net centripetal force to the car's net centripetal force?

(b) At what new speed v_{truck} will the net centripetal force acting on the truck equal the net centripetal force acting on the car still moving at the original speed v?

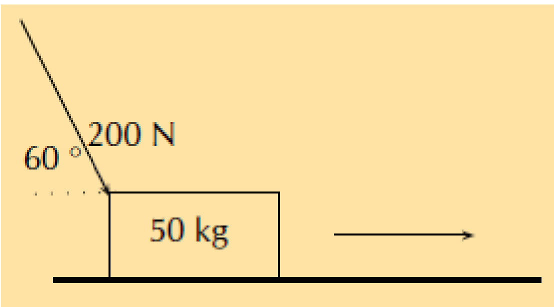
10. Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?

4.8 Problems

1. Determine the acceleration of a mass of 24 kg when a force of magnitude 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved?
2. A mass of 8 kg is accelerating at 5 m/s^2.
   (a) Determine the resultant force that is causing the acceleration.
   (b) What acceleration would be produced if we doubled the force and reduced the mass by half?
3. Find the magnitude of the two forces such that it they are at right angles, their resultant is 10 N. But if they act at 600, their resultant is 13 N.
4. A block on an inclined plane experiences a force due to gravity of 300 N straight down. If the slope is inclined at 600 to the horizontal, what is the component of the force due to gravity perpendicular and parallel to the slope? At what angle would the perpendicular and parallel components of the force due to gravity be equal?
5. The following forces act at a point:
   (a) 20 N inclined at 300 towards North of East
   (b) 25 N towards North
   (c) 30 N inclined at 450 towards North of West
   (d) 35 N inclined at 400 towards South of West.

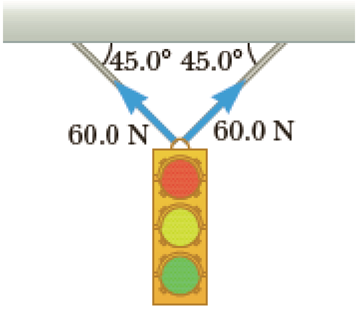
Find the magnitude and direction of the resultant force.

6. A force of 200 N, acting at 600 to the horizontal, accelerates a block of mass 50 kg along a horizontal plane as shown in the Figure.

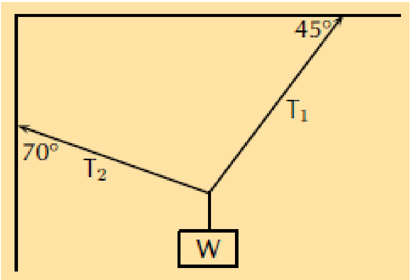


   (a) Calculate the component of the 200 N force that accelerates the block horizontally.
   (b) If the acceleration of the block is 1.5 m/s2, calculate the magnitude of the frictional force on the block.
   (c) Calculate the vertical force exerted by the block on the plane.

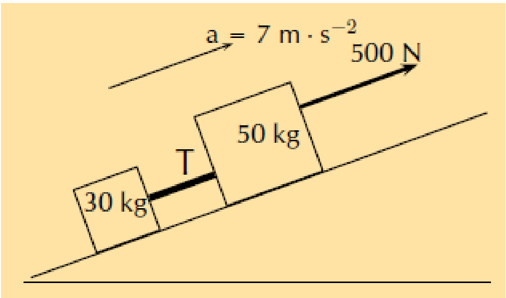
7. (a) What is the resultant force exerted by the two cables supporting the traffic light shown below in the Figure? (b) What is the weight of the light?



8. An object of weight W is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are T_1 and T_2, respectively. If tension T_1 = 1200 N, determine the tension T_2 and the weight W of the object.



9. Two crates of masses 30 kg and 50 kg are connected with a thick rope as shown in the diagram. If they are dragged up an incline such that the ratio of the parallel and perpendicular components of the gravitational force on each block are 3:5. The boxes move with an acceleration of 7 m/s2 up the slope. The ratio of the frictional forces on the two crates is the same as the ratio of their masses. The magnitude of the force due to gravity on the 30 kg crate is 294 N and on the 50 kg crate is 490 N. Calculate:



(a) the magnitude and direction of the total frictional force present.

(b) the magnitude of the tension in the rope at T.

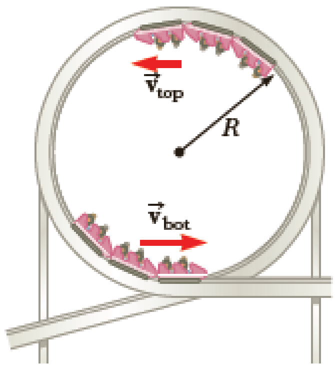
10. A car of mass 875 kg is traveling 30.0 m/s when the driver applies the brakes, which lock the wheels. The car skids for 5.60 s in the positive x-direction before coming to rest.

(a) What is the car's acceleration?

(b) What magnitude force acted on the car during this time?

(c) How far did the car travel?

11. A student of mass 60.0 kg, starting at rest, slides down a slide 20.0 m long, tilted at an angle of 30.00 with respect to the horizontal. If the coefficient of kinetic friction between the student and the slide is 0.120, find
    (a) the force of kinetic friction,
    (b) the acceleration, and
    (c) the speed she is traveling when she reaches the bottom of the slide.
12. A man exerts a horizontal force of 125 N on a crate with a mass of 30.0 kg.
    (a) If the crate doesn't move, what is the magnitude of the static friction force?
    (b) What is the minimum possible value of the coefficient of static friction between the crate and the floor?
13. A 75-kg man standing on a scale in an elevator notes that as the elevator rises, the scale reads 825 N. What is the acceleration of the elevator?
14. An elevator is required to lift a body of mass 65 kg. Find the acceleration of the elevator, which could cause a reaction of 800 N on the floor.
15. A toy rocket experiences a force due to gravity of magnitude 4.5 N is supported vertically by placing it in a bottle. The rocket is then ignited. Calculate the force that is required to accelerate the rocket vertically upwards at 8 m/s2.
16. At what angle must a railway track with a bend of radius 880 m be banked for the safe running of a train at a velocity of 44 m/s?
17. A 55.0 kg ice skater is moving at 4.0 m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.80 m around the pole.
    (a) Determine the force exerted by the horizontal rope on her arms.
    (b) Compare this force with her weight.
18. A 40.0 kg child swings in a swing supported by two chains, each 3.0 m long. The tension in each chain at the lowest point is 350 N. Find
    (a) the child's speed at the lowest point and
    (b) the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)
19. A certain light truck can go around a flat curve having a radius of 150 m with a maximum speed of 32.0 m/s. With what maximum speed can it go around a curve having a radius of 75.0 m?
20. A roller-coaster car is moving around a circular loop of radius R; as shown in the Figure.



21. Figure: A roller coaster traveling around a circular track.
    (a) What speed must the car have at the top of the loop so that it will just make it over the top without any assistance from the track?
    (b) What speed will the car subsequently have at the bottom of the loop?
    (c) What will be the normal force on a passenger at the bottom of the loop if the loop has a radius of 10.0 m?

5 Gravitation and Kepler's Laws of Motion

Learning Outcome

After completing this Chapter, students are expected to:

- Apply Newton's law of gravitation to relate the gravitational force between two particles to their masses and their separation.
- Identify that a uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.
- Draw a free-body diagram to indicate the gravitational force on a particle due to other particles or a uniform spherical distribution of matter
- Identify that a uniform shell of matter exerts no net gravitational force on a particle located inside it.
- Calculate the gravitational potential energy of a system of particles (or uniform sphere that can be treated as particles).
- State Kepler's three laws

Introduction

In this chapter, we study Newton's law of universal gravitation. We emphasize on the description of planetary motion because astronomical data provide an important test of this law's validity. We then show that the laws of planetary motion developed by Johannes Kepler follow from the law of universal gravitation and the principle of conservation of angular momentum. We conclude the chapter by deriving a general expression for the gravitational potential energy of a system and examining the energetics of planetary and satellite motion.

5.1 Newton's Law of Gravitation

Learning outcome

After completing this section, students are expected to:

- Calculate the attractive force between two point masses
- Explain the relation between the force between two point masses and the separation between the masses

Prior to 1686, a great deal of data had been collected on the motions of the Moon and planets, but no one had a clear understanding of the forces affecting them. In that year, Isaac Newton provided the key that unlocked the secrets of the heavens. He knew from the first law that a net force had to be acting on the Moon. If it were not, the Moon would move in a straight-line path rather than in its almost circular orbit around Earth. Newton reasoned that it was the same kind of force that attracted objects—such as apples—close to the surface of the Earth. He called it the force of gravity.

In 1687 Newton published his work on the law of universal gravitation, which is stated as:

Gravitational force, $\vec{F}_g$, of attraction between two particles, with masses m_1 and m_2 and separated by a distance r, is directly proportional to the product of their masses and inversely proportional to the square of their separation. Moreover, the gravitational force is directed along the line joining the masses. That is;

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{e}_r \qquad (5\text{-}1)$$

where $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$ is a constant of proportionality called the **universal gravitational constant**. The gravitational force is always attractive.

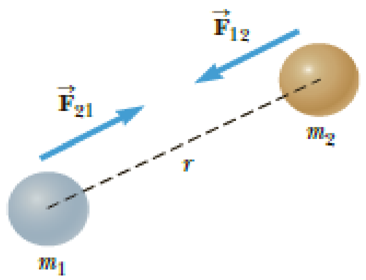


Figure 5-1: The gravitational force between two particles is attractive and acts along the line joining the particles.

The force law given by Eqn. (5.1) is an example of an inverse - square law, in that it varies as one over the square of the distance between particles. From Newton's third law, we know that the force exerted by m_1 on m_2, designated $\vec{F}_{12}$ in Fig 5.1, is equal in magnitude but opposite in direction to the force $\vec{F}_{21}$ exerted by m_2 on m_1, forming an action–reaction pair. That is,

$$\vec{F}_{12} = -\vec{F}_{21}$$

Another important fact is that the gravitational force exerted by a uniform sphere on a particle outside the sphere is the same as the force exerted if the entire mass of the sphere were concentrated at its center. This is called Gauss' law.

Examples

1. Calculate the net gravitational force that mass $m_2 = 20kg$ and $m_3 = 10kg$ exerts on mass $m_1 = 20kg$ for the case shown in Fig. 5.2.

Solution:

The net gravitational force $\vec{F}_1$ on m_1 is the vector sum of the forces due to m_2 and m_3. The magnitude of force $\vec{F}_{12}$ on particle 1 by particle 2 is

$$\vec{F}_{12} = \frac{Gm_1m_2}{(2a)^2}\hat{\imath} = \frac{6.67 \times 10^{-11}Nm^2kg^{-2} \times 20.0kg \times 20.0kg}{4 \times 0.25m^2} = 2.67 \times 10^{-8}N\hat{\imath}$$

Similarly, the magnitude of the $\vec{F}_{13}$ on particle 1 from particle 3 is

$$\vec{F}_{13} = \frac{Gm_1m_3}{(a)^2}\hat{\jmath} = \frac{6.67 \times 10^{-11}Nm^2kg^{-2} \times 10.0kg \times 20.0kg}{0.25m^2} = 5.34 \times 10^{-8}N\hat{\jmath}$$

The two forces are at right angle to each other. That is the force $\vec{F}_{12}$ directed along positive x-axis and $\vec{F}_{13}$ is directed along the positive-y-axis. The magnitude of resultant or the net force that the two masses exert on particle 1 is

$$F_1 = \sqrt{\left|\vec{F}_{12}\right|^2 + \left|\vec{F}_{13}\right|^2} = \sqrt{(2.67N)^2 + (5.34N)^2} = 5.97 \times 10^{-8}N$$

The direction of the force is determined by calculating the angle θ that the resultant force makes with the positive x-axis and is given by $\theta = tan^{-1}\left(\frac{5.34\times10^{-8}}{2.67\times10^{-8}}\right) = 63.4°$.

2. The Earth-Sun distance is about $1.5 \times 10^{11}m$ = (one astronomical unit). The mass of the Earth is $6.0 \times 10^{24}kg$ and the mass of the Sun is $2.0 \times 10^{30}kg$. What is the magnitude of the force the Earth exerts on the Sun?

Solution:

$$\vec{F} = \frac{Gm_Em_S}{r^2} = \frac{6.67 \times 10^{-11}Nm^2kg^{-2} \times 6.0 \times 10^{24}kg \times 2.0 \times 10^{30}kg}{(1.5 \times 10^{11}m)^2} = 3.56 \times 10^{21}N$$

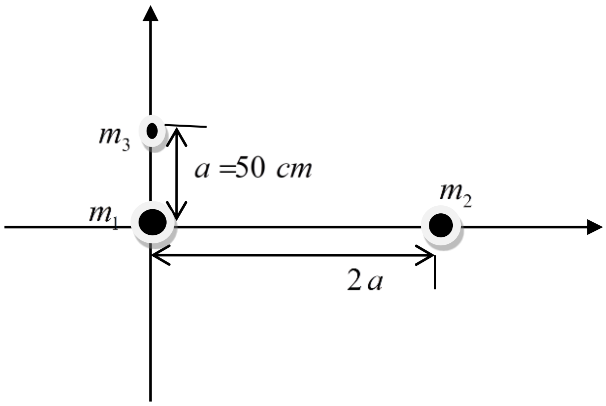


Figure 5-2: arrangement of three masses

Exercises

1. Comparing the results obtained in examples 1 and 2 above, determine in which case the gravitational force is significant?
2. Two objects attract each other with a gravitational force of magnitude $1.0 \times 10^{-8} N$ when separated by 20.0 cm. If the total mass of the objects is 5.00 kg, what is the mass of each?
3. (a) Find the magnitude of the gravitational force between a planet with mass $7.5 \times 10^{24} kg$ and its moon, with mass$2.70 \times 10^{22} kg$, if the average distance between their centers is 2.80 3 108 m. (b) What is the acceleration of the moon towards the planet? (c) What is the acceleration of the planet towards the moon?
4. A square edge length 20. 0 cm is formed by four spheres of masses $m_1 = 5.00g$, $m_2 = 3.00g$, $m_3 = 1.00g$ and $m_4 = 5.00g$ (see Fig. 5.3). In unit-vector notation, what is the net gravitational force from them on a central sphere with mass $m_5 = 2.500g$?

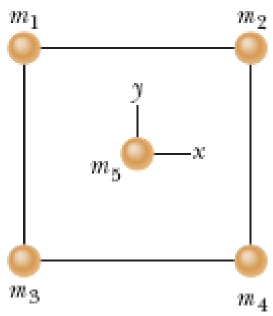


Figure 5-3: Gravitational force on the central sphere by the corner spheres.

5.1.1 Free-Fall Acceleration and the Gravitational force

Learning outcome

After completing this section, students are expected to:

- Distinguish between the free-fall acceleration and the gravitational acceleration

- Calculate the gravitational acceleration at different heights above the surface of the Earth.

The *gravitational acceleration* g of a particle (of mass *m*) near the surface of Earth is due solely to the gravitational force acting on it. We recall that gravitational force, $\vec{F}_g$, on a particle of mass *m* placed on(near) the surface of the earth is equal to the weight of the particle. Moreover, the net force on the particle is gravitational attraction exerted by earth, which is given by Eq. (5.1). Thus, by Newton's second law,

$$F_g = mg = \frac{GmM_E}{R_E^2}$$

$$g = \frac{GM_E}{R_E^2} \tag{5-2}$$

where M is mass of the Earth and R_E is its radius. Equation 5.2 relates the free-fall acceleration g to physical parameters of the Earth – its mass and radius – and explains the origin of the value of 9.80 m/s^2 that we have used in earlier chapters. Now consider an object of mass m located a distance h above the Earth's surface or a distance r from the Earth's center, where $r = R_E + h$. The magnitude of the gravitational force acting on this object is

$$F_g = \frac{GmM_E}{r^2} = \frac{GmM_E}{(R_E + h)^2}$$

The magnitude of the gravitational force acting on the object at this position is also $F_g = mg'$, where g' is the value of the free-fall acceleration at the altitude *h.* Substituting this expression for F_g into the last equation shows that g' is given by

$$g' = \frac{GM_E}{(R_E+h)^2} \tag{5-3}$$

The variation of free-fall acceleration g with altitude above the Earth is shown in Table 5-1.

Table 5-1: Free-Fall acceleration g at various altitudes

Altitude above the surface of the Earth (km)	$g(m/s^2)$
1000	7.33
2000	5.68
3000	4.53
4000	3.70
5000	3.08
6000	2.60
7000	2.20
8000	1.93

9000	1.69
10000	1.49
50000	0.13

Examples:

1. The International Space Station operates at an altitude of 350 km. Plans for the final construction show that material of weight $4.22 \times 10^6 N$, measured at the Earth's surface, will have been lifted off the surface by various spacecraft. What is the weight of the space station when in orbit?

Solution:

First, we calculate the mass of material from its weight when it was at the ground**.**

$$\boldsymbol{F = mg = weight \Rightarrow m = \frac{weight}{g} = \frac{4.22 \times 10^6 N}{9.80 ms^{-2}} = 4.31 \times 10^5 kg}$$

Next determine the gravitational acceleration g at the height of 350 km using equation (5.3).

$$g = \frac{GM_E}{(R_E + h)^2} = \frac{6.67 \times 10^{-11} Nm^2 kg^{-2} \times 6.0 \times 10^{24} kg}{(6.4 \times 10^6 m + 3.5 \times 10^5 m)^2} = 8.82 ms^{-2}$$

Hence, $weight = W = mg = 4.31 \times 10^5 kg \times 8.82 ms^{-2} = 3.8 \times 10^6 N$

2. At what altitude above the Earth's surface does the gravitational acceleration reduced by half?

Solution:

$$g_h = \frac{g}{2} = \frac{GM_E}{(h + R_E)^2}$$

$$\Rightarrow ((h + R_E)^2) = \frac{2}{g} GM_E = \frac{2}{9.8\ ms^{-2}} \times 6.67 \times 10^{-11} Nm^2 kg^{-2} \times 6.0 \times 10^{24} kg$$

$h^2 + 2 \times R_E h + R_E^2 = 8.18 \times 10^{13} m^2$

Substituting the value of R_E and solving the quadratic equation, we obtain

$$h = -R_E \pm \frac{\sqrt{4R_E^2 + 4 \times 8.18 \times 10^{13} m^2 - 4R_E^2}}{2} = 2.64 \times 10^6 m$$

Exercises

1. The mass of Jupiter is $1.9 \times 10^{27} kg$and its radius is $7.1 \times 10^{7} m$. Calculate the gravitational acceleration at the surface of Jupiter.
2. When a falling meteoroid is at a distance above the Earth's surface of 3.00 times the Earth's radius, what is its acceleration due to the Earth's gravitation?
3. (a) What will an object weigh on the Moon's surface if it weighs 100 N on Earth's surface? (b) How many Earth radii must this same object be from the center of Earth if it is to weigh the same as it does on the Moon?
4. An astronaut standing on the surface of Ceres, the largest asteroid, drops a rock from a height of 10.0 m. It takes 8.06 s to hit the ground. (a) Calculate the acceleration of gravity on Ceres. (b) Find the mass of Ceres, given that the radius of Ceres is $R_C = 5.1 \times 10^{2} km$. (c) Calculate the gravitational acceleration 50.0 km from the surface of Ceres.

5.1.2 Gravitational Potential Energy

Learning Outcome

After completing this Chapter, students are expected to:

- Determine the gravitational potential energy at height h above the surface of the Earth
- Discuss the effect of distance between two masses on gravitational potential energy they possess

The common concept of gravitational potential energy associated with an object of mass *m* near the surface of the Earth could be calculated from the relation $PE = mgh$, where *h* is the height of the object above or below some reference level. This equation, however, is valid *only* when the object is near Earth's surface. For objects high above Earth's surface, such as a satellite, an alternative relation must be used because g varies with distance from the surface, as shown in Table 5.1

The gravitational potential energy associated with an object of mass *m* at a distance *r* from the center of Earth is

$$PE = -\frac{GmM_E}{r} \tag{5-4}$$

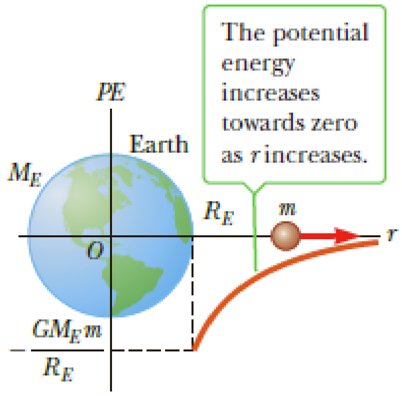


Figure 5-4: As a mass *m* moves radially away from the Earth, the potential energy of the Earth-mass system, which is $PE = -G(mM_E)/R_E$ at Earth's surface, increases toward a limit of zero as the mass *m* travels away from Earth, as shown in the graph.

Example

How much work is done by the gravitational field in moving a mass of 20.0 kg, from infinity to a point A, 5.0 m from a mass of 1000 kg?

Solution

The work done in moving an object from infinity to a given point is just the negative of the potential energy of the object when it was at infinity. Hence

$$W = -PE = \frac{Gm_1m_2}{r} = \frac{6.67 \times 10^{-11} Nm^2kg^{-2} \times 20.0kg \times 1000.0kg}{5.0m}$$

$$W = 2.7 \times 10^{-7} J$$

Exercises

1. What must the separation be between a 5.2 kg particle and a 2.4 kg particle for their gravitational attraction to have a magnitude of $2.3 \times 10^{-12} N$? What is the gravitational potential energy of the two-particle system? If you triple the separation between the particles, how much work is done (b) by the gravitational force between the particles and (c) by you?
2. Figure 5.5 gives the potential energy function PE(r) of a projectile, plotted outward from the surface of a planet of radiusR_S. What least kinetic energy is required of a projectile launched at the surface if the projectile is to “escape” the planet?

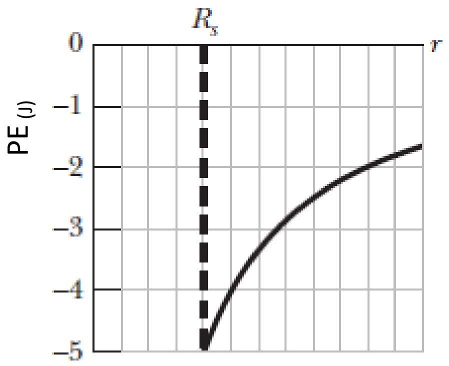


Figure 5-5: Relates to Exercise 2

5.1.3 Escape Speed

Learning outcome

After completing this Chapter, students are expected to:

- Define escape speed
- Calculate the escape speed of a given satellite
- Analyse the correlation between escape speed and the mass of the projected object

If an object is projected upward from Earth's surface with a large enough speed, it can soar off into space and never return. This speed is called Earth's escape speed. (It is also commonly called the *escape velocity,* but in fact is more properly a speed.) The escape speed form the Earth's surface can be found by applying conservation of energy. Suppose an object of mass *m* is projected vertically upward from Earth's surface with an initial speed v_i. The initial mechanical energy (kinetic plus potential energy) of the object–Earth system is given by

$$\boldsymbol{KE_i + PE_i = \frac{1}{2} m v_i^2 - \frac{G m M_E}{R_E}} \qquad (5\text{-}5)$$

In this expression the air resistance was neglected and the initial speed is just large enough to allow the object to reach infinity with a speed of zero. This initial the speed (v_i) of the object is the escape speed v_{esc} of the object. When the object is at an infinite distance from Earth, its kinetic energy is zero because $v_f = 0$, and the gravitational potential energy is also zero because $1/r$ goes to zero as r goes to infinity. Hence the total mechanical energy is zero, and the law of conservation of energy gives

$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = \frac{1}{2}mv_{esc}^2 - \frac{GmM_E}{R_E} = 0$$

$$\frac{1}{2}mv_{esc}^2 = \frac{GmM_E}{R_E}$$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} \tag{5-6}$$

Table *5-2: Escape speed for the planets and the moon*

Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1

Examples

If the spacecraft leaves the cannon (on the surface of the Earth) at escape speed, at what speed is it moving when $2\times10^5\ km$ from the center of Earth? Neglect any friction effects.

Solution

Here the escape seed is that of the Earth's escape speed, 11.2 km/s, and applying the law of conservation of energy

$$\frac{1}{2}mv_{esc}^2 - \frac{GmM_E}{R_E} = \frac{1}{2}mv_f^2 - \frac{GmM_E}{r}$$

where $r = 2\times10^5 km$.

$$v_{esc}^2 - \frac{2GmM_E}{R_E} = v_f^2 - \frac{2GmM_E}{r}$$

$$v_f^2 = v_{esc}^2 - \frac{2GM_E}{R_E} + \frac{2GM_E}{r}$$

$$v_f = \sqrt{v_{esc}^2 - \frac{2GM_E}{R_E} + \frac{2GM_E}{r}}$$

$$v_f = \sqrt{(1.12 \times 10^4 m/s)^2 + 2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}\left(\frac{1}{2 \times 10^8 m} - \frac{1}{6.4 \times 10^6 m}\right)}$$

$$v_f = 2.1 \times 10^3 m/s$$

Exercise

1. (a) What is the escape speed on a spherical asteroid whose radius is 500 km and whose gravitational acceleration at the surface is 3.0 m/s^2? (b) How far from the surface will a particle go if it leaves the asteroid's surface with a radial speed of 1000 m/s? (c) With what speed will an object hit the asteroid if it is dropped from 1000 km above the surface?
2. What multiple of the energy needed to escape from Earth gives the energy needed to escape from (a) the Moon and (b) Jupiter?
3. A projectile is fired straight upward from the Earth's surface at the South Pole with an initial speed equal to one third the escape speed. (a) Ignoring air resistance, determine how far from the center of the Earth the projectile travels before stopping momentarily. (b) What is the altitude of the projectile at this instant?

5.2 Kepler's Law and the Motion of Planets

Learning outcome

After completing this Chapter, students are expected to:

- Distinguish between the three Kepler's law of planetary motion
- apply Kepler's laws to an orbiting natural or artificial satellite.

There are three planetary laws proposed by German astronomer Johannes Kepler. These laws are:

1. The law of orbits

All planets move in elliptical orbits with the Sun at one of the focal points.

2. The law of area

A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.

3. The law of periods

The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun.

Newton later demonstrated that these laws are consequences of the gravitational force that exists between any two objects. Newton's law of universal gravitation, together with his laws of motion, provides the basis for a full mathematical description of the motions of planets and satellites.

5.2.1 Kepler's First Law

The first law arises as a natural consequence of the inverse square nature of Newton's law of gravitation. Any object bound to another by a force that varies as $1/r^2$ will move in an elliptical orbit. As shown in Figure 5.6a, an ellipse is a curve drawn so that the sum of the distances from any point on the curve to two internal points called *focal points* or *foci* (singular, *focus*) is always the same. The semi-major axis a is half the length of the line that goes across the ellipse and contains both foci. For the Sun–planet configuration (Fig. 5.6b), the Sun is at one focus and the other focus is empty. Because the orbit is an ellipse, the distance from the Sun to the planet continuously changes.

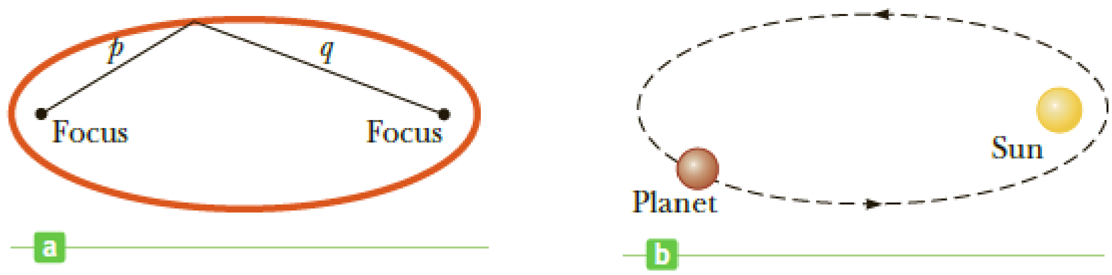


Figure 5-6: (a) The sum *p* + *q* is the same for every point on the ellipse. (b) In the Solar System, the Sun is at one focus of the elliptical orbit of each planet and the other focus is empty.

5.2.2 Kepler's second Law

Kepler's second law states that a line drawn from the Sun to any planet sweeps out equal areas in equal time intervals. Consider a planet in an elliptical orbit about the Sun, as in Figure 5.7. In a given period Δt, the planet moves from point Ⓐ to point Ⓑ. The planet moves more slowly on that side of the orbit because it's farther away from the sun. On the opposite side of its orbit, the planet moves from point Ⓒ to point Ⓓ in the same amount of time, Δt, moving faster because it's closer to the sun. Kepler's second law says that any two wedges formed as in Figure 5.7 will always have the same area.

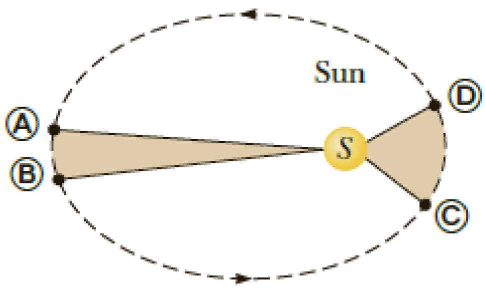


Figure 5-7: The two areas swept out by the planet in its elliptical orbit about the Sun are equal if the time interval between point Ⓐ is and Ⓑ is equal to the time interval between pointsⒸ and Ⓓ.

5.2.3 Kepler's third Law:

The derivation of Kepler's third law is simple enough to carry out for the special case of a circular orbit. Consider a planet of mass $M_P$moving around the Sun, which has a mass of M_S, in a circular orbit. Because the orbit is circular, the planet moves at a constant speed v. Newton's second law, his law of gravitation, and centripetal acceleration then give the following equation:

$$M_P a_c = \frac{M_p v^2}{r} = \frac{G M_S M_P}{r^2}$$

But $v = \frac{2\pi r}{T}$, where T is the period of revolution and r is the radius of the orbit.

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right) r^3$$

$$\boldsymbol{T^2 = K_S r^3} \quad \text{(5-7)}$$

This leads to

$$K_S = \frac{4\pi^2}{GM_S} = 2.97 \times 10^{-19} s^2/m^3$$

This equation is also valid for elliptical orbits if we replace r with the length a of the semimajor axis

$$\boldsymbol{T^2 = \left(\frac{4\pi^2}{GM_S}\right) a^3 = K_S a^3} \quad \text{(5-8)}$$

Table 5.3: Use ful planetary data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Mean Distance from Sun (m)	$\frac{T^2}{r^3} 10^{-19} \left(\frac{s^2}{m^3}\right)$
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}	2.97
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99
Earth	5.98×10^{24}	6.38×10^{6}	3.156×10^{7}	1.496×10^{11}	2.97
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}	2.95
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}	2.99
Pluto[a]	1.27×10^{23}	1.14×10^{6}	7.82×10^{9}	5.91×10^{12}	2.96
Moon	7.36×10^{22}	1.74×10^{6}	—	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—	—

Comets and asteroids usually have elliptical orbits. For these orbits, the radius r must be replaced with *a,* the semi-major axis – half the longest distance across the elliptical orbit.

The last column in Table 5.3 confirms that T^2/r^3 is very nearly constant.

Examples

1. For geostationary satellite, calculate (a) The height above the Earth's surface and (b) The speed in orbit

(radius of Earth, $R_E = 6.4 \times 10^6\ m$; mass of the earth = 6.0 x 10^{24} kg)

Solution

(a) The period of the satellite is 24 hours = 6.64 x 10^4 s.

Using $T^2/r^3 = \frac{4\pi^2}{GM_E}$, we get

$$r = \left(\frac{6.67 \times 10^{-11} Nm^2kg^2 \times 6 \times 10^{24} \times (8.64 \times 10^4 s)^2}{4\pi^2}\right)^{\frac{1}{3}} = 4.23 \times 10^7 m$$

(b) $\boldsymbol{v = \frac{2\pi r}{T} = \frac{2 \times 4.23 \times 10^7 m}{8.64 \times 10^4 s} = 3070 m/s}$

2. The radius of the Moon's orbit 3.84 x 108 m, and its period is 27.4 days. Use Kepler's law to calculate the period of the orbit of a satellite orbiting the Earth just above the Earth's (radius

of Earth's radius is 6.4 x 106 m).

Solution

First convert 27.4 days to second: $27.4 \times 24 \times 60 \times 60s = 2367360s$. Then use the relation

$$\frac{T_M^2}{r_M^3} = \frac{T_S^2}{r_S^3}$$

$$T_S = \sqrt{\frac{T_M^2 r_S^3}{r_M^3}} = \sqrt{\frac{(2367360s)^2 \times (6.4 \times 10^6 m)^3}{(3.84 \times 10^8 m)^3}} = 1.4hour$$

Exercises

1. (a) What linear speed must an Earth satellite have to be in a circular orbit at an altitude of 160 km above Earth's surface? (b)What is the period of revolution?
2. Satellite is put in a circular orbit about Earth with a radius equal to one-half the radius of the Moon's orbit. What is its period of revolution in lunar months? (A lunar month is the period of revolution of the Moon.)
3. The Martian satellite Phobos travels in an approximately circular orbit of radius 9.4 x 106 m with a period of 7 h 39 min. Calculate the mass of Mars from this information

5.3 Summary

Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{e}_r$$

where $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$ is the universal gravitational constant. This equation enables us to calculate the force of attraction between masses under many circumstances.

An object at a distance h above the Earth's surface experiences a gravitational force of magnitude mg, where g is the free-fall acceleration at that elevation:

$$F_g = \frac{GmM_E}{r^2} = \frac{GmM_E}{(R_E + h)^2}$$

In this expression, M_E is the mass of the Earth and R_E is its radius. Therefore, the weight of an object decreases as the object moves away from the Earth's surface.

Kepler's laws of planetary motion state:

1. All planets move in elliptical orbits with the Sun at one focus.
2. The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet is proportional to the cube of the sem-imajor axis of the elliptical orbit.

Kepler's third law can be expressed as

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right)a^3 = K_S a^3$$

where M_S is the mass of the Sun and *a* is the semi-major axis. For a circular orbit, *a* can be replaced in Equation 5.8 by the radius *r.* Most planets have nearly circular orbits around the Sun.

The gravitational potential energy associated with a system of two particles separated by a distance *r* is

$$PE = -\frac{GmM_E}{r}$$

where *PE* is taken to be zero as *r* tends to infinity.

If an isolated system consists of an object of mass *m* moving with a speed *v* in the vicinity of a massive object of mass *M,* the total energy *E* of the system is the sum of the kinetic and potential energies:

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

The total energy of the system is a constant of the motion. If the object moves in an elliptical orbit of semi-major axis *a* around the massive object and M>>m, the total energy of the system is

$$E = -\frac{GmM}{2a}$$

For a circular orbit, this same equation applies with $a = r$

The escape speed for an object projected from the surface of a planet of mass *M* and radius *R* is

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

5.4 Conceptual Questions

1. A parachute is falling toward the ground. Which of the following statements are false? (a) The force that the parachute exerts on Earth is equal in magnitude to the force that Earth exerts on the parachute. (b) The parachute undergoes the same acceleration as Earth. (c) The magnitude of the force the Earth exerts on the parachute is greater than the magnitude of the force the parachute exerts on the Earth.
2. Planet X is located at a distance $r$from the sun and planet Y is located at a distance $2r$ from the sun. Which planet exerts a bigger force on the sun? by what factor the force is bigger?

3. As the asteroid approaches Earth, does the gravitational potential energy associated with the asteroid Earth system (a) increase, (b) decrease, (c) remain the same?
4. A satellite in low-Earth orbit is not truly traveling through a vacuum. Rather, it moves through very thin air. Does the resulting air friction cause the satellite to slow down?
5. The escape speed is independent of the direction in which the object leaves Earth's surface. Why is this?
6. The satellites of Jupiter follow Kepler's third law: The Square of their periods divided by the radius of their orbits cubed is cubed. Is this the same constant as the planets moving around the sun?
7. If Earth were a perfect sphere, would you weigh more or less at the equator than at the poles?
8. How do satellites set into circular orbits?
9. If there were no gravitational attraction forces between the sun and the planets what may happen?
10. Why we do not feel the attraction forces between us?

5.5 Problems

1. (a) Find the magnitude of the gravitational force between a planet with mass 7.50 x 10^{24} kg and its moon, with mass 2.70 x 10^{22} kg, if the average distance between their centers is 2.80 x 10^{8} m. (b) What is the acceleration of the moon towards the planet? (c) What is the acceleration of the planet towards the moon?
2. An artificial satellite circling the Earth completes each orbit in 110 minutes. (a) Find the altitude of the satellite. (b) What is the value of g at the location of this satellite?
3. Two neutron stars are separated by a distance 1.0×10^{10}m. They each have a mass of 1.0×10^{30}kg and a radius of 1.0×10^{5}m. They are initially at rest with respect to each other. As measured from that rest frame, how fast are they moving when (a) their separation has decreased to one-half its initial value and (b) they are about to collide?
4. At what altitude above Earth's surface would the gravitational acceleration be 4.9 m/s^2?
5. I_0, a satellite of Jupiter, has an orbital period of 1.77 days and an orbital radius of 4.22×10^{5} km. From these data, determine the mass of Jupiter.
6. The Sun, which is 2.2 x 10^{20} m from the center of the Milky Way galaxy, revolves around that center once every 2.5 x 10^{8} years. Assuming each star in the Galaxy has a mass equal to the Sun's mass of 2.0 x 10^{30} kg, the stars are distributed uniformly in a sphere about the galactic center, and the Sun is at the edge of that sphere, estimate the number of stars in the Galaxy.
7. A 20 kg satellite has a circular orbit with a period of 2.4 h and a radius of 8.0 x 10^{6} m around a planet of unknown mass. If the magnitude of the gravitational acceleration on the surface of the planet is 8.0 m/s^2, what is the radius of the planet?

6 Work and Energy

Learning Outcome

After completing this Chapter, students are expected to:

- Understand the concepts of work, energy, and power.
- Express these quantities in mathematical formula.
- Solve problems based on these concepts.

Introduction

This chapter introduces: meanings of work done, kinetic and potential energies, and how each of these are calculated using a given quantities. Problems are considered from different corners of real life within the context of simplified models.

6.1 The Concept of Work

Work and energy are important concepts in physics as well as in our everyday lives. In physics, a force does work if its point of application moves through a distance and there is a component of the force in the direction of the velocity of the force's point of application. For a constant force in one dimension, the work done equals the force component in the direction of the displacement times the displacement. (This differs somewhat from the everyday use of the word work. When you study hard for an exam, the only work you do according to the use of the word in physics, is in pushing your pencil on the paper, or turning the pages of your book.)

Energy is closely associated with work. When work is done by one system on another, energy is transferred between the two systems. For example, when you do work pushing a swing, chemical energy of your body is transferred to the swing and appears as kinetic energy of motion or as gravitational potential energy of the earth-swing system. There are many forms of energy. Kinetic energy is associated with the motion of an object. Potential energy is associated with the configuration of a system, such as the separation distance between two objects that attract each other. Thermal energy is associated with the random motion of the molecules within a system and is closely connected with the temperature of the system.

6.1.1 Work done by a constant force

We consider an object that is displaced along the *x* axis by Δx by applying a constant force $\vec{F}$ that makes an angle $\boldsymbol{\theta}$ with the direction of motion, as shown in Fig. 6.1.

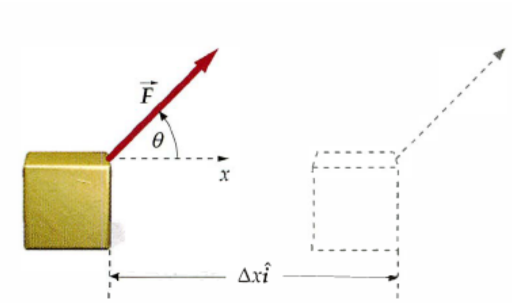


Fig. 6.1 Work done by a constant force

Then the work done on the object by the constant force (constant in both magnitude and direction) is defined as the product of the magnitude of the displacement times the component of the force parallel to the displacement. That is,

$$W = \Delta x F cos\theta, \qquad (6.1)$$

where $Fcos\theta$ is component of the force in the direction of the displacement, W is work done, and Δx is the magnitude of the displacement. We notice that work done can be positive, negative or zero depending on the angle θ between the force and the displacement. The work done is positive when the force has a component in the same direction as the displacement ($0^0 \leq \theta < 90^0$). On the other hand, when the force has a component opposite to the displacement ($90^0 < \theta \leq 180^0$) , the work done is negative. When the force is perpendicular to the displacement, $\theta = 90^0$ and the work done by the force is zero.

Work is said to be done on an object by a force if (a) the force is not perpendicular to the displacement ($\theta \neq 90^0$); and (b) the force displaces the object. For instant, a person pushing a fixed wall is not doing work since there is no displacement. Moreover, a person carrying a quintal of teff and moving horizontally is not doing work. In this case the force applied on the quintal is vertically upward and the displacement is in the horizontal direction.

6.1.2 Kinetic energy and work-energy theorem

The total work done on a body by external forces is related to the body's displacement-that is, to changes in its position. But the total work is also related to changes in the speed of the body. Consider a particle with mass m moving along the x-axis under the action of a constant net force with magnitude F directed along the positive x-axis (Fig. 6.2). Suppose the speed changes from v_1 to v_2 while the particle undergoes a displacement $\boldsymbol{s} = \boldsymbol{x_1} - \boldsymbol{x_2}$ from point x_1 to x_2.

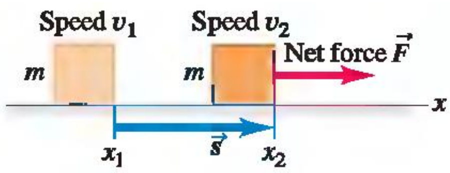


Fig. 6. 2

This constant net force on the object is given by Newton's second law as

$$\boldsymbol{F} = m\boldsymbol{a} \tag{6.2}$$

Now since the force is constant, the acceleration will also remain constant and thus can be written as

$$a = \frac{v_2^2 - v_1^2}{2s} \tag{6.3}$$

Upon combining Eqs. (6.2) and (6.3), the net force is expressible as

$$F = m\left(\frac{v^2{}_2 - v^2{}_1}{2s}\right)$$

or

$$Fs = \frac{1}{2}mv^2{}_2 - \frac{1}{2}mv^2{}_1 \tag{6.4}$$

The product Fs is the work done by the net force and the quantity

$$KE = \frac{1}{2}mv^2 \tag{6.5}$$

is called **kinetic energy** of a mass m moving with speed v. Thus, $\frac{1}{2}mv^2{}_2$ is the final kinetic energy of the mass and $\frac{1}{2}mv^2{}_1$ is its initial kinetic energy. We therefore observe that Eq. (6.4) relates the work done on an object by a net force to the change in its kinetic energy.

Then work-energy theorem can then be stated as

The work done by a net force on an object is equal to the change in its kinetic energy.

$$W_{net} = KE_f - KE_i \qquad (6.6)$$

where $KE_f = \frac{1}{2}mv_2^2$ is the final kinetic energy of the particle and $KE_i = \frac{1}{2}mv_1^2$ is its initial kinetic energy.

The work–kinetic energy theorem indicates that the speed of an object *increases* if the net work done on it is *positive* because the final kinetic energy is greater than the initial kinetic energy. The speed *decreases* if the net work is *negative* because the final kinetic energy is less than the initial kinetic energy. On the other hand, no work is done by the net force in moving an object with a constant speed.

Example

A 61 kg skier on level snow coasts 184 m to a stop from a speed of $\mathbf{12.0\ m/s}$. (a) Use the work–energy principle to find the coefficient of kinetic friction between the skis and the snow. (b) Suppose a 75 kg skier with twice the starting speed coasted the same distance before stopping. Find the coefficient of kinetic friction between that skier's skis and the snow.

Solution:

(a) $m = 61\ kg, \quad s = 184m, \ v_i = 12.0\ m/s, \ v_f = 0\ m/s, \quad \mu_k = ?$

According to the work-energy theorem, the change in the kinetic energy of the skis is equal to the work done by the net force acting the skis. That is,

$$W_{net} = KE_f - KE_i \qquad (1)$$

The net horizontal force on the skis is the kinetic frictional force,

$$f_k = \mu_k F_N = \mu_k mg \qquad (2)$$

Consequently, the work done by the net force is equal to the work done by frictional force. We thus have

$$W_{net} = -f_k s = -\mu_k m \qquad (3)$$

Equating Eqs. (1) and (3) and using the given values, we obtain

$$-\frac{1}{2}mv_i^2 = -\mu_k mgs$$

or

$$\mu_k = \frac{v_i^2}{2gs} = 0.039.$$

(b) $m = 75\,kg,\ s = 184m, v_i = 24.0\,m/s,\ v_f = 0\,m/s,\quad \mu_k = ?$

Following the same procedure, one can verify that

$$\mu_k = \frac{v_i^2}{2gs} = 0.156.$$

6.1.3 Elastic potential energy

When a spring is stretched (or compressed) from its equilibrium position, it has ability to do work as it returns to the equilibrium position. Thus, the spring may have the potential for doing work because of its stretch (or compression).

Energy that is stored in an elastic object when you stretch, compress, twist, or otherwise deform it is called elastic potential energy. Consider an object of mass $\boldsymbol{m}$ attached to a spring of spring constant $\boldsymbol{k}$, as shown in Fig.6.3.

For a force that is a linear function of position, such as spring force ($F_{sp} = -kx$), the work done by the force is the average force multiplied by the displacement. Accordingly, the work done by spring force when the spring is stretched from x_i to x_f is the product of the average spring force and the displacement. That is;

$$W_{sp} = -\tfrac{1}{2}k\left(x_f + x_i\right)\left(x_f - x_i\right) = -\tfrac{1}{2}k({x^2}_f - {x^2}_i) \qquad (6.7)$$

The quantity $U_{ela} = \frac{1}{2}kx^2$ is elastic potential energy of spring mass system when the spring is stretched or compressed by x. We thus notice that $U_{elaf} = \frac{1}{2}k{x^2}_f$ is the final elastic potential energy and $U_{elai} = \frac{1}{2}k{x^2}_i$ is the initial elastic potential energy. Accordingly, we observe from Eq. (6.7) that the work done by spring force when the spring is stretched from x_i to x_f is equal to the negative of the change in elastic potential energy. That is,

$$W_{sp} = -(U_{elaf} - U_{elai}) = -\Delta U_{ela}. \qquad (6.8)$$

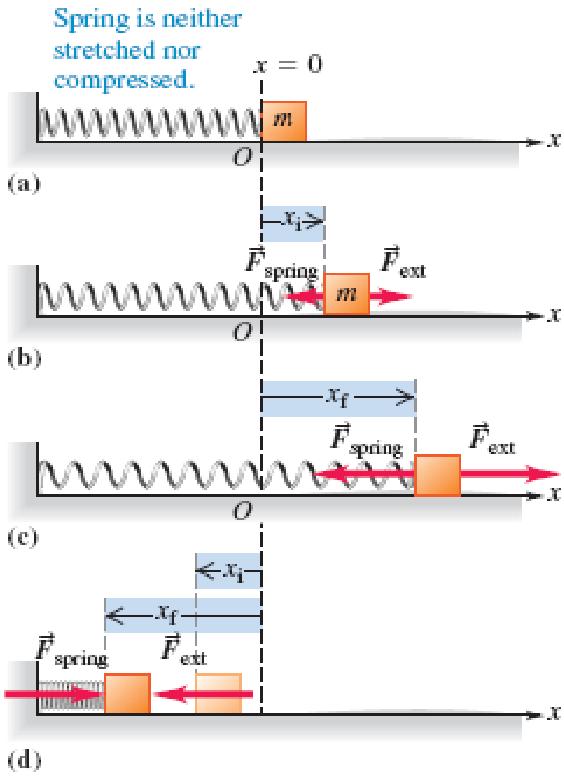


Fig. 6.3: The forces acting on the mass attached to the spring are spring force F_{spring} and external force F_{ext}.

Exercise

Plot the elastic potential energy, U_{ela} versus position, x and discuss about the total mechanical energy.

6.1.4 Gravitational potential energy

In this section we will discuss another form of mechanical energy, called ***potential energy***, associated with the position or configuration of object. Thus, the potential energy of a system of interacting objects represents the ability of the system to do work because of its position or configuration.

To define gravitational potential energy, we consider a physics book of mass m lifted from an initial height $\boldsymbol{y_i}$ to a final height y_f above the surface of the earth, as indicated in Fig. 6.4.

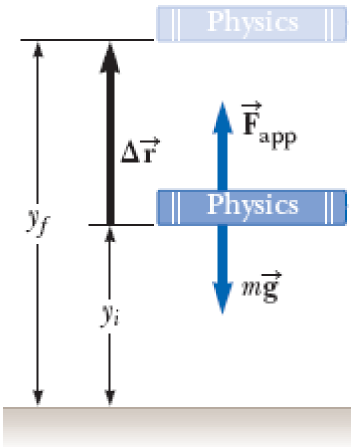


Fig. 6.4: Relating potential energy to work done by a force

The work done by gravitational force as the book is raised from the initial height y_i to a final height y_f from the ground is given by

$$W_g = |\vec{F}_g||\Delta\vec{r}|\cos\theta, \quad (6.9)$$

where $\vec{F}_g = -mg\hat{j}$ is the gravitational force on the book and $\Delta\vec{r} = (y_f - y_i)\hat{j}$ is the displacement and θ is the angle between the gravitational force and the displacement. In Figure 6.4, $\theta = 180°$. It then follows that

$$W_g = -[mgy_f - mgy_i]. \quad (6.10)$$

The quantity $U_g = mgy$ is called gravitational potential energy. Thus $U_{gf} = mgy_f$ is the final gravitational potential energy and $U_{gi} = mgy_i$ is the initial gravitational potential energy of the earth-book system. Accordingly, the work done by gravitational force can be expressed as

$$W_g = -[U_{gf} - U_{gi}] = -\Delta U_g. \quad (6.11)$$

We observe that work done by gravitational force is equal to the negative of the change in gravitational potential energy. When the object moves down, *y* decreases, the gravitational force does *positive* work, and the potential energy *decreases.* When the object moves *up*, the work done by the gravitational force is *negative,* and the potential energy *increases.*

Examples

1. A force $\vec{F} = (4\hat{\imath} + 3\hat{\jmath})N$ acts on an object of mass $m = 2kg$, moving the object by dragging it from origin to $x = 5.0m$. Find the work done on the object and the angle θ between the force and the displacement.

Solution:

The displacement is in the positive x-direction:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (5.0\ m)\hat{\imath} - \mathbf{0} = (5.0m)\hat{\imath}\ ,\ \Delta x = 5m$$

Only the component of the force in the displacement direction does work. Therefore,

$$W = F_x \Delta x = 4N \times 5m = 20J.$$

Using $W = Fd\cos\theta$

$$\theta = cos^{-1}\left(\frac{W}{Fd}\right) = cos^{-1}\left(\frac{20}{25}\right) = 37^0.$$

2. In the above problem, the contact surface on which the object is dragged is rough with coefficient of friction $\mu_k = 0.2$, What is the value of the total work done?

Solution

The friction force acts in opposite direction to the displacement and is given as

Exercises

1. A box is dragged horizontally across a floor by a 100 N force acting parallel to the floor. What is the work done by the force in moving it through a distance of 8 m?
2. A box is dragged across a floor by a 100 N force directed 60° above the horizontal. How much work does the force do in pulling the object 8 m?
3. A horizontal force F pulls a 10 kg cartoon across the floor at constant speed. If the coefficient of sliding friction between the carton and the floor is 0.30, how much work is done by F in moving the carton by 5m?

6.2 Conservation of Energy

Learning Outcomes

After completing this section, students are expected to

- Describe the law of conservation of mechanical energy
- Apply the law of conservation of mechanical energy in solving problems
- Identify whether mechanical energy of a system is conserved or not

In general terms, energy is neither created nor destroyed. This means there is conservation of energy within a universe. However, since energy exists in various forms and it can transform from one form to another, conservation of energy may also be regarded to mean conservation of magnitude of a given form energy.

The law of conservation of mechanical energy states that in the absence of dissipative force such as friction, mechanical energy of a system remains constant or conserved.

That is;

$$\Delta ME = 0$$

Or $ME_f = ME_i$

$$KE_f + U_f = KE_i + U_i \qquad (6.12)$$

On the other hand, in the presence of friction, the change in mechanical energy is equal to the work done by friction. That is;

$$\Delta ME = W_f, \qquad (6.13)$$

where $W_f = -F_f s = -(\mu_k F_N)s$ is work done by friction, in which μ_k is coefficient of kinetic friction and F_N is normal force.

When a particle is under an only gravitational force field, the potential energy is all potential, $U = U_g$ and when the particle is under the action of an only elastic restoring force field, $U = U_{ela}$. For a particle under the action of both gravitational and elastic forces, $U = U_g + U_{ela}$.

Examples

1. A 40.0-kg box initially at rest is pushed 5.00 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. The coefficient of friction between box and floor is 0.300. Find (a) the work done by the applied force, , (b) the work done by the normal force, (c) the work done by the gravitational force, (d) the work done by frictional force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.

Solution

$F_g = mg = 400N, s = 5.00m, \; F_{ap} = 130N, \mu_k = 0.300, v_i = 0,$

a. Since the applied force is constant, work done by this force is given by $W_{ap} = F_{ap} s cos\theta$. Moreover, the applied force is parallel to the direction of motion. Thus, $\theta = 0^0$ and $cos\theta = 1$. The work done by the applied constant horizontal force is $W_{ap} = F_{ap} s = 650J$.
b. The normal force is perpendicular to the direction of motion and thus does no work. That is, work done by the normal force F_N is $W_N = F_N s cos90 = 0$.
c. Just like the normal force, the gravitational force is perpendicular to the direction of motion, and hence $W_g = F_g s cos90 = 0$.
d. The work done by frictional force f_k is $W_f = f_k s cos\theta$, where θ is the angle between the displacement and frictional force. Frictional force is always opposite to the direction of motion and hence $\theta = 180^0$, $cos180^0 = -1$. The work done by friction is thus $W_f = -f_k s = -\mu_k F_N s$. In this example, the normal force on the box is equal to its weight. It then follows
$$W_f = -\mu_k F_N s = -\mu_k mgs = -600J.$$
e. According to the work-energy theorem, the change in the kinetic energy of the box is equal to the total work done on (or work done by net force) on it. That is,
$$\Delta KE = W_{total} = W_{ap} + W_N + W_g + W_f = 50J.$$
f. From $\Delta KE = \frac{1}{2} m\left(v_f^2 - v_i^2\right) = 50J$, we obtain $v_f = 5\frac{m}{s}$.

2. A 10.0-kg block is released from rest at point A in Figure 6.5. The track is frictionless except for the portion between points B and C, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2250 N/m, and compresses the

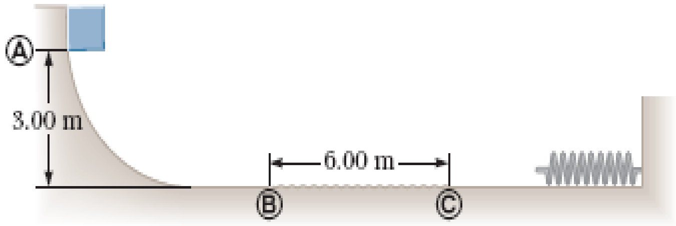


Fig.6.5: Block sliding down a frictionless track toward a rough level surface

spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine (a)The speed of the block at point B, (b)The speed of the block at point C, (c) the coefficient of kinetic friction between the block and the rough surface between points B and C, and (d) The speed of the block after travelling 2.0m on the rough plane.

Solution

$m = 10.0\ kg, v_A = 0, \quad k = 2250\ N/m, \quad x = 0.300m$

(a) The portion of the track from point A to B is frictionless and thus mechanical energy is conserved. That is,

$ME_A = ME_B$ or $KE_A + U_A = KE_B + U_B$.

Choosing the gravitational potential energy to be zero on the horizontal surface, we see that $U_B = 0$. Moreover, since $v_A = 0$, $KE_A = \frac{1}{2}mv_A^2 = 0$. We therefore notice that

$$U_A = KE_B$$

or

$$mgh_A = \frac{1}{2}mv_B^2.$$

Using $h_A = 3.00m$ and $g = 10\ m/s^2$, we obtain $v_B = 7.746\frac{m}{s}$.

(b) Consider motion of the block from point C to a point G, where it momentarily comes to rest after compressing the spring. Since there is no frictional force, mechanical energy of the block-spring system is conserved. That is,

$ME_C = ME_G$.

We observe that $ME_C = \frac{1}{2}mv_c^2$ and $ME_G = \frac{1}{2}kx^2$. The speed at point C is then $v_c = 4.5\frac{m}{s}$.

(c) The coefficient of kinetic friction can be obtained applying the work-energy theorem to the motion from point B to C. One observes that

$KE_C - KE_B = W_{net}$.

Since the net horizontal force on the block is the kinetic friction $f_k = \mu_k mg$, the work done by the net force is

$W_{net} = -\mu_k mgs = -600\mu_k J$.

Using $v_B = 7.746\ m/s$ and $v_c = 4.5\ m/s$, we obtain

$$\boldsymbol{KE_C - KE_B = \frac{1}{2}mv_c^2 - \frac{1}{2}mv_B^2 = -198.75J = -600\mu_k J}$$

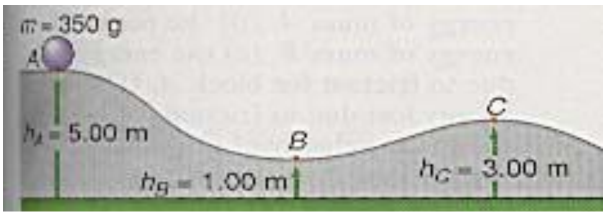


Fig. 6.6: A ball rolling down a smooth track

$\boldsymbol{\mu_k = 0.33}$.

(d) The speed of the block at a point D $2m$ from point B is obtainable using the work-energy theorem. We can then write

$$KE_D - KE_B = W_{net}$$

or

$$\frac{1}{2}mv_D^2 - \frac{1}{2}mv_B^2 = -\mu_k mgs'.$$

Using $s' = 2.0m$, we get $v_D = 6.84\frac{m}{s}$

3. A ball of mass 350 g starts from rest at position A at the top of the smooth track. Find (a) the total energy at A, (b) the total energy at B, (c) the velocity of the ball at B, and (d) the velocity of the ball at C.

Exercises

1. If it takes 4.00 J of work to stretch a Hooke's law spring 10.0 cm from its unstretched length, determine the extra work required to stretch it an additional10.0 cm.
2. An inclined plane of angle $\theta = 30°$ has a spring of force constant k = 500N/m fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure 6.7. Use m=3.5 Kg, a distance d=0.300 m from the spring and neglect friction. From this position, the block is projected downward toward the spring with speed v = 0.750 m/s. By what distance is the spring compressed when the block momentarily comes to rest?

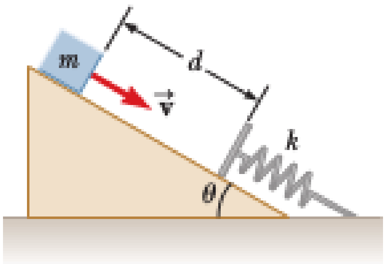


Fig. 6.7

3. In problem 2 above, how would the value of x change if the inclined plane is rough surface with $\mu_k = 0.25$?.

4. Two objects with masses $m_1 = 4Kg$ and $m_2 = 5Kg$, slide past a fixed pulley and are attached together by inextensible rope of length $l = 3m$ (Figure 6.8). If initially the masses are at the same vertical height above ground and are then set into motion, what is the total kinetic energy of the masses when the vertical separation between them becomes 0.6m?

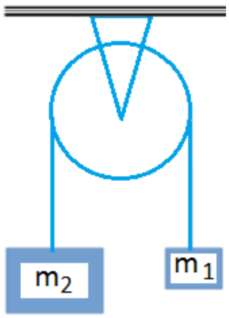


Fig. 6.8

6.3 Power

Time considerations aren't involved directly in the definition of work. If you lift a barbell weighing 400 N through a vertical distance of 0.5 m at constant velocity, you do 200 J of work on it, whether it takes you 1 second, 1 hour, or 1 year to do it. Often, though, we need to know how quickly work is done. The time rate at which work is done or energy is transferred is called power. Like work and energy, power is a scalar quantity. We define average power as follows:

When a quantity of work ΔW is done during a time interval $\Delta \boldsymbol{t}$, the average power P_{ave}, or work per unit time, is defined as

$$P_{ave} = \frac{\Delta W}{\Delta t} \qquad (6.13)$$

The SI unit of power is watts (W), where 1W=1J/s. For many applications, power is measured in kilowatts or megawatts (1 MW = 106 W). Power units can be used to define new units of work or energy. The kilowatt-hour (kWh) is the usual commercial unit of electrical energy. One kilowatt-hour is the total work done in 1 hour (3600 s) when the power is 1 kilowatt (10^3 J/s). so

$$1kWh = (10^3 J/s)\,(3600\ s) = 3.6\ MJ.$$

The kilowatt-hour is a unit of work or energy, not power.

In the limit of , the average power becomes instantaneous power. Suppose a force F parallel to the displacement is applied on an object. The work done on the object by this force when the object is displaced by $\Delta \boldsymbol{s}$ is $\Delta W = F(\Delta s)$ and the average power is expressible as

$$P_{ave} = \frac{\Delta W}{\Delta t} = F\left(\frac{\Delta s}{\Delta t}\right) = F v_{ave}\ .$$

The instantaneous power is the limit of P_{ave} as the time interval $\Delta \boldsymbol{t}$ approaches zero. It the follows that

$$P(t) = Fv(t) \tag{6.14}$$

where v is the instantaneous speed.

6.4 Chapter Summary

The work done on a particle by a constant force $\vec{F}$ during displacement $\vec{s}$ is $W = Fscos\theta$ (work done by a constant force), in which θ is the constant angle between the directions of $\vec{F}$ and $\vec{s}$.Only the component of $\vec{F}$ that is along the displacement $\vec{s}$ can do work on the object.

Work done can be positive, negative or zero depending on the angle θ between the force and the displacement. The work done is positive when the force has a component in the same direction as the displacement ($0^0 \leq \theta < 90^0$). When the force has a component opposite to the displacement ($90^0 < \theta \leq 180^0$) , the work done is negative. When the force is perpendicular to the displacement, $\theta = 90^0$ and the work done by the force is zero.

Work is said to be done on an object by a force if (a) the force is not perpendicular to the displacement ($\theta \neq 90^0$); and (b) the force displaces the object.

Work-energy theorem: For a particle, a change ΔKE in the kinetic energy equals the work W_{net} done by the net force on the particle:

$$\Delta KE = KE_f - KE_i = W_{net} \text{ (work-kinetic energy theorem)},$$

in which KE_i is the initial kinetic energy of the particle and KE_f is the final kinetic energy.

Work done by a spring force: If an object is attached to the spring's free end, the work W_{sp} done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_{sp} = -\left[\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right].$$

Elastic potential energy of a spring stretched or compressed by an amount x is

$$U_{ela} = \frac{1}{2}kx^2$$

Gravitational potential energy of a mass m at a height h from the surface of the earth is $U_g = mgh$

Work done by gravitational force is equal to the negative of the change in gravitational potential energy. That is, $W_g = -[mgy_f - mgy_i] = -\Delta U_g$.

The law of conservation of mechanical energy states that "In the absence of friction force, the total mechanical energy (kinetic plus potential) is constant"; that is,

$$KE_f + U_f = KE_i + U_i,$$

where *U* may include both gravitational and elastic potential energies.

When friction forces exist, the change in mechanical energy of a system is equal to the work done by the frictional force. That is,

$\Delta KE + \Delta U = W_f$.

Power is the rate of doing work.

6.5 Conceptual Questions

1. Frictional force is an example of a non-conservative force. Describe a situation in work done by frictional forces are useful and harmful.
2. A man carries a hand bag by hanging on his hand and moves horizontally where the bag does not move up or down. What is the work done on the bag? The man gets tired after some time of the movement. Why?

6.6 Problems

1. Persons A, B ,andC can do a piece of work in 20, 30, and 60 days, respectively. In how many days can A do the work if he is assisted by B and C on every third day? a) 12 days b) 15 days c) 16 days d) 18 days.
2. In which case in Figure H1 would the work done be zero, negative? (F denotes force andd denotes displacement).

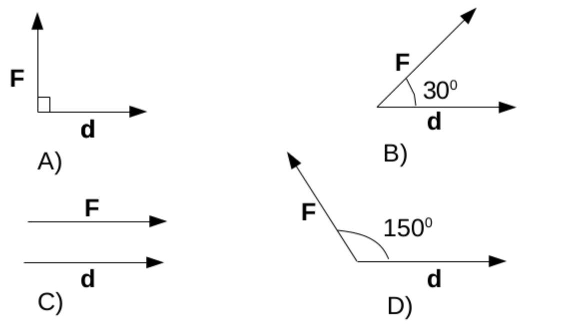


Figure H1.

3. Determine the potential energies at points A, B, C, D, and E in Figure H2. Assume the ball bounces down the stairs from a potential energy at the top of the plane being 50 J.

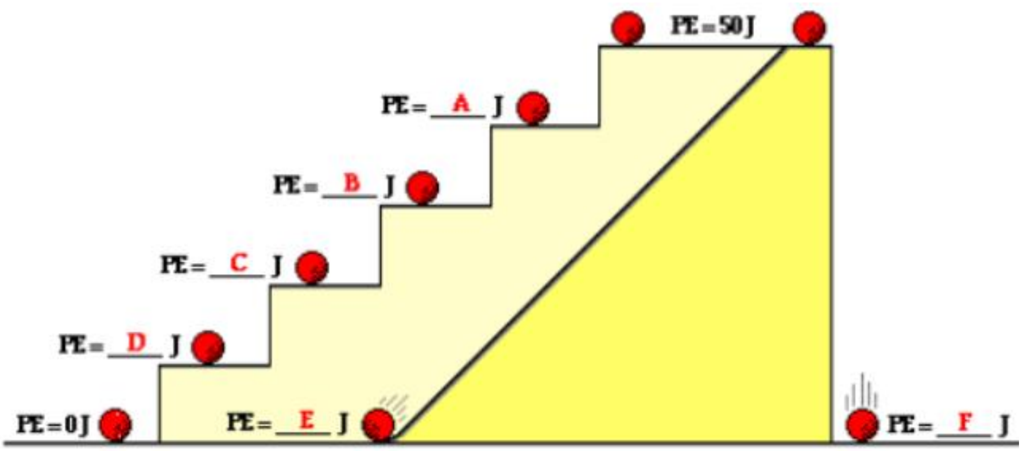


Figure H2.

4. Compare the work done values indicated in Figure H3. Assume the block starts from rest at the top of the hill and skids on a frictionless surface with constant velocity after reaching the bottom of the hill.

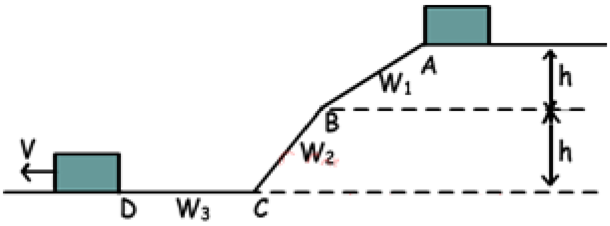


Figure H3.

5. A 5.00- kg particle starts from the origin at time zero. Its velocity as a function of time is given by $\vec{v} = 6t^2\hat{\imath} + 2t\hat{\jmath}$ where $\vec{v}$ is in meters per second and t is in seconds. Determine the power injected into the system of the particle as a function of time.
6. An 8.0 kg block is moving at 3.2 m/s. A net force of 10 N is constantly applied on the block in the direction of its movement, until it has moved 16 m. What is the approximate final velocity of the block?
7. Suppose a man is pulling a loaded 50.0-kg sled on a snow. The coefficient of kinetic friction between the sled and snow is 0.2. (a) If the man again pulls the sled 5.00 m by exerting a force of 1.20 x 10^2 N at an angle of 0°. Find the work done on the sled by friction, and the net work done. (b) Repeat the calculation if the applied force is exerted at an angle of 30.0° with the horizontal.
8. A constant force of 35 N is applied to an object at an angle of $45^0$with the horizontal. If the object is pulled 12 m at an angle of $15^0$with the horizontal, how much work is done in the process of moving the object?
9. What is the minimum amount of work in Joules that a 60 kg man must do in order to climb up a tree house that is 3 meters high?
10. A body of mass m=2 kg is dropped from h=70 cm above a horizontal platform that is fixed to one end of an elastic spring, as shown in the Figure E1. As a result, the spring is compressed by an amount of Δy = 20 cm. What is the spring constant of the spring?
11. The gravitational acceleration is g=9.8 m/s2 and the air resistance is negligible. What is the total mechanical energy of the ball by the time the spring is compressed by 10 cm?

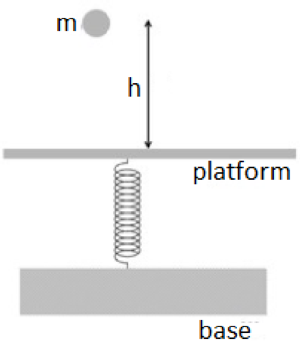


Figure E1.

12. A mass m=3 kg that lies on a horizontal surface is connected to another mass M=6 kg, as shown in Figure E2. The coefficient of kinetic friction between mass m and the surface is μk=0.1. If the system is released from rest, what is the squared velocity of mass m, when mass M as descended a distance of h=9m? The gravitational acceleration is g=9.8 m/s2.

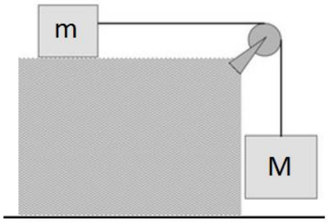


Figure E2.

13. In the Figure E3, a 2.0kg package slides along a floor with speedv1 = 4.0m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic friction force from the floor, of magnitude 15 N, acts on it. The spring constant is 10,000 N/m. By what distance d is the spring compressed when the package stops? What is the average value of power dissipated by the friction force during this time interval?

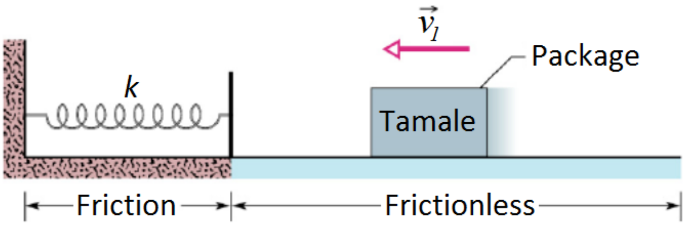


Figure E3

14. Two celestial bodies have masses $m_1 = 6x10^{24}Kg$ and $m_2 = 7.3x10^{22}Kg$ have the distance between them changed from $r_1 = 1700Km$ to $r_2 = 1750Km$ during a period of one quarter of a year. Determine the work done by the gravitational interaction force and the average power during the duration.

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7 Linear Momentum

Learning Outcome

After completing this Chapter, students are expected to:

- Define linear momentum,
- Explain the relationship between momentum and force,
- State kinetic energy of motion in terms of momentum
- Relate collision and momentum
- Solve problems related to various types of collision

Introduction

What happens when two automobiles collide? To begin answering such questions, we introduce *momentum.* Intuitively, anyone or anything that has a lot of momentum is going to be hard to stop. Physically, the more momentum an object has, the more force has to be applied to stop it in a given time. This concept leads to one of the most powerful principles in physics: *conservation of momentum.* Using this law, complex collision problems can be solved without knowing much about the forces involved during contact. We'll also be able to derive information about the average force delivered in an impact. With conservation of momentum, we'll have a better understanding of what choices to make when designing an automobile or a rocket, or when addressing a football or basketball training.

7.1 The Concept of Momentum and Impulse

Learning outcome

After completing this section, students are expected to:

- Define linear impulse.
- Explain the relationship between momentum and impulse.
- Calculate momentum given mass and velocity.
- Describe effects of impulses in everyday life.
- Solve simple problems involving momentum and impulse

The linear momentum (or simply momentum) $\vec{\boldsymbol{p}}$ of a body of mass *m* moving with velocity $\vec{\boldsymbol{v}}$ is defined as

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}} \tag{7-1}$$

Doubling either the mass or the velocity of an object doubles its momentum; doubling both quantities quadruples its momentum. Momentum is a vector quantity. Its components in Cartesian coordinates are

$$p_x = mv_x, \; p_y = mv_y \; \text{ and } \; p_z = mv_z \tag{7-2}$$

where $p_x$is the momentum of the object in the *x* – direction, $p_y$its momentum in the *y* – direction and $p_z$its momentum in the *z* – direction.

The magnitude of the momentum

$$p = \sqrt{{p_x}^2 + {p_y}^2 + {p_z}^2} \tag{7-3}$$

of an object of mass m can be related to its kinetic energy E_K:

$$E_K = \frac{p^2}{2m} \tag{7-4}$$

This relationship is easy to prove using the definitions of kinetic energy and momentum and is valid for objects travelling at speeds much less than the speed of light. Curiously, there is no named SI unit for measuring momentum. Momentum in SI units is measured in units of kg m/s.

Changing the momentum of an object requires the application of a force. This is, in fact, how Newton originally stated his second law of motion. Starting from the more common version of the second law, we have

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$$

where the mass m and the force are assumed constants. The quantity in parentheses is just the momentum, so we have the following result:

The change in an object's momentum $\Delta\vec{p}$ divided by the elapsed time Δt equals the constant net force $\vec{F}_{net}$ acting on the object:

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change of mumentum}}{\text{time interval}} = \vec{F}_{net} \tag{7-5}$$

This equation is also valid when the forces are not constant, provided the limit is taken as Δt becomes infinitesimally small. Equation (7.5) says that if the net force on an object is zero, the object's momentum doesn't change. In other words, the linear momentum of an object is conserved when $\vec{F}_{net} = 0$. Equation (7.5) also shows us that changing an object's momentum requires the continuous application of a force over a period of time Δt leading to the definition of impulse. If a constant force $\vec{F}$ acts on an object, the impulse $\vec{I}$ delivered to the object over a time interval Δt is given by

$$\vec{I} = \vec{F}\Delta t \tag{7-6}$$

Impulse is a vector quantity with the same direction as the constant force acting on the object. When a single constant force $\vec{F}$ acts on an object, (7.6) can be written as

$$\vec{I} \equiv \vec{F}\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i \qquad (7\text{-}7)$$

which is a special case of the impulse–momentum theorem. Equation (7.7) shows that the impulse of the force acting on an object equals the change in momentum of that object. That equality is true even if the force is not constant, as long as the time interval Δt is taken to be arbitrarily small.

In real-life situations, the force on an object is only rarely constant. To analyze the case of variable force with rather complex interaction of bodies, it's useful to define an average force $\vec{F}_{av}$. The average force is the constant force delivering the same impulse to the object in the time interval Δt as the actual time-varying force. We can then write the impulse–momentum theorem as

$$\vec{F}_{av}\Delta t = \Delta\vec{p} \qquad (7\text{-}8)$$

Eample7.1 Teeing off:

A golf ball with mass 5.0×10^{-2}kg is struck with a club as in Figure 7.1. The force on the ball varies from zero when contact is made up to some maximum value (when the ball is maximally deformed) and then back to zero when the ball leaves the club. Assume that the ball leaves the club face with a velocity of 44 m/s. (a) Find the magnitude of the impulse due to the collision. (b) Estimate the duration of the collision and the average force acting on the ball.

Figure 7-1 (Example 7.1) During impact, the club head momentarily flattens the side of the golf ball

Solution

(a) Find the impulse delivered to the ball. The problem is essentially one dimensional. Note that v_i= 0, and calculate the change in momentum, which equals the impulse:

$$I = \Delta p = p_f - p_i = (5.0 \times 10^{-2}\text{kg})\left(44\frac{\text{m}}{\text{s}}\right) - 0 = 2.2 \text{ kg m/s}$$

(b) Estimate the duration of the collision and the average force acting on the ball. Estimate the time interval of the collision, Δt, using the approximate displacement (radius of the ball) and its average speed (half the maximum speed):

$$\Delta t = \frac{\Delta x}{v_{av}} = \frac{2 \times 10^{-2}\text{m}}{22 \text{ m/s}} = 9.1 \times 10^{-4}\text{s}$$

Estimate the average force from Equation (7.8):

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{2.2 \text{ kg m/s}}{9.1 \times 10^{-4}\text{s}} = 2.4 \times 10^{3}\text{N}$$

REMARKS: This estimate shows just how large such contact forces can be. A good golfer achieves maximum momentum transfer by shifting weight from the back foot to the front foot, transmitting the body's momentum through the shaft and head of the club. This timing, involving a short movement of the hips, is more effective than a shot powered exclusively by the arms and shoulders. Following through with the swing ensures that the motion isn't slowed at the critical instant of impact.

Exercises

1. What average club speed would double the average force? (Assume the final velocity is unchanged.)
2. A 0.150-kg baseball, thrown with a speed of 40.0 m/s, is hit straight back at the pitcher with a speed of 50.0 m/s. (a) What is the magnitude of the impulse delivered by the bat to the baseball? (b) Find the magnitude of the average force exerted by the bat on the ball if the two are in contact for 2.00×10^{-3}s. ANSWERS (a) 13.5 kg m/s (b) 6.75 kN

BOXING AND BRAIN INJURY (application of impulse)

Boxers in the nineteenth century used their bare fists. In modern boxing, fighters wear padded gloves. How do gloves protect the brain of the boxer from injury? Also, why do boxers often "roll with the punch"?

EXPLANATION: The brain is immersed in a cushioning fluid inside the skull. If the head is struck suddenly by a bare fist, the skull accelerates rapidly. The brain matches this acceleration only because of the large impulsive force exerted by the skull on the brain. This large and sudden force (large F_{av} and small Δt) can cause severe brain injury. Padded gloves extend the time Δt over which the force is applied to the head. For a given impulse $F_{av}\Delta t$, a glove results in a longer time interval than a bare fist, decreasing the average force. Because the average force is decreased, the acceleration of the skull is decreased, reducing (but not eliminating) the chance of brain injury. The same argument can be made for "rolling with the punch": If the head is held steady while being struck, the time interval over which the force is applied is relatively short and the average force is large. If the head is allowed to move in the same direction as the punch, the time interval is lengthened and the average force reduced.

Eample7.2

How good are the bumpers? In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in Figure 7.2a. The initial and final velocities of the car are v_i=-15.0 m/s and v_f= 2.60 m/s, respectively. If the collision lasts for 0.150 s, find (a) the impulse delivered to the car due to the collision and (b) the magnitude and direction of the average force exerted on the car.

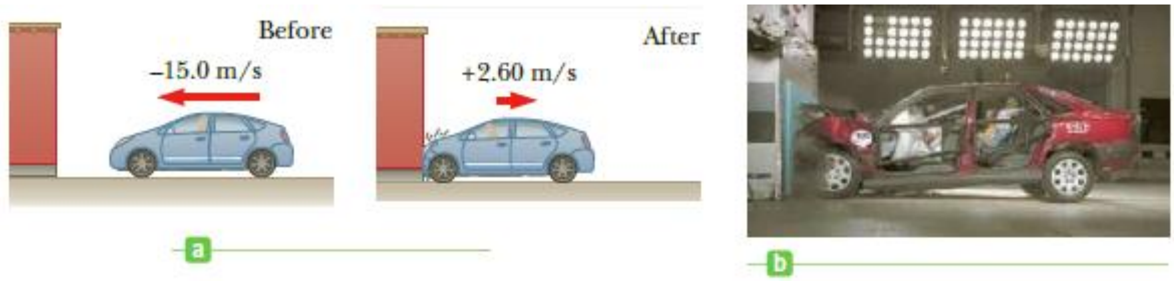


Figure 7-2 Example 7.2) (a) This car's momentum changes as a result of its collision with the wall. (b) In a crash test (an inelastic collision), much of the car's initial kinetic energy is transformed into the energy it took to damage the vehicle

Solution:

(a) Find the impulse delivered to the car.

Calculate the initial and final momenta of the car

$$\boldsymbol{p_i = mv_i = (1.5 \times 10^3\ \text{kg})(-15.0\ \text{m/s}) = -2.25 \times 10^4\ \text{kg m/s}}$$

$$\boldsymbol{p_f = mv_f = (1.5 \times 10^3\ \text{kg})(2.6.0\ \text{m/s}) = 0.39 \times 10^4\ \text{kg m/s}}$$

The impulse is just the difference between the final and initial momenta:

$$\boldsymbol{I = p_f - p_i = [(0.39) - (-2.25)] \times 10^4\ \text{kg m/s} = 2.64 \times 10^4\ \text{kg m/s}}$$

(b) Find the average force exerted on the car. Apply Equation (7.8), the impulse–momentum theorem

$$\boldsymbol{F_{av} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4\ \text{kg m/s}}{0.150\ \text{s}} = 1.76 \times 10^5\ \text{N}}$$

REMARKS: If the car doesn't rebound off the wall, the average force exerted on the car is smaller than the value just calculated. With a final momentum of zero, the car undergoes a smaller change in momentum. During the collision the state of the wall is hardly changed due to the fact it has much larger mass compared to that of the car. Similarly, although a ball has greater velocity, a player has much greater mass, and thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Exercises

1. When a person is involved in a car accident, why is the likelihood of injury greater in a head-on collision as opposed to being hit from behind? Answer using the concepts of relative velocity, momentum, and average force.
2. Suppose the car doesn't rebound off the wall, but the time interval of the collision remains at 0.150 s. In this case, the final velocity of the car is zero. Find the average force exerted on the car. ANSWER $+\mathbf{1.5} \times \mathbf{10^5}\ \mathbf{N}$

7.2 Conservation of Momentum

Learning outcome

After completing this section, students are expected to:

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Solve simple problems involving conservation of momentum

When a collision occurs in an isolated system, the total momentum of the system doesn't change with the passage of time. Instead, it remains constant both in magnitude and in direction. The momenta of the individual objects in the system may change, but the vector sum of all the momenta will not change. The total momentum is therefore said to be conserved. In this section, we will see how the laws of motion lead us to this important conservation law.

A collision may be the result of physical contact between two objects, as illustrated in Figure 7.3a. This is a common macroscopic event, as when a pair of billiard balls or a baseball and a bat strike each other. By contrast, because contact on a submicroscopic scale is hard to define accurately, the notion of collision must be generalized to that scale. Forces between two objects arise from the electrostatic interaction of the electrons in the surface atoms of the objects. Electric charges are either positive or negative. Charges with the same sign repel each other, while charges with opposite sign attract each other. To understand the distinction between macroscopic and microscopic collisions, consider the collision between two positive charges, as shown in Figure 7.3b. Because the two particles in the figure are both positively charged, they repel each other. During such a microscopic collision, particles need not touch in the normal sense in order to interact and transfer momentum.

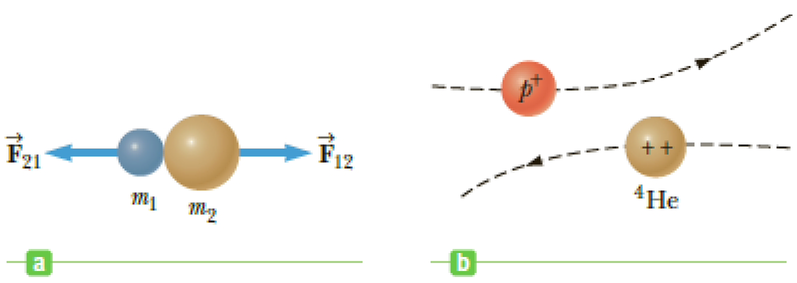


Figure 7-3(a) A collision between two objects resulting from direct contact. (b) A collision between two charged objects (in this case, a proton and a helium nucleus).

Figure 7.4 shows an isolated system of two particles before and after they collide. By "isolated," we mean that no external forces, such as the gravitational force or friction, act on the system. Before the collision, the velocities of the two particles are $\vec{v}_{1i}$ and $\vec{v}_{2i}$; after the collision, the velocities are $\vec{v}_{1f}$ and $\vec{v}_{2f}$. The impulse–momentum theorem applied to m_1 becomes

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

Likewise, for m_2, we have

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

where $\vec{F}_{21}$ is the average force exerted by m_2 on m_1 during the collision and $\vec{F}_{12}$ is the average force exerted by m_1 on m_2 during the collision, as in Figure 7.3a.

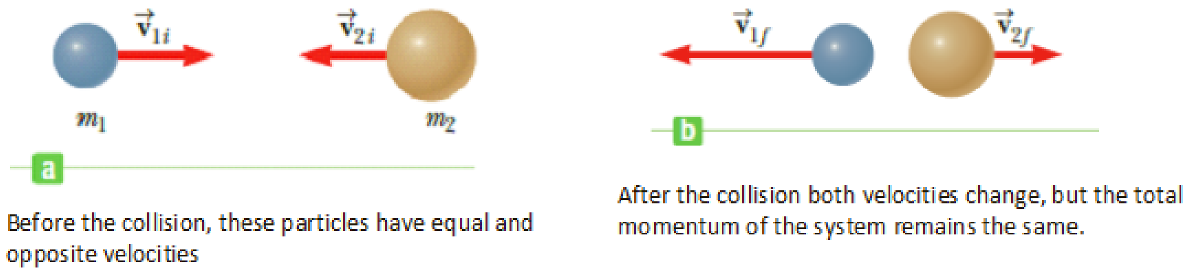


Figure*7-4:* Before and after a head-on collision between two particles. The momentum of each object changes during the collision, but the total momentum of the system is constant. Notice that the magnitude of the change of velocity of the lighter particle is greater than that of the heavier particle, which is true in general.

We use average values for $\vec{F}_{21}$ and $\vec{F}_{12}$ even though the actual forces may vary in time in a complicated way. Newton's third law states that at all times these two forces are equal in magnitude and opposite in direction: $\vec{F}_{21} = -\vec{F}_{12}$.

In addition, the two forces act over the same time interval. As a result, we have

$$\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$$

or

$$m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$$

after substituting the expressions obtained for $\vec{F}_{21}$ and $\vec{F}_{12}$. This equation can be rearranged to give the following important result:

$$m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i} \qquad (7\text{-}9)$$

This result is a special case of the law of conservation of momentum and is true of isolated systems containing any number of interacting objects.

When no net external force acts on a system, the total momentum of the system remains constant in time.

Defining the isolated system is an important feature of applying this conservation law. A cheerleader jumping upwards from rest might appear to violate conservation of momentum, because initially her momentum is zero and suddenly she's leaving the ground with velocity *v*. The flaw in this reasoning lies in the fact that the cheerleader isn't an isolated system. In jumping, she exerts a downward force on Earth, changing its momentum. This change in Earth's momentum isn't noticeable, however, because of Earth's gargantuan mass compared to the cheerleader's. When we define the system to be *the cheerleader and Earth,* momentum is conserved.

Action and reaction, together with the accompanying exchange of momentum between two objects, is responsible for the phenomenon known as *recoil.* Everyone knows that throwing a baseball while standing straight up, without bracing one's feet against Earth, is a good way to fall over backwards. This reaction, an example of recoil, also happens when you fire a gun or shoot an arrow. Conservation of momentum provides a straightforward way to calculate such effects, as the next example shows.

Example 7.3 The Archer: An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60.00 kg. (See Fig. 7.5.) (a) If the archer fires a 0.030 0-kg arrow horizontally at 50.0 m/s in the positive *x* - direction, what is his subsequent velocity across the ice? (b) He then fires a second identical arrow at the same speed relative to the ground but at an angle of 30.0° above the horizontal. Find his new speed. (c) Estimate the average normal force acting on the archer as the second arrow is accelerated by the bowstring. Assume a draw length of 0.800 m

Figure 7-5(Example 7.3) An archer fires an arrow horizontally to the right. Because he is standing on frictionless ice, he will begin to slide to the left across the ice.

Solution

(a) Find the archer's subsequent velocity across the ice. Write the conservation of momentum equation for the *x* - direction. $\boldsymbol{p_i} = \boldsymbol{p_f}$

Let m_1 and v_{1f} be the archer's mass and velocity after firing the arrow, respectively, and m_2 and v_{2f} the arrow's mass and velocity. Both velocities are in the *x* - direction. Substitute $p_i = 0$ and expressions for the final momenta:

$$\mathbf{0 = m_1 v_{1f} + m_2 v_{2f}}$$

Solve for v_{1f} and substitute m_1 = 59.97 kg, m_2 = 0.030 0 kg, and v_{2f} = 50.0 m/s:

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\left(\frac{0.0300\ \mathrm{kg}}{59.97\ \mathrm{kg}}\right) 50.0\ \frac{\mathrm{m}}{\mathrm{s}} = -0.0250\ \frac{\mathrm{m}}{\mathrm{s}}$$

(b) Calculate the archer's velocity after he fires a second arrow at an angle of 30.0° above the horizontal.

Write the *x* - component of the momentum equation with m_1 again the archer's mass after firing the first arrow as in part (a) and m_2 the mass of the next arrow:

$$m_1\, v_{1i} = (m_1 - m_2) v_{1f} + m_2 v_{2f} \cos\theta$$

Solve for v_{1f}, the archer's final velocity, and substitute: m_1 = 59.97 kg, m_2 = 0.030 0 kg, and v_{2f} = 50.0 m/s, $\boldsymbol{v_{1i} = 0.0250\ \mathrm{m/s}}$

$$v_{1f} = \frac{m_1}{(m_1 - m_2)} v_{1i} - \frac{m_2}{(m_1 - m_2)} v_{2f} \cos\theta$$

$$v_{1f} = \frac{59.97\ \mathrm{kg}}{59.94\ \mathrm{kg}}\left(-0.0250\frac{\mathrm{m}}{\mathrm{s}}\right) - \frac{0.0300\ \mathrm{kg}}{59.94\ \mathrm{kg}}\left(50\frac{\mathrm{m}}{\mathrm{s}}\right)\cos 30 = -0.0467\frac{\mathrm{m}}{\mathrm{s}}$$

(c) Estimate the average normal force acting on the archer as the arrow is accelerated by the bowstring.

Use kinematics in one dimension to estimate the acceleration of the arrow:

$$v^2 - v_0^2 = 2a\Delta x$$

Solve for the acceleration and substitute values setting v=v_{2f}, the final velocity of the arrow:

$a = \frac{v^2 - v_0^2}{2\Delta x} + \frac{(50\ \mathrm{m/s})^2 + 0}{2(0.800\ \mathrm{m})} = 1.56 \times 10^3 \mathrm{m/s^2}$

Find the time the arrow is accelerated using

$$v = at + v_0 : t = \frac{v - v_0}{a} = \frac{50\ \mathrm{m/s}}{1.56 \times 10^3 \mathrm{m/s^2}} = 0.0320\ \mathrm{s}:$$

Write the y - component of the impulse–momentum theorem:

$$F_{y,av}\Delta t = \Delta p_y,\ F_{y,av} = \frac{\Delta p_y}{\Delta t} = \frac{m_2 v_{2f} \sin\theta}{\Delta t}$$

$$F_{y,av} = \frac{(0.0300\ \mathrm{kg})(50.0\ \mathrm{m/s})\sin 30^0}{0.0320\ \mathrm{s}} = 23.4\ \mathrm{N}$$

The average normal force is given by the archer's weight plus the reaction force R of the arrow on the archer: $\sum F_y = N - mg - R = 0$,

$$N = mg + R = (59.94\ \mathrm{kg})\left(9.80\frac{\mathrm{m}}{\mathrm{s^2}}\right) + (23.4\ \mathrm{N}) = 6.11 \times 10^2\ \mathrm{N}$$

REMARKS: The negative sign on v_{1f}indicates that the archer is moving opposite the arrow's direction, in accordance with Newton's third law. Because the archer is much more massive than the arrow, his acceleration and velocity are much smaller than the acceleration and velocity of the arrow. A technical point: the second arrow was fired at the same velocity relative to the ground, but because the archer was moving backwards at the time, it was travelling slightly faster than the first arrow relative to the archer. Velocities must always be given relative to a frame of reference.

Notice that conservation of momentum was effective in leading to a solution in parts (a) and (b). The final answer for the normal force is only an average because the force exerted on the arrow is unlikely to be constant. If the ice really were frictionless, the archer would have trouble standing. In general, the coefficient of static friction of ice is more than sufficient to prevent sliding in response to such small recoils.

Exercise

1. Would firing a heavier arrow necessarily increase the recoil velocity? Explain.
2. A 70.0-kg man and a 55.0-kg woman holding a 2.50-kg purse on ice skates stand facing each other. (a) If the woman pushes the man backwards so that his final speed is 1.50 m/s, with what average force did she push him, assuming they were in contact for 0.500 s? (b) What is the woman's recoil speed? (c) If she now throws her 2.50-kg purse at him at a 20.0° angle

above the horizontal and at 4.20 m/s relative to the ground, what is her subsequent speed?
ANSWERS (a) 2.10×10^2 N(b) 1.83 m/s (c) 2.09 m/s

7.3 Collisions in One Dimension and in Two Dimensions

Learning outcome

After completing this section, students are expected to:

- Identify the types of collision
- Define inelastic collision.
- Explain perfectly inelastic collision
- Apply an understanding of collisions to everyday life
- Describe an elastic collision of two objects in one dimension.
- Solve simple problems involving collisions

We have seen that for any type of collision, the total momentum of the system just before the collision equals the total momentum just after the collision as long as the system may be considered isolated. The total kinetic energy, on the other hand, is generally not conserved in a collision because some of the kinetic energy is converted to internal energy, sound energy, and the work needed to permanently deform the objects involved, such as cars in a car crash. We define an inelastic collision as a collision in which momentum is conserved, but kinetic energy is not. The collision of a rubber ball with a hard surface is inelastic, because some of the kinetic energy is lost when the ball is deformed during contact with the surface. When two objects collide and stick together, the collision is called *perfectly inelastic.* For example, if two pieces of putty collide, they stick together and move with some common velocity after the collision. If a meteorite collides head on with Earth, it becomes buried in Earth and the collision is considered perfectly inelastic. Only in very special circumstances is all the initial kinetic energy lost in a perfectly inelastic collision.

An elastic collision is defined as one in which both momentum and kinetic energy are conserved. Billiard ball collisions and the collisions of air molecules with the walls of a container at ordinary temperatures are highly elastic. Macroscopic collisions such as those between billiard balls are only approximately elastic, because some loss of kinetic energy takes place—for example, in the clicking sound when two balls strike each other. Perfectly elastic collisions do occur, however, between atomic and subatomic particles. Elastic and perfectly inelastic collisions are *limiting* cases; most actual collisions fall into a range in between them.

As a practical application, an inelastic collision is used to detect glaucoma, a disease in which the pressure inside the eye builds up and leads to blindness by damaging the cells of the retina. In this application, medical professionals use a device called a *tonometer* to measure the pressure inside the eye. This device releases a puff of air against the outer surface of the eye and measures the speed of the air after reflection from the eye. At normal pressure, the eye is slightly spongy, and the pulse is reflected at low speed. As the pressure inside the eye increases, the outer surface becomes more rigid, and the speed of the reflected pulse increases. In this way, the speed of the reflected puff of air can measure the internal pressure of the eye.

We can summarize the types of collisions as follows:

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved but kinetic energy is not.
- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.

In the remainder of this section, we will treat perfectly inelastic collisions and elastic collisions in one dimension

Before a perfectly inelastic collision the objects move independently.

After the collision the objects remain in contact. System momentum *is* conserved, but system energy is *not* conserved

Figure 7-6(a) Before and (b) after a perfectly inelastic head-on collision between two objects.

7.3.1 Perfectly Inelastic Collisions

Consider two objects having masses m_1 and m_2 moving with known initial velocity components $v_{1i}$and v_{2i}along a straight line, as in Figure 7.6a. If the two objects collide head-on, stick together, and move with a common velocity component v_f after the collision, then the collision is perfectly inelastic (Fig. 7.6b). Because the total momentum of the two-object isolated system before the collision equals the total momentum of the combined-object system after the collision, we can solve for the final velocity using conservation of momentum alone:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad (7\text{-}10)$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} \qquad (7\text{-}11)$$

It's important to notice that $\boldsymbol{v_{1i}}$, $\boldsymbol{v_{2i}}$, and $\boldsymbol{v_f}$represent the *x* - components of the velocity vectors, so care is needed in entering their known values, particularly with regard to signs. For example, in Figure 7.6a, $\boldsymbol{v_{1i}}$would have a positive value (m_1 moving to the right), whereas $\boldsymbol{v_{2i}}$would have a negative value (m_2 moving to the left). Once these values are entered, Equation (7.11) can be used to find the correct final velocity, as shown in Examples 7.4 and 7.5.

Example 7.4

A Truck Versus a Compact: A pickup truck with mass $\mathbf{1.80 \times 10^3\ kg}$ traveling eastbound at +15.0 m/s, while a compact car with mass $\mathbf{9.00 \times 10^2\ kg}$ is traveling westbound at -15.0 m/s. (See Fig. 7.7.) The vehicles collide head-on, becoming entangled. (a) Find the speed of the entangled vehicles after the collision. (b) Find the change in the velocity of each vehicle. (c) Find the change in the kinetic energy of the system consisting of both vehicles.

Figure 7-7(Example 7.4)

Solution

(a) Find the final speed after collision.

Let m_1 and v_{1i} represent the mass and initial velocity of the pickup truck, while m_2 and v_{2i} pertain to the compact. Apply conservation of momentum: $\boldsymbol{p_i = p_f}$

Substitute the values and solve for the final velocity,

$$\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

$$\boldsymbol{v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)} = \frac{(1.80 \times 10^3\ \mathrm{kg})(15.0\ \mathrm{m/s}) + (9.00 \times 10^2\ \mathrm{kg})(215.0\ \mathrm{m/s})}{(1.80 \times 10^3\ \mathrm{kg} + 9.00 \times 10^2\ \mathrm{kg})} = 5.0\ \mathrm{m/s}}$$

(b) Find the change in velocity for each vehicle.

Change in velocity of the pickup truck is

$$\boldsymbol{\Delta v_1 = v_f - v_{1i} = 5.00\frac{\mathrm{m}}{\mathrm{s}} - 15.0\frac{\mathrm{m}}{\mathrm{s}} = -10.0\ \mathrm{m/s}}$$

Change in velocity of the compact car is

$$\boldsymbol{\Delta v_2 = v_f - v_{2i} = 5.00\frac{\mathrm{m}}{\mathrm{s}} - (-15.0)\frac{\mathrm{m}}{\mathrm{s}} = +20.0\ \mathrm{m/}}$$

(c) Find the change in kinetic energy of the system.

Calculate the initial kinetic energy of the system:

$$\boldsymbol{E_{Ki} = \frac{1}{2}\left(m_1 v_{1i}^2 + m_2 v_{2i}^2\right) = \frac{1}{2}\left((1.80 \times 10^3\ \mathrm{kg})(15.0\ \mathrm{m/s})^2 + (9.0 \times 10^2\ \mathrm{kg})(215.0\ \mathrm{m/s})^2\right)}$$

$$E_{Ki} = 33.04 \times 10^5 \text{J}$$

Calculate the final kinetic energy of the system and the change in kinetic energy,

$$E_{Kf} = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}\left((1.80 \times 10^3 \text{ kg}) + (9.0 \times 10^2 \text{ kg})(5.0 \text{ m/s})^2\right)$$

$$E_{Kf} = 3.38 \times 10^4 \text{J}$$

$$\Delta E_{Ki} = E_{Kf} - E_{Ki} = -2.70 \times 10^5 \text{J}$$

REMARKS: During the collision, the system lost almost 90% of its kinetic energy. The change in velocity of the pickup truck was only 10.0 m/s, compared to twice that for the compact car. This example underscores perhaps the most important safety feature of any car: its mass. Injury is caused by a change in velocity, and the more massive vehicle undergoes a smaller velocity change in a typical accident.

Exercises

1. If the mass of both vehicles were doubled, how would the final velocity be affected? The change in kinetic energy?
2. Suppose the same two vehicles are both travelling eastward, the compact car leading the pickup truck. The driver of the compact car slams on the brakes suddenly, slowing the vehicle to 6.00 m/s. If the pickup truck travelling at 18.0 m/s crashes into the compact car, find (a) the speed of the system right after the collision, assuming the two vehicles become entangled, (b) the change in velocity for both vehicles, and (c) the change in kinetic energy of the system, from the instant before impact (when the compact car is travelling at 6.00 m/s) to the instant right after the collision. ANSWERS (a) 14.0 m/s (b) pickup truck: $\Delta v_1 = -4.0 \text{ m/s}$ compact car: $\Delta v_2 = 8.0 \text{ m/s}$(c) -4.32×10^4 J

Example 7.5 The Ballistic Pendulum

The ballistic pendulum (Fig. 7.8) is a device used to measure the speed of a fast - moving projectile such as a bullet. The bullet is fired into a large block of wood suspended from some light wires. The bullet embeds in the block, and the entire system swings up to a height *h*. It is possible to obtain the initial speed of the bullet by measuring *h* and the two masses. As an example of the technique, assume that the mass of the bullet, m_1, is 5.00 g, the mass of the pendulum, m_2, is 1.000 kg, and *h* is 5.00 cm. (a) Find the velocity of the system after the bullet embeds in the block. (b) Calculate the initial speed of the bullet

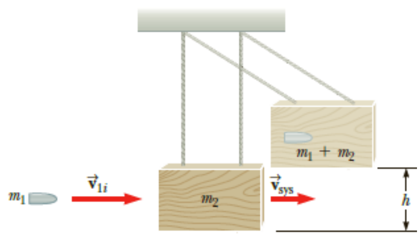


Figure 7-8 Example 7.5(Example 7.5) (a) Diagram of a ballistic pendulum. Note that $\vec{v}_{sys}$is the velocity of the system just *after* the perfectly inelastic collision

Solution

(a) Find the velocity of the system after the bullet embeds in the block. Apply conservation of energy to the block–bullet system after the collision:

$$(E_K + E_P)_{\text{after collision}} = (E_K + E_P)_{\text{top}}$$

Substitute expressions for the kinetic and potential energies. Note that both the potential energy at the bottom and the kinetic energy at the top are zero:

$$\frac{1}{2}(m_1 + m_2)v_{sys}^2 + 0 = 0 + (m_1 + m_2)gh$$

Solve for the final velocity of the block–bullet system, v_{sys} from $v_{sys}^2 = 2gh$,

$$v_{sys} = \sqrt{2(9.8\ \text{m/s}^2)5 \times 10^{-2}\text{m}} = 0.99\ \text{m/s}$$

(b) Calculate the initial speed of the bullet. Write the conservation of momentum equation and substitute expressions.

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_{sys}$$

Solve for the initial velocity of the bullet, and substitute values:

$$v_{1i} = \frac{m_1 + m_2}{m_1}v_{sys} = \frac{(1.005\ \text{kg})(0.990\ \text{m/s})}{5.00 \times 10^{-3}\ \text{kg}} = 199\ \text{m/s}$$

REMARKS Because the impact is inelastic, it would be incorrect to equate the initial kinetic energy of the incoming bullet to the final gravitational potential energy associated with the bullet–block combination. The energy isn't conserved!

Exercises

1. List three ways by which mechanical energy can be lost from the system in this experiment.
2. A bullet with mass 5.00 g is fired horizontally into a 2.000-kg block attached to a horizontal spring. The spring has a constant $6.00\ \times 10^2\ \text{N/m}$ and reaches a maximum compression of 6.00 cm. (a) Find the initial speed of the bullet–block system. (b) Find the speed of the bullet. ANSWERS (a) 1.04 m/s (b) 417 m/s

7.3.2 Elastic Collisions

Now consider two objects that undergo an elastic head-on collision (Fig. 7.9). In this situation, both the momentum and the kinetic energy of the system of two objects are conserved. We can write these conditions as

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad (7\text{-}12)$$

and

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 \qquad (7\text{-}13)$$

where *v* is positive if an object moves to the right and negative if it moves to the left. In a typical problem involving elastic collisions, there are two unknown quantities, and Equations (7.12) and (7.13) can be solved simultaneously to find them.

These two equations are linear and quadratic, respectively. An alternate approach simplifies the quadratic equation to another linear equation, facilitating solution. Cancelling the factor ½ in Equation (7.11), we rewrite the equation as

$$m_1\left(v_{1i}^2 - v_{1f}^2\right) + m_2 = m_2\left(v_{2f}^2 - v_{2i}^2\right)$$

Here we have moved the terms containing m_1 to one side of the equation and those containing m_2 to the other. Next, we factor both sides of the equation:

$$m_1\left(v_{1i} + v_{1f}\right)\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) \qquad \text{(7-14)}$$

Now we separate the terms containing m_1 and m_2 in the equation for the conservation of momentum (7.12) to get

$$m_1\left(v_{1i} - v_{1f}\right) = m_2\left(v_{2f} - v_{2i}\right) \qquad \text{(7-15)}$$

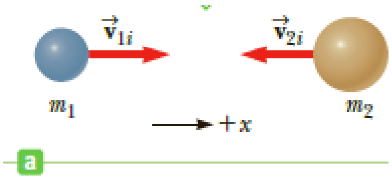
Next, we divide Equation (7.14) by Equation (7.15), producing

$$v_{1i} + v_{1f} = v_{2f} + v_{2i} \qquad \text{(7-16)}$$

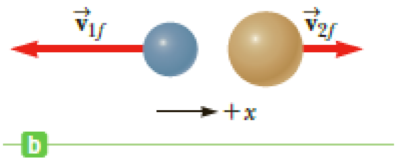
Gathering initial and final values on opposite sides of the equation gives

$$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right). \qquad \text{(7-17)}$$

This equation, in combination with Equation (7.12), will be used to solve problems dealing with perfectly elastic head-on collisions. Equation (7.16) shows that the sum of the initial and final velocities for object 1 equals the sum of the initial and final velocities for object 2. According to Equation (7.17), the relative velocity of the two objects before the collision, $v_{1i} - v_{2i}$, equals the negative of the relative velocity of the two objects after the collision, $-\left(v_{1f} - v_{2f}\right)$. To better understand the equation, imagine that you are riding along on one of the objects. As you measure the velocity of the other object from your vantage point, you will be measuring the relative velocity of the two objects. In your view of the collision, the other object comes toward you and bounces off, leaving the collision with the same speed, but in the opposite direction. This is just what Equation (7.17) states.



Before an elastic collision the two objects move independently.



After the collision the object velocities change, but *both* the energy and momentum of the system are conserved.

Figure 7-9(a) Before and (b) after an elastic head - on collision between two hard spheres. Unlike an inelastic collision, both the total momentum and the total energy are conserved.

ROBLEM-SOLVING STRATEGY - One-Dimensional Collisions

The following procedure is recommended for solving one - dimensional problems involving collisions between two objects:

1. **Coordinates.** Choose a coordinate axis that lies along the direction of motion.
2. **Diagram**. Sketch the problem, representing the two objects as blocks and labelling velocity vectors and masses.
3. **Conservation of Momentum**. Write a general expression for the *total* momentum of the system of two objects *before* and *after* the collision, and equate the two, as in Equation (7.12). On the next line, fill in the known values.
4. **Conservation of Energy**. If the collision is elastic, write a general expression for the total energy before and after the collision, and equate the two quantities, as in Equation (7.13) or (preferably) Equation (7.17). Fill in the known values. (*Skip* this step if the collision is *not* perfectly elastic.)
5. **Solve the equations simultaneously**. Equations (7.12) and (7.17) form a system of two linear equations and two unknowns. If you have forgotten Equation (7.17), use Equation (7.13) instead.

Steps 1 and 2 of the problem-solving strategy are generally carried out in the process of sketching and labelling a diagram of the problem. This is clearly the case in our next example,

EXAMPLE 7.6 LET'S PLAY POOL

Two billiard balls of identical mass move toward each other as in Figure 7.9, with the positive *x* - axis to the right (steps 1 and 2). Assume that the collision between them is perfectly elastic. If the initial velocities of the balls are 130.0 cm/s and 220.0 cm/s, what are the velocities of the balls after the collision? Assume friction and rotation are unimportant.

SOLUTION

Write the conservation of momentum equation. Because $m_1 = m_2$, we can cancel the masses, then substitute $v_{1i} = +30.0$ cm/s and $v_{2i} = -20.0$ cm/s (Step 3):

$$\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

$$\mathbf{30.0cm/s - 20.0cm/s} = \boldsymbol{v_{1f} + v_{2f}}$$

(1) $$\mathbf{10.0cm/s} = \boldsymbol{v_{1f} + v_{2f}}$$

Next, apply conservation of energy in the form of Equation 7.17 (Step 4):

(2) $$\boldsymbol{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})}$$

$$\mathbf{30.0cm/s - (-20.0cm/s)} = \boldsymbol{-(v_{1f} - v_{2f})}$$

(3) $$\mathbf{50.0cm/s} = \boldsymbol{-(v_{1f} - v_{2f})}$$

Now solve Equations (1) and (3) simultaneously by adding them together (Step 5):

$$\boldsymbol{2v_{2f}} = \mathbf{60.0cm/s}\boldsymbol{v_{2f}} = \mathbf{30.0cm/s}$$

Substitute the answer for $\boldsymbol{v_{2f}}$into Equation (1) to get $\boldsymbol{v_{1f}}$:

$$\mathbf{10.0cm/s} = \boldsymbol{v_{1f}} + \mathbf{30.0cm/s}\boldsymbol{v_{1f}} = \mathbf{-20.0cm/s}$$

REMARKS Notice the balls exchanged velocities—almost as if they'd passed through each other. This is always the case when two objects of equal mass undergo an elastic head-on collision.

QUESTION 7.6 In this example, is it possible to adjust the initial velocities of the balls so that both are at rest after the collision? Explain.

EXERCISE 7.6 Find the final velocities of the two balls if the ball with initial velocity $v_{2i} = -20\text{cm/s}$ has a mass equal to onehalf that of the ball with initial velocity $v_{1i} = +30.0$ cm/s.

ANSWER $v_{1f} = -3.33\text{cm/s}$; $v_{2f} = +46.7\text{cm/s}$

7.3.3 Two dimensional (Glancing) Collisions

In Section 7.2 we showed that the total linear momentum of a system is conserved when the system is isolated (i.e., when no external forces act on the system). For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the total momentum of the system in each direction is conserved. However, an important subset of collisions takes place in a plane. The game of billiards is a familiar example involving multiple collisions of objects moving on a two-dimensional surface. We restrict our attention to a single two-dimensional collision between two objects that takes place in a plane, and ignore any possible rotation. For such collisions, we obtain two component equations for the conservation of momentum:

$$\boldsymbol{m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}}$$
$$\boldsymbol{m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}}$$

We must use three subscripts in this general equation, to represent, respectively, (1) the object in question, and (2) the initial and final values of the components of velocity.

Now, consider a two-dimensional problem in which an object of mass m_1 collides with an object of mass $m_2$that is initially at rest, as in Figure 7.10. After the collision, object 1 moves at an angle $\boldsymbol{\theta}$ with respect to the horizontal, and object 2 moves at an angle ϕ with respect to the horizontal. This is called a *glancing* collision. Applying the law of conservation of momentum in component form, and noting that the initial *y*-component of momentum is zero, we have *x* - component:

$$\text{x} - \text{component} \quad m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi \qquad \text{(7-18)}$$

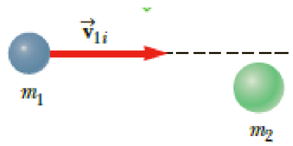
$$\text{y} - \text{component} \quad 0 + 0 = m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\varphi \qquad (7\text{-}19)$$

If the collision is elastic, we can write a third equation, for conservation of energy, in the form

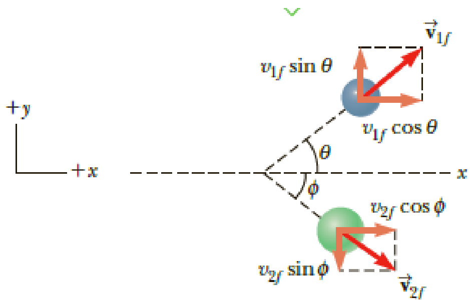
$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad (7\text{-}20)$$

If we know the initial velocity v_{1i} and the masses, we are left with four unknowns (v_{1f}, v_{2f}, θ, and φ). Because we have only three equations, one of the four remaining quantities must be given in order to determine the motion after the collision from conservation principles alone.

If the collision is inelastic, the kinetic energy of the system is *not* conserved, and Equation (7.20) does *not* apply.



Before the collision



After the collision

Figure 7-10A glancing collision between two objects.

Example 7.7: Collision at an Intersection

A car with mass 1.50×10^3 kg travelling east at a speed of 25.0 m/s collides at an intersection with a 2.50×10^3 - kg pickup truck travelling north at a speed of 20.0 m/s, as shown in Figure 7.11. Find the magnitude and direction of the velocity of the wreckage immediately after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together) and assuming that friction between the vehicles and the road can be neglected.

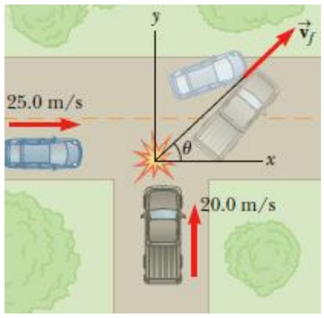


Figure 7-11(Example 7.7) A top view of a perfectly inelastic collision between a car and a pickup truck

Solution

Find the *x* - components of the initial and final total momenta:

$$\sum p_{xi} = m_{car} v_{car} = (1.50 \times 10^3 \text{ kg})\left(25.0 \frac{\text{m}}{\text{s}}\right) = 3.75 \times 10^4 \text{ kg m/s}$$

$$\sum p_{xf} = (m_{\text{car}} + m_{\text{truk}})v_f \cos\theta = (4 \times 10^3 \text{ kg})v_f \cos\theta$$

Set the initial *x* - momentum equal to the final *x* - momentum:

$$(1) \qquad 3.75 \times 10^4 \text{ kg m/s} = (4 \times 10^3 \text{ kg})v_f \cos\theta$$

Set the initial *y* - momentum equal to the final *y* - momentum:

$$\sum p_{yi} = m_{\text{truk}}v_{\text{truk}} = (2.5 \times 10^3 \text{ kg})(20.0 \text{ m/s}) = 5.00 \times 10^4 \text{ kg m/s}$$

$$\sum p_{yf} = (m_{\text{car}} + m_{\text{truk}})v_f \sin\theta = (4 \times 10^3 \text{ kg})v_f \sin\theta$$

Set the initial *y* - momentum equal to the final *y* - momentum:

$$(2) \qquad 5.00 \times 10^4 \text{ kg m/s} = (4 \times 10^3 \text{ kg})v_f \sin\theta$$

Divide Equation (2) by Equation (1) and solve for θ:

$$\tan\theta = \frac{5.00 \times 10^4 \text{ kg}\frac{\text{m}}{\text{s}}}{3.75 \times 10^4 \text{ kg}\frac{\text{m}}{\text{s}}} = 1.55, \ \theta = 53.1°$$

Substitute this angle back into Equation (2) to find v_f:

$$v_f = \frac{5.00 \times 10^4 \text{ kg m/s}}{(4 \times 10^3 \text{ kg}) \sin 53.1°} = 15.6 \text{ m/s}$$

REMARKS: It's also possible to first find the *x* - and *y* - components v_{fx} and v_{fy} of the resultant velocity. The magnitude and direction of the resultant velocity can then be found with the Pythagorean theorem, $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$, and the inverse tangent function $\theta = \tan^{-1}(v_{fy}/v_{fx})$. Setting up this alternate approach is a simple matter of substituting $v_{fx} = v_f \cos\theta$ and $v_{fy} = v_f \sin\theta$ into Equations (1) and (2).

Exercise

1. If the car and truck had identical mass and speed, what would the resultant angle have been?
2. A 3.00 - kg object initially moving in the positive *x* - direction with a velocity of +5.00 m/s collides with and sticks to a 2.00-kg object initially moving in the negative *y* - direction with a velocity of –3.00 m/s. Find the final components of velocity of the composite object. Answer v_{fx} = 3.00 m/s; $v_{fx} = -1.20$ m/s

7.4 The Concept of Center of Mass

Learning outcome

After completing this section, students are expected to:

- Define point masses.

- Define the center of mass of an object.
- Apply the concept of center of mass to everyday life
- Solve simple problems involving momentum and collisions of rigid bodies

In this section, we describe the overall motion of a system in terms of a special point called the center of mass of the system. The system can be either a small number of particles or an extended, continuous object, such as a gymnast leaping through the air. We shall see that the translational motion of the center of mass of the system is the same as if all the mass of the system were concentrated at that point. That is, the system moves as if the net external force were applied to a single particle located at the center of mass. This model, the *particle model,* was introduced in Chapter 2. This behavior is independent of other motion, such as rotation or vibration of the system or deformation of the system (for instance, when a gymnast folds her body).

Consider a system consisting of a pair of particles that have different masses and are connected by a light, rigid rod (Fig. 7.12). The position of the center of mass of a system can be described as being the *average position* of the system's mass. The center of mass of the system is located somewhere on the line joining the two particles and is closer to the particle having the larger mass. If a single force is applied at a point on the rod above the center of mass, the system rotates clockwise (see Fig. 7.12a). If the force is applied at a point on the rod below the center of mass, the system rotates counterclockwise (see Fig. 7.12b). If the force is applied at the center of mass, the system moves in the direction of the force without rotating (see Fig. 7.12c). The center of mass of an object can be located with this procedure.

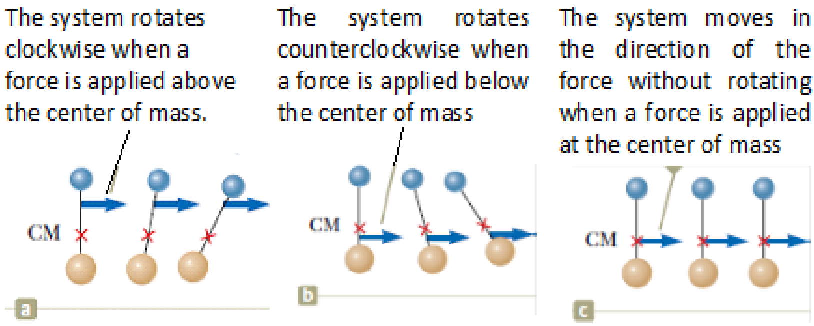


Figure 7-12A force is applied to a system of two particles of unequal mass connected by a light, rigid rod..

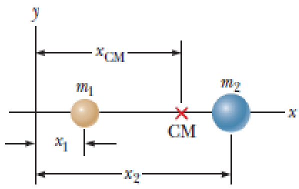


Figure 7-13The center of mass of two particles of unequal mass on the x axis is located at x_{CM}, a point between the particles, closer to the one having the larger mass.

The center of mass of the pair of particles described in Figure 7.13 is located on the *x* axis and lies somewhere between the particles. Its *x* coordinate is given by

$$x_{CM} = \frac{m_1x_1+m_2x_2}{m_1+m_2} \tag{7-21}$$

For example, if $x_1 = 0$, $x_2 = d$, and $m_2 = 2m_1$, we find that $x_{CM} = \frac{2}{3}d$. That is, the center of mass lies closer to the more massive particle. If the two masses are equal, the center of mass lies midway between the particles.

We can extend this concept to a system of many particles with masses m_i in three dimensions. The *x* coordinate of the center of mass of *n* particles is defined to be

$$x_{CM} = \frac{m_1x_1+m_2x_2+m_3x_3+\cdots+m_nx_n}{m_1+m_2+m_3+\cdots+m_nx_n} = \frac{\sum_i m_ix_i}{\sum_i m_i} = \frac{1}{M}\sum_i m_ix_i \quad (7\text{-}22)$$

where $x_i$is the *x* coordinate of the *i*th particle and the total mass is $M = \sum_i m_i$where the sum runs over all *n* particles. The *y* and *z* coordinates of the center of mass are similarly defined by the equations

$$y_{CM} = \frac{1}{M}\sum_i m_iy_i, \quad z_{CM} = \frac{1}{M}\sum_i m_iz_i \quad (7\text{-}23)$$

The center of mass can be located in three dimensions by its position vector $\vec{r}_{CM}$. The components of this vector are x_{CM}, y_{CM}, and z_{CM}, defined in Equations (7.22) and (7.23). Therefore,

$$\vec{r}_{CM} = y_{CM}\hat{\imath} + y_{CM}\hat{\jmath} + z_{CM}\hat{k} = \frac{1}{M}\left(\sum_i m_ix_i\,\hat{\imath} + \sum_i m_iy_i\,\hat{\jmath} + \sum_i m_iz_i\,\hat{k}\right)$$

or

$$\vec{r}_{CM} = \frac{1}{M}\sum_i m_i\vec{r}_i \quad (7\text{-}24)$$

Where $\vec{r}_i$ is the position vector of the *i*th particle, defined by $\vec{r}_i = y_i\hat{\imath} + y_i\hat{\jmath} + z_i\hat{k}$.

It's often possible to guess the location of the center of mass. The center of mass of a homogeneous, symmetric body must lie on the axis of symmetry.

For example, the center of mass of a homogeneous rod lies midway between the ends of the rod, and the center of mass of a homogeneous sphere or a homogeneous cube lies at the geometric center of the object.

Several examples involve homogeneous, symmetric objects where the centers of mass coincide with their geometric centers. A rigid object in a uniform gravitational field can be balanced by a single force equal in magnitude to the weight of the object, as long as the force is directed upward through the object's center of mass.

Example 7.8: Where is the Center Of Mass?

(a) Three objects are located in a coordinate system as shown in Figure 7.14a. Find the center of mass.

(b) How does the answer change if the object on the left is displaced upward by 1.00 m and the object on the right is displaced downward by 0.500 m (Fig. 7.14b)? Treat the objects as point particles

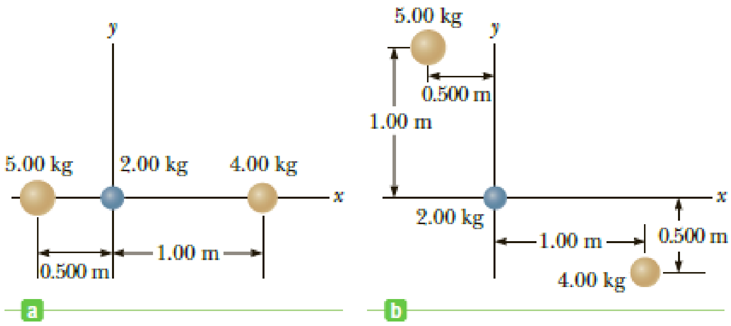


Figure 7-14(Example 7.8) Locating the center of mass of a system of three particles.

Solution

(a) Find the center of mass of the system in Figure 7.13a. Apply Equation (7.22) to the system of three objects:

$$(1) \qquad x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3},$$

Compute the numerator and the denominator of Equation (1):

$$\sum_i m_i x_i = 1.50 \text{ kg m}, \qquad \sum_i m_i = 11.00 \text{ kg},$$

substitute into Equation (1).$x_{CM} = \frac{1.50 \text{ kg m}}{11.00 \text{ kg}} = 0.136 \text{ m}$

b) How does the answer change if the positions of the objects are changed as in Figure 7.13b? Because the *x*-coordinates have not been changed, the *x*-coordinate of the center of mass is also unchanged:

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}$$

$$\sum_i m_i y_i = (5.00 \text{ kg})(1.00 \text{ m}) + (2.00 \text{ kg})(0 \text{ m}) + (4.00 \text{ kg})(-0.500\text{m}) = 5.00 \text{ kg m}$$

$$x_{CM} = \frac{3.00 \text{ kg m}}{11.00 \text{ kg}} = 0.273 \text{ m}$$

REMARKS: Notice that translating objects in the *y* - direction doesn't change the *x*-coordinate of the center of mass. The three components of the center of mass are each independent of the other two coordinates.

Exercises

1. If 1.00 kg is added to the masses on the left and right in Figure 7.13a, does the center of mass (a) move to the left, (b) move to the right, or (c) remain in the same position?
2. If a fourth particle of mass 2.00 kg is placed at (0, 0.25 m) in Figure 7.13a, find the *x*- and *y* -coordinates of the center of mass for this system of four particles.

Answer *x*cm 5 0.115 m; *y*cm 5 0.038 5 m

Example 7.9 Tug of War at The Ice Fishing Hole

Bob and Sherry are lying on the ice, a fishing hole of radius 1.00 m cut in the ice halfway between them. A rope of length 10.0 m lies between them, and they both grip it and begin pulling, as in Figure 7.15a. Bob has mass of m_B= 85.0 kg and Sherry has mass of m_S= 48.0 kg, so Sherry reaches the hole, first. Where is Bob at that time? Assume the hole is centered on the origin and that Bob and Sherry start at x_B= 5.00 m and x_S= –5.00 m, respectively. Neglect forces of friction.

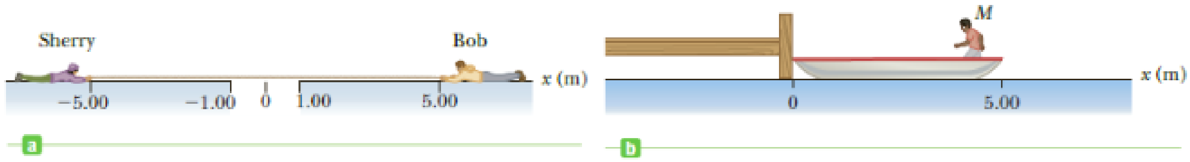


Figure 7-15 (Example 7.9) (a) Bob and Sherry engage in a tug - of war.(b) (Exercise 7.9)

Solution

Calculate the center of mass using the initial positions:

$$x_{CM} = \frac{m_S x_S + m_B x_B}{m_S + m_B} = \frac{(48.0\ \mathrm{kg})(-5.00\mathrm{m}) + (85.0\ \mathrm{kg})(5.00\mathrm{m})}{48.0\ \mathrm{kg} + 85.0\ \mathrm{kg}} = 1.39\ \mathrm{m}$$

Find Bob's position using the center of mass equation and the fact that Sherry reaches the hole first:

$$1.39\ \mathrm{m} = \frac{m_S x_S + m_B x_B}{m_S + m_B} = \frac{(48.0\ \mathrm{kg})(-1.00\mathrm{m}) + (85.0\ \mathrm{kg})x_B}{133\ \mathrm{kg}}$$

$$x_B = 2.74\ \mathrm{m}$$

REMARKS: So when Sherry reaches the edge of the fishing hole, Bob is still nearly two meters away from the edge on his side of the hole.

Exercises

1. How do the speeds v_S of Sherry and v_B of Bob compare during the motion?
2. A man of mass *M* 5 75.0 kg is standing in a canoe of mass 40.0 kg that is 5.00 m long, as in Figure 7.15b. The far end of the canoe is next to a dock. From a position 0.500 m from his end of the canoe, he walks to the same position at the other end of the canoe. (a) Find the center of mass of the canoe–man system, taking the end of the dock as the origin. (b) Neglecting drag forces, how far is he from the dock? (*Hint:* the final location of the canoe's center of mass will be 2.00 m farther from the dock than the man's final position, which is unknown.) ANSWERS (a) 3.80 m (b) 3.10 m

7.5 Summary

The linear momentum $\vec{p}$ of an object of mass *m* moving with velocity $\vec{v}$ is defined as

$$\vec{p} \equiv m\vec{v}$$

Momentum carries units of kg # m/s. The impulse $\vec{I}$of a constant force $\vec{F}$ delivered to an object is equal to the product of the force and the time interval during which the force acts:

$$\vec{I} \equiv \vec{F}\Delta t$$

These two concepts are unified in the impulse– momentum theorem, which states that the impulse of a constant force delivered to an object is equal to the change in momentum of the object:

$$\vec{I} = \vec{F}\Delta t = \Delta\vec{p} = m\vec{v}_{\mathrm{f}} - m\vec{v}_{\mathrm{i}}$$

Solving problems with this theorem often involves estimating speeds or contact times (or both), leading to an average force.

When no net external force acts on an isolated system, the total momentum of the system is constant. This principle is called conservation of momentum. In particular, if the isolated system consists of two objects undergoing a collision, the total momentum of the system is the same before and after the collision (Fig. 7.4). Conservation of momentum can be written mathematically for this case as

$$m_1\vec{v}_{1f} + m_2\vec{v}_{2f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$$

Collision and recoil problems typically require finding unknown velocities in one or two dimensions. Each vector component gives an equation, and the resulting equations are solved simultaneously.

In an inelastic collision, the momentum of the system is conserved, but kinetic energy is not. In a perfectly inelastic collision, the colliding objects stick together and the common final velocity is given by

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{(m_1 + m_2)}.$$

In an elastic collision, both the momentum and the kinetic energy of the system are conserved.

A one - dimensional elastic collision between two objects can be solved by using the conservation of momentum and conservation of energy equations:

$$\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

and

$$\boldsymbol{\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2}$$

The following equation, derived from Equations (7.12) and (7.13), is usually more convenient to use than the original conservation of energy equation:

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

These equations can be solved simultaneously for the unknown velocities. Energy is not conserved in inelastic collisions, so such problems must be solved with Eq. (7.12) alone.

In glancing collisions, conservation of momentum can be applied along two perpendicular directions: an *x* – axis and a *y* - axis. Problems can be solved by using the *x* – and *y* - components of Equation (7.9). Elastic two - dimensional collisions will usually require Equation (7.12) as well. (Equation (7.17) doesn't apply to two dimensions.) Generally, one of the two objects is taken to be traveling along the *x* - axis, undergoing a deflection at some angle u after the collision. The final velocities and angles can be found with elementary trigonometry.

The motion of extended object is described by the motion of the object's center of mass. The *x*-, *y* -, and *z*-of an object's center of mass are given by

$$x_{CM} = \frac{1}{M}\sum_i m_i x_i, \qquad y_{CM} = \frac{1}{M}\sum_i m_i y_i \text{ and} z_{CM} = \frac{1}{M}\sum_i m_i z_i$$

7.6 Conceptual Questions

Q7.1. A batter bunts a pitched baseball, blocking the ball without swinging. (a) Can the baseball deliver more kinetic energy to the bat and batter than the ball carries initially? (b) Can the baseball deliver more momentum to the bat and batter than the ball carries initially? Explain each of your answers.

Q7.2. If two objects collide and one is initially at rest, (a) is it possible for both to be at rest after the collision? (b) Is it possible for only one to be at rest after the collision? Explain.

Q7.3. Two carts on an air track have the same mass and speed and are traveling towards each other. If they collide and stick together, find (a) the total momentum and (b) total kinetic energy of the system. (c) Describe a different colliding system with this same final momentum and kinetic energy.

Q7.4. Two identical ice hockey pucks, labeled A and B, are sliding towards each other at speed *v*. Which one of the following statements is true concerning their momenta and kinetic energies? (a) $\vec{p}_A = \vec{p}_B$ and $E_{KA} = E_{KB}$ (b) $\vec{p}_A = -\vec{p}_B$ and $E_{KA} = -E_{KB}$(c) $\vec{p}_A = -\vec{p}_B$ and $E_{KA} = E_{KB}$ (d) $\vec{p}_A = \vec{p}_B$ and $E_{KA} = -E_{KB}$

Q7.5. A ball of clay of mass *m* is thrown with a speed *v* against a brick wall. The clay sticks to the wall and stops. Is the principle of conservation of momentum violated in this example?

Q7.6. A skater is standing still on a frictionless ice rink. Her friend throws a Frisbee straight to her. In which of the following cases is the largest momentum transferred to the skater? (a) The skater catches the Frisbee and holds onto it. (b) The skater catches the Frisbee momentarily, but then drops it vertically downward. (c) The skater catches the Frisbee, holds it momentarily, and throws it back to her friend.

Q7.7. A baseball is thrown from the outfield toward home plate. (a) True or False: Neglecting air resistance, the momentum of the baseball is conserved during its flight. (b) True or False: Neglecting air resistance, the momentum of the baseball–Earth system is conserved during the baseball's flight.

Q7.8. (a) If two automobiles collide, they usually do not stick together. Does this mean the collision is elastic? (b) Explain why a head-on collision is likely to be more dangerous than other types of collisions.

Q7.9. Your physical education teacher throws you a tennis ball at a certain velocity, and you catch it. You are now given the following choice: The teacher can throw you a medicine ball (which is much more massive than the tennis ball) with the same velocity, the same momentum, or the same kinetic energy as the tennis ball. Which option would you choose in order to make the easiest catch, and why?

Q7.10. Two carts move in the same direction along a frictionless air track, each acted on by the same constant force for a time interval D*t*. Cart 2 has twice the mass of cart 1. Which one of the following statements is true? (a) Each cart has the same change in momentum. (b) Cart 1 has the greater change in momentum. (c) Cart 2 has the greater change in momentum. (d) The changes in momenta depend on the initial velocities.

Q7.11. For the situation described in the previous question, which cart experiences the greater change in kinetic energy? (a) Each cart has the same change in kinetic energy. (b) Cart 1 (c) Cart 2 (d) It's impossible to tell without knowing the initial velocities

Q7.12. At a bowling alley, two players each score a spare when their bowling balls make head-on, approximately elastic collisions at the same speed with identical pins. After the collisions, the pin hit by ball A moves much more quickly than the pin hit by ball B. Which ball has more mass?

Q7.13. An open box slides with constant speed across the frictionless surface of a frozen lake. If water from a rain shower falls vertically downward into it, does the box: (a) speed up, (b) slow down, or (c) continue to move with constant speed?

Q7.14. Does a larger net force exerted on an object always produce a larger change in the momentum of the object, compared to a smaller net force? Explain.

Q7.15. Does a larger net force always produce a larger change in kinetic energy than a smaller net force? Explain.

Q7.16. If two particles have equal momenta, are their kinetic energies equal? (a) yes, always (b) no, never (c) no, except when their masses are equal (d) no, except when their speeds are the same (e) yes, as long as they move along parallel lines.

Q7.17. Two particles of different mass start from rest. The same net force acts on both of them as they move over equal distances. How do their final kinetic energies compare? (a) The particle of larger mass has more kinetic energy. (b) The particle of smaller mass has more kinetic energy. (c) The particles have equal kinetic energies. (d) Either particle might have more kinetic energy.

Q7.18. (a) Does the center of mass of a rocket in free space accelerate? Explain. (b) Can the speed of a rocket exceed the exhaust speed of the fuel? Explain.

7.7 Problems

P7.1. Calculate the magnitude of the linear momentum for the following cases: (a) a proton with mass equal to 1.67×10^{-27} kg, moving with a speed of 5.00×10^{6} m/s; (b) a 15.0-g bullet moving with a speed of 300 m/s; (c) a 75.0-kg sprinter running with a speed of 10.0 m/s; (d) the Earth (mass =5.98×10^{24} kg) moving with an orbital speed equal to 2.98×10^{4} m/s.

P7.2. A 0.280-kg volleyball approaches a player horizontally with a speed of 15.0 m/s. The player strikes the ball with her fist and causes the ball to move in the opposite direction with a speed of 22.0 m/s. (a) What impulse is delivered to the ball by the player? (b) If the player's fist is in contact with the ball for 0.060 0 s, find the magnitude of the average force exerted on the player's fist.

P7.3. Drops of rain fall perpendicular to the roof of a parked car during a rainstorm. The drops strike the roof with a speed of 12 m/s, and the mass of rain per second striking the roof is 0.035 kg/s. (a) Assuming the drops come to rest after striking the roof, find the average force exerted by the rain on the roof. (b) If hailstones having the same mass as the raindrops fall on the roof at the same rate and with the same speed, how would the average force on the roof compare to that found in part (a)?

P7.4. A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?

P7.5. V A 65.0-kg basketball player jumps vertically and leaves the floor with a velocity of 1.80 m/s upward. (a) What impulse does the player experience? (b) What force does the floor exert on the player before the jump? (c) What is the total average force exerted by the floor on the player if the player is in contact with the floor for 0.450 s during the jump?

P7.6. T The front 1.20 m of a 1 400-kg car is designed as a "crumple zone" that collapses to absorb the shock of a collision. If a car traveling 25.0 m/s stops uniformly in 1.20 m, (a) how long does the collision last, (b) what is the magnitude of the average force on the car, and (c) what is the acceleration of the car? Express the acceleration as a multiple of the acceleration of gravity.

P7.7. A pitcher throws a 0.14-kg baseball toward the batter so that it crosses home plate horizontally and has a speed of 42 m/s just before it makes contact with the bat. The batter then hits the ball straight back at the pitcher with a speed of 48 m/s. Assume the ball travels along the same line leaving the bat as it followed before contacting the bat. (a) What is the magnitude of the impulse delivered by the bat to the baseball? (b) If the ball is in contact with the bat for 0.005 0 s, what is the magnitude of the average force exerted by the bat on the ball? (c) How does your answer to part (b) compare to the weight of the ball?

P7.8. V High - speed stroboscopic photographs show that the head of a 2.00×10^2 - g golf club is traveling at 55.0 m/s just before it strikes a 46.0 - g golf ball at rest on a tee. After the collision, the club head travels (in the same direction) at 40.0 m/s. Find the speed of the golf ball just after impact.

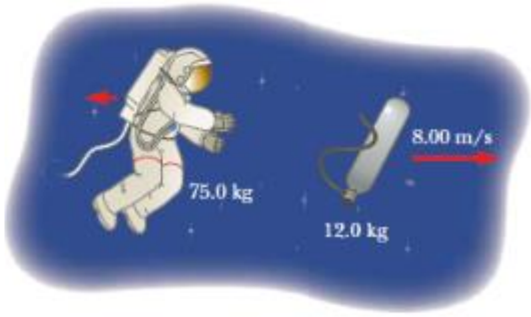
P7.9. A 45.0 - kg girl is standing on a 150. - kg plank. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless surface. The girl begins to walk along the plank at a constant velocity of 1.50 m/s to the right relative to the plank. (a) What is her velocity relative to the surface of the ice? (b) What is the velocity of the plank relative to the surface of the ice?

P7.10. *This is a symbolic version of Problem 9.* A girl of mass m_G is standing on a plank of mass m_P. Both are originally at rest on a frozen lake that constitutes a frictionless, flat surface. The girl begins to walk along the plank at a constant velocity v_{GP} to the right relative to the plank. (The subscript *GP* denotes the girl relative to plank.) (a) What is the velocity v_{PI} of the plank relative to the surface of the ice? (b) What is the girl's velocity v_{GI} relative to the ice surface?

P7.11. Squids are the fastest marine invertebrates, using a powerful set of muscles to take in and then eject water in a form of jet propulsion that can propel them to speeds of over 11.5 m/s. What speed would a stationary 1.50 - kg squid achieve by ejecting 0.100 kg of water (not included in the squid's mass) at 3.25 m/s? Neglect other forces, including the drag force on the squid.

P7.12. A 65.0 - kg person throws a 0.045 0 - kg snowball forward with a ground speed of 30.0 m/s. A second person, with a mass of 60.0 kg, catches the snowball. Both people are on skates. The first person is initially moving forward with a speed of 2.50 m/s, and the second person is initially at rest. What are the velocities of the two people after the snowball is exchanged? Disregard friction between the skates and the ice.

P7.13. An astronaut in her space suit has a total mass of 87.0 kg, including suit and oxygen tank. Her tether line loses its attachment to her spacecraft while she's on a spacewalk. Initially at rest with respect to her spacecraft, she throws her 12.0 - kg oxygen tank away from her spacecraft with a speed of 8.00 m/s to propel herself back toward it (Fig. P7.1). (a) Determine the maximum distance she can be from the craft and still return within 2.00 min (the amount of time the air in her helmet remains breathable). (b) Explain in terms of Newton's laws of motion why this strategy works.



P 7-1

P7.14. Gayle runs at a speed of 4.00 m/s and dives on a sled, initially at rest on the top of a frictionless, snow- covered hill. After she has descended a vertical distance of 5.00 m, her brother, who is initially at rest, hops on her back, and they continue down the hill together. What is their speed at the bottom of the hill if the total vertical drop is 15.0 m? Gayle's mass is 50.0 kg, the sled has a mass of 5.00 kg, and her brother has a mass of 30.0 kg.

P7.15. A 75.0-kg ice skater moving at 10.0 m/s crashes into a stationary skater of equal mass. After the collision, the two skaters move as a unit at 5.00 m/s. Suppose the average force a skater can experience without breaking a bone is 4 500 N. If the impact time is 0.100 s, does a bone break?

P7.16. A railroad car of mass M moving at a speed v_1 collides and couples with two coupled railroad cars, each of the same mass M and moving in the same direction at a speed v_2. (a) What is the speed v_f of the three coupled cars after the collision in terms of v_1 and v_2? (b) How much kinetic energy is lost in the collision? Answer in terms of M, v_1 and v_2.

P7.17. Consider the ballistic pendulum device discussed in Example 7.5 and illustrated in Figure 7.8. (a) Determine the ratio of the momentum immediately after the collision to the momentum immediately before the collision. (b) Show that the ratio of the kinetic energy immediately after the collision to the kinetic energy immediately before the collision is $m_1 /(m_1 + m_2)$.

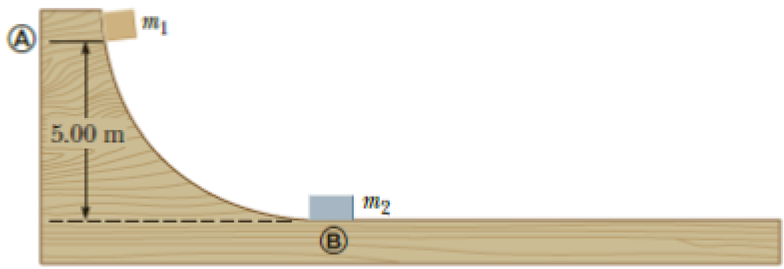
P7.18. In a Broadway performance, an 80.0-kg actor swings from a 3.75-m-long cable that is horizontal when he starts. At the bottom of his arc, he picks up his 55.0-kg costar in an inelastic collision. What maximum height do they reach after their upward swing?

P7.19. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30.0° with respect to the original line of motion. (a) Find the velocity (magnitude and direction) of the second ball after collision. (b) Was the collision inelastic or elastic?

P7.20. A typical person begins to lose consciousness if subjected to accelerations greater than about 5g (49.0 m/s^2) for more than a few seconds. Suppose a 3.00×10^4-kg manned spaceship's engine has an exhaust speed of 2.50×10^3 m/s. What maximum burn rate $|\Delta M/\Delta t|$ could the engine reach before the ship's acceleration exceeded 5g and its human occupants began to lose consciousness?

P7.21. A spaceship's orbital maneuver requires a speed increase of 1.20×10^3 m/s. If its engine has an exhaust speed of 2.50×10^3 m/s, determine the required ratio M_i/M_f of its initial mass to its final mass. (The difference M_i - M_f equals the mass of the ejected fuel.)

P7.22. In research in cardiology and exercise physiology, it is often important to know the mass of blood pumped by a person's heart in one stroke. This information can be obtained by means of a *ballistic-ardiograph.* The instrument works as follows: The subject lies on a horizontal pallet floating on a film of air. Friction on the pallet is negligible. Initially, the momentum of the system is zero. When the heart beats, it expels a mass *m* of blood into the aorta with speed *v*, and the body and platform move in the opposite direction with speed *V*. The speed of the blood can be determined independently (e.g., by observing an ultrasound Doppler shift). Assume that the blood's speed is 50.0 cm/s in one typical trial. The mass of the subject plus the pallet is 54.0 kg. The pallet moves at a speed of 6.00 3 1025 m in 0.160 s after one heartbeat. Calculate the mass of blood that leaves the heart. Assume that the mass of blood is negligible compared with the total mass of the person. This simplified example illustrates the principle of ballistocardiography, but in practice a more sophisticated model of heart function is used.

P7.23. Consider a frictionless track as shown in Figure P7.2. A block of mass m_1 = 5.00 kg is released from A. It makes a head-on elastic collision at B with a block of mass m_2= 10.0 kg that is initially at rest. Calculate the maximum height to which $m_1$rises after the collision.	 P 7-2

P7.24. An unstable nucleus of mass 1.7 × 10^{-26} kg, initially at rest at the origin of a coordinate system, disintegrates into three particles. One particle, having a mass of m_1 = 5.0 × 10^{-27} kg, moves in the positive *y*- direction with speed v_1 = 6.0 × 10^6 m/s. Another particle, of mass m_2 = 8.4 × 10^{-27} kg, moves in the positive *x* - direction with speed v_2 = 4.0 × 10^6 m/s. Find the magnitude and direction of the velocity of the third particle.

P7.25. The mass of the Earth is 5.97 3 1024 kg, and the mass of the Moon is 7.35 × 10^{22} kg. The distance of separation, measured between their centers, is 3.84 × 10^8 m. Locate the center of mass of the Earth–Moon system as measured from the center of the Earth.

P7.26. Two blocks of masses m_1 = 2.00 kg and m_2 = 4.00 kg are each released from rest at a height of *h* = 5.00 m on a frictionless track, as shown in Figure P7.3, and undergo an elastic headon collision. (a) Determine the velocity of each block just before the collision. (b) Determine the velocity of each block immediately after the collision. (c) Determine the maximum heights to which m_1 and m_2 rise after the collision.

P 7-3

P7.27. A wooden block of mass *M* rests on a table over a large hole as in Figure P7.4. A bullet of mass *m* with an initial velocity $v_i$is fired upward into the bottom of the block and remains in the block after the collision. The block and bullet rise to a maximum height of *h.* (a) Describe how you would find the initial velocity of the bullet using ideas you have learned in this topic. (b) Find an expression for the initial velocity of the bullet.

P7.28. V A 1.25-kg wooden block rests on a table over a large hole as in Figure P7.4. A 5.00-g bullet with an initial velocity *vi* is fired upward into the bottom of the block and remains in the block after the collision. The block and bullet rise to a maximum height of 22.0 cm. (a) Describe how you would find the initial velocity of the bullet using ideas you have learned in this topic. (b) Calculate the initial velocity of the bullet from the information provided.

P 7-4 *Problems 7.27 and 28*

P7.29. Two objects of masses m and $3m$ are moving toward each other along the x-axis with the same initial speed v_0. The object with mass m is traveling to the left, and the object with mass $3m$ is traveling to the right. They undergo an elastic glancing collision such that m is moving downward after the collision at right angles from its initial direction. (a) Find the final speeds of the two objects. (b) What is the angle $\boldsymbol{\theta}$ at which the object with mass $3m$ is scattered?

8 Fluid mechanics

Learning Outcomes

After completing this chapter, students are expected to:

- Define density of static fluids
- Define pressure of static fluids
- Explain relation between pressure and depth
- Apply to hydraulic lift
- Explain the principle of pressure measurement

Introduction

There are four known states of matter: solids, liquids, gases, and plasmas. In the Universe at large, plasmas—systems of charged particles interacting electromagnetically—are the most common. In our environment on Earth, solids, liquids, and gases predominate.

An understanding of the fundamental properties of these different states of matter is important in all the sciences, in engineering, and in medicine. Forces put stresses on solids, and stresses can strain, deform, and break those solids, whether they are steel beams or bones.

Fluids under pressure can perform work or carry nutrients and essential solutes, like the blood flowing through our arteries and veins. Flowing gases cause pressure differences that can lift a massive cargo plane or the roof off a house in a hurricane. High-temperature plasmas created in fusion reactors may someday allow humankind to harness the energy source of the Sun.

The study of any one of these states of matter is itself a vast discipline. Here, we'll introduce basic properties of solids and liquids, the latter including some properties of gases.

8.1 Density and Pressure in Static Fluids

8.1.1 Density

The density $\boldsymbol{\rho}$ of an object having uniform composition is its mass M divided by its volume V:

$$\boldsymbol{\rho} \equiv \frac{\boldsymbol{M}}{\boldsymbol{V}} \tag{8.1}$$

SI unit: kilogram per meter cubed (kg/m^3)

The specific gravity of a substance is the ratio of its density to the density of water at 4°C, which is 1.0×10^3 kg/m^3. (The size of the kilogram was originally defined to make the density of water 1.0×10^3 kg/m^3 at 4°C.) By definition, specific gravity is a dimensionless quantity. For example, if the specific gravity of a substance is 3.0, its density is 3.0 (1.0×10^3 kg/m^3) $= 1.0 \times 10^3$ kg/m^3.

8.1.2 Pressure

If F is the magnitude of a force exerted perpendicular to a given surface of area A, then the average pressure P is the force divided by the area:

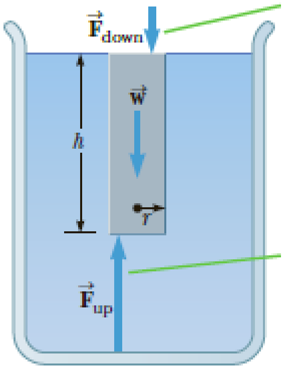
$$\boldsymbol{P = \frac{F}{A}} \tag{8.2}$$

SI unit: pascal (Pa = N/m^2)

Pressure can change from point to point, which is why the pressure in the above equation is called an average. Because pressure is defined as force per unit area, it has units of Pascal (newton per square meter).

Example 8.1: Pressure and weight of water

GOAL: Relate density, pressure, and weight.



(a) Calculate the weight of a cylindrical column of water with height h = 40.0 m and radius r = 1.00 m.

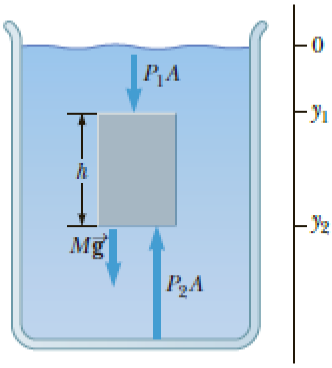
(b) (b) Calculate the force exerted by air on a disk of radius 1.00 m at the water's surface. (c) What pressure at a depth of 40.0 m supports the water column?

Solution:

For part (a), calculate the volume and multiply by the density to get the mass of water, then multiply the mass by g to get the weight. Part (b) requires substitution into the definition of pressure. Adding the results of parts (a) and (b) and dividing by the area gives the pressure of water at the bottom of the column.

8.1.3 Variation of pressure with depth

When a fluid is at rest in a container, all portions of the fluid must be in static equilibrium—at rest with respect to the observer. Furthermore, all points at the same depth must be at the same pressure.



Next, let's examine the fluid contained within the volume indicated by the darker region in the figure. This region has cross-sectional area A and extends from position y_1 to position y_2 below the surface of the liquid. Three external forces act on this volume of fluid: the force of gravity, Mg; the upward force P_2A exerted by the liquid below it; and a downward force P_1A exerted by the fluid above it. Because the given volume of fluid is in equilibrium, these forces must add to zero, so we get

$$\boldsymbol{P_2A - P_1A - Mg = 0} \tag{8.3}$$

From the definition of density, we have

$$\boldsymbol{M = \rho V = \rho A(y_1 - y_2)} \tag{8.4}$$

Substituting Equation 8.4 into Equation 8.3, canceling the area *A*, and rearranging terms, we get

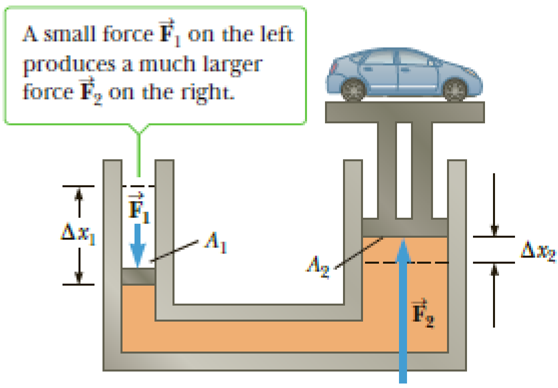
$$\boldsymbol{P_2 = P_1 + \rho g(y_1 - y_2)} \quad 8.5$$

Atmospheric pressure is also caused by a piling up of fluid—in this case, the fluid is the gas of the atmosphere. The weight of all the air from sea level to the edge of space results in an atmospheric pressure of $P_0 = 1.013 \times 10^5$ Pa at sea level. This result can be adapted to find the pressure P at any depth $h = (y_1 - y_2) = (0 - y_2)$ below the surface of the water:

$$\boldsymbol{P = P_0 + \rho g h} \quad 8.6$$

According to Equation 8.6, the pressure *P* at a depth *h* below the surface of a liquid open to the atmosphere is greater than atmospheric pressure by the amount ρgh. Moreover, the pressure isn't affected by the shape of the vessel. Equation 8.6 is often called the *equation of hydrostatic equilibrium*.

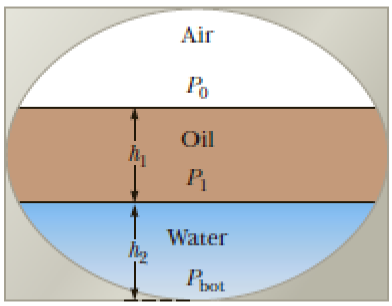
8.1.3.1 Application: Hydraulic lift



An important application of Pascal's principle is the hydraulic press (see figure). A downward force F_1 is applied to a small piston of area A_1. The pressure is transmitted through a fluid to a larger piston of area A_2. As the pistons move and the fluids in the left and right cylinders change their relative heights, there are slight differences in the pressures at the input and output pistons. Neglecting these small differences, the fluid pressure on each of the pistons may be taken to be the same; $P_1 = P_2$. From the definition of pressure, it then follows that $F_1/A_1 = F_2/A_2$. Therefore, the magnitude of the force F_2 is larger than the magnitude of F_1 by the factor A_2/A_1. That's why a large load, such as a car, can be moved on the large piston by a much smaller force on the smaller piston. Hydraulic brakes, car lifts, hydraulic jacks, forklifts, and other machines make use of this principle.

Example 8.2: Oil and water

GOAL Calculate pressures created by layers of different fluids.



In a huge oil tanker, salt water has flooded an oil tank to a depth of h_2 = 5.00 m. On top of the water is a layer of oil h_1 = 8.00 m deep, as in the cross-sectional view of the tank in Figure. The oil has a density of 0.700 g/cm^3. Find the pressure at the bottom of the tank. (Take 1 025 kg/m^3 as the density of salt water.)

Solution:

Equation 8.6 must be used twice. First, use it to calculate the pressure P_1 at the bottom of the oil layer. Then use this pressure in place of P_0 in Equation 8.6 and calculate the pressure P_{bot} at the bottom of the water layer.

Example 8.3: Car lift

GOAL Apply Pascal's principle to a car lifts, and shows that the input work is the same as the output work.

In a car lift used in a service station, compressed air exerts a force on a small piston of circular cross section having a radius of r_1 = 5.00 cm. This pressure is transmitted by an incompressible liquid to a second piston of radius r_2 = 15.0 cm.

(a) What force must the compressed air exert on the small piston in order to lift a car weighing 13 300 N? Neglect the weights of the pistons.

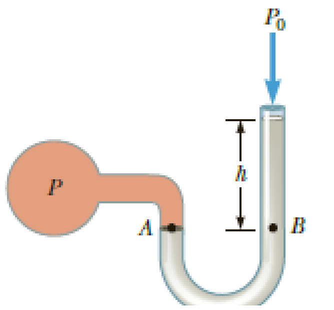
(b) What air pressure will produce a force of that magnitude?

(c) Show that the work done by the input and output pistons is the same.

Solution:

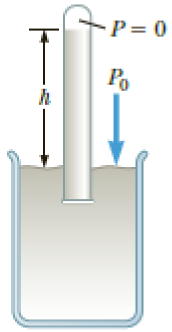
Substitute into Pascal's principle in part (a), while recognizing that the magnitude of the output force, F_2, must be equal to the car's weight in order to support it. Use the definition of pressure in part (b). In part (c), use $W = Fx$ to find the ratio W_1/W_2, showing that it must equal 1. This requires combining Pascal's principle with the fact that the input and output pistons move through the same volume.

8.1.4 Pressure measurements



A simple device for measuring pressure is the open-tube manometer. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure *P*. The pressure at point *B* equals $P_0 + \rho g h$, where ρ is the density of the fluid. The pressure at *B*, however, equals the pressure at *A*, which is also the unknown pressure *P*. We conclude that $P = P_0 + \rho g h$.

The pressure *P* is called the absolute pressure, and $P - P_0$ is called the gauge pressure. If *P* in the system is greater than atmospheric pressure, *h* is positive. If *P* is less than atmospheric pressure (a partial vacuum), *h* is negative, meaning that the right-hand column is lower than the left-hand column.



Another instrument used to measure pressure is the barometer, invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury. The closed end of the tube is nearly a vacuum, so its pressure can be taken to be zero. It follows that $P_0 = \rho g h$, where ρ is the density of mercury and *h* is the height of the mercury column. Note that the barometer measures the pressure of the atmosphere, whereas the manometer measures pressure in an enclosed fluid.

8.2 Buoyant Forces, Archimedes' Principle

Learning Outcomes

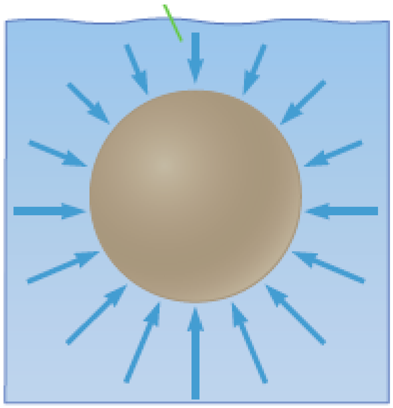
After completing this section, students are expected to:

- Explain Archimedes' principle
- Explain and evaluate buoyant force

A fundamental principle affecting objects submerged in fluids was discovered by Greek mathematician and natural philosopher Archimedes. Archimedes' principle can be stated as follows:

Any object completely or partially submerged in a fluid is buoyed up by a force with magnitude equal to the weight of the fluid displaced by the object.

In the figure, the cannon ball is pressed on all sides by the surrounding fluid. Arrows indicate the forces arising from the pressure. Because pressure increases with depth, the arrows on the underside are larger than those on top.



Adding them all up, the horizontal components cancel, but there is a net force upward. This force, due to differences in pressure, is the buoyant force B. The sphere of water neither rises nor falls, so the vector sum of the buoyant force and the force of gravity on the sphere of fluid must be zero, and it follows that *B* 5 *Mg*, where *M* is the mass of the fluid. The buoyant force, therefore, is equal in magnitude to the weight of the displaced fluid.

Archimedes' principle can also be obtained from Equation 8.3, relating pressure and depth, using figure in section 8.1.3. Horizontal forces from the pressure cancel, but in the vertical direction P_2A acts upward on the bottom of the block of fluid, and P_1A and the gravity force on the fluid, *Mg*, act downward, giving

$$\boldsymbol{B = P_2A - P_1A = Mg} \quad 8.7a$$

where the buoyancy force has been identified as the result of differences in pressure and is equal in magnitude to the weight of the displaced fluid. This buoyancy force remains the same regardless of the material occupying the volume in question because it's due to the *surrounding* fluid. Using the definition of density, Equation 8.7a becomes

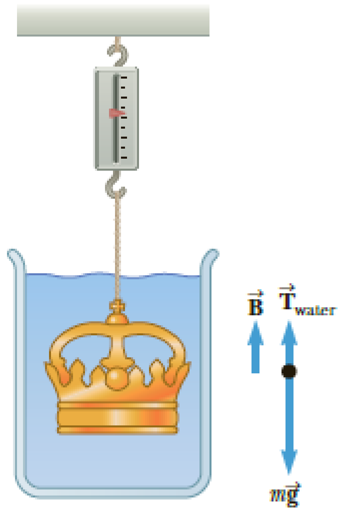
$$\boldsymbol{B = \rho_{fluid} V_{fluid} g} \quad 8.7b$$

where ρ_{fluid} is the density of the fluid and V_{fluid} is the volume of the displaced fluid.

Example 8.4: A red-tag special on crowns

GOAL Apply Archimedes' principle to a submerged object.

A bargain hunter purchases a "gold" crown at a flea market. After she gets home, she hangs it from a scale and finds its weight to be 7.84 N. She then weighs the crown while it is immersed in water, as in the figure, and now the scale reads 6.86 N. Is the crown made of pure gold?



STRATEGY The goal is to find the density of the crown and compare it to the density of gold. We already have the weight of the crown in air, so we can get the mass by dividing by the acceleration of gravity. If we can find the volume of the crown, we can obtain the desired density by dividing the mass by this volume.

Example 8.5: Floating down the river

GOAL Apply Archimedes' principle to a partially submerged object.

A raft is constructed of wood having a density of 6.00×10^2 kg/m^3. Its surface area is 5.70 m^2, and its volume is 0.60 m^3. When the raft is placed in fresh water, to what depth *h* is the bottom of the raft submerged?

If the raft is placed in salt water, which has a density greater than fresh water, would the value of *h* (a) decrease, (b) increase, or (c) not change?

Solution:

There are two forces acting on the raft: the buoyant force of magnitude *B*, acting upward, and the force of gravity, acting downward. Because the raft is in equilibrium, the sum of these forces is zero. The buoyant force depends on the submerged volume $V_{water} = Ah$. Set up Newton's second law and solve for *h*, the depth reached by the bottom of the raft.

8.3 Moving Fluids and Bernoulli's Equation

Learning Outcomes

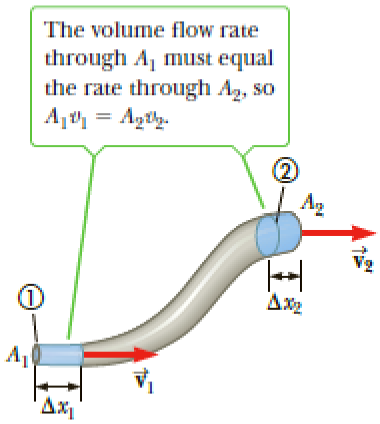
After completing this section, students are expected to:

- Define laminar and turbulent flow
- Explain and apply the equation of continuity
- Explain and apply Bernoulli's equation
- Calculate the speed of a fluid

When a fluid is in motion, its flow can be characterized in one of two ways. The flow is said to be streamline, or laminar, if every particle that passes a particular point moves along exactly the same smooth path followed by previous particles passing that point.

In contrast, the flow of a fluid becomes irregular, or turbulent, above a certain velocity or under any conditions that can cause abrupt changes in velocity.

8.3.1 Equation of continuity



Let us consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move along the streamlines in steady-state flow. In a small time interval ?t, the fluid entering the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$, where v_1 is the speed of the fluid at that location. If A_1 is the cross-sectional area in this region, then the mass contained in the bottom blue region is $\Delta M_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$, where ? is the density of the fluid. Similarly, the fluid that moves out of the upper end of the pipe in the same time interval ?t has a mass of $\Delta M_2 = \rho A_2 v_2 \Delta t$. However, because mass is conserved and because the flow is steady, the mass that flows into the bottom of the pipe through A_1 in the time ?t must equal the mass that flows out through A_2 in the same interval. Therefore, $\Delta M_1 = \Delta M_2$, or

$$\boldsymbol{A_1 v_1 = A_2 v_2} \qquad 8.8$$

This expression is called the equation of continuity.

Example 8.6: Niagara Falls

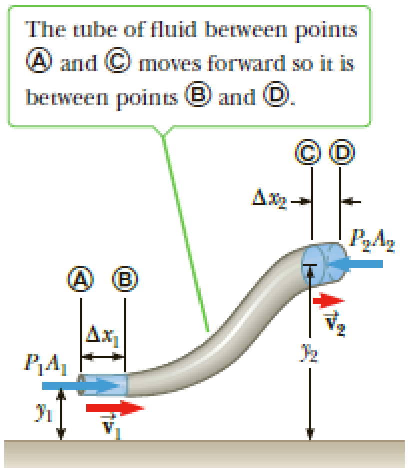
GOAL Apply the equation of continuity.

Each second, 5 525 m^3 of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. Estimate its speed at that instant. By what factor would the range be changed if the flow rate were doubled?

Solution:

This is an estimate, so only one significant figure will be retained in the answer. The volume flow rate is given, and, according to the equation of continuity, is a constant equal to *Av.* Find the cross-sectional area, substitute, and solve for the speed.

8.3.2 Bernoulli's equation



In deriving Bernoulli's equation, we again assume the fluid is incompressible, non-viscous, and flows in a non-turbulent, steady-state manner. Consider the flow through a non-uniform pipe in the time Δt, as in the figure. The force on the lower end of the fluid is P_1A_1, where P_1 is the pressure at the lower end. The work done on the lower end of the fluid by the fluid behind it is

$$\boldsymbol{W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 V}$$

where V is the volume of the lower blue region in the figure. In a similar manner, the work done on the fluid on the upper portion in the time Δt is

$$W_2 = -P_2 A_2 \Delta x_2 = -P_2 V$$

The volume is the same because, by the equation of continuity, the volume of fluid that passes through A_1 in the time Δt equals the volume that passes through A_2 in the same interval. The work W_2 is negative because the force on the fluid at the top is opposite its displacement. The net work done by these forces in the time Δt is

$$W_{fluid} = P_1 V - P_2 V$$

Part of this work goes into changing the fluid's kinetic energy, and part goes into changing the gravitational potential energy of the fluid–Earth system. If *m* is the mass of the fluid passing through the pipe in the time interval Δt, then the change in kinetic energy of the volume of fluid is

$$\Delta KE = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

The change in the gravitational potential energy is

$$\Delta PE = mgy_2 - mgy_1$$

Because the net work done by the fluid on the segment of fluid shown changes the kinetic energy and the potential energy of the latter, we have

$$W_{fluid} = \Delta KE + \Delta PE$$

Substituting expressions for each of the terms gives

$$P_1 V - P_2 V = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

If we divide each term by *V* and recall that □ = *m*/*V*, this expression becomes

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g y_2 - \rho g y_1$$

Rearrange the terms as follows:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \qquad 8.9$$

This is Bernoulli's equation, often expressed as

$$P + \frac{1}{2} \rho v^2 + \rho g y = constant \qquad 8.10$$

Bernoulli's equation states that the sum of the pressure *P*, the kinetic energy per unit volume, $1/2\rho v^2$, and the potential energy per unit volume, $\rho g y$, has the same value at all points along a streamline.

Example 8.7: Shoot-out at the old water tank

GOAL Apply Bernoulli's equation to find the speed of a fluid.

A nearsighted sheriff fires at a cattle rustler with his trusty six-shooter. Fortunately for the rustler, the bullet misses him and penetrates the town water tank, causing a leak.

(a) If the top of the tank is open to the atmosphere, determine the speed at which the water leaves the hole when the water level is 0.500 m above the hole.

(b) Where does the stream hit the ground if the hole is 3.00 m above the ground?

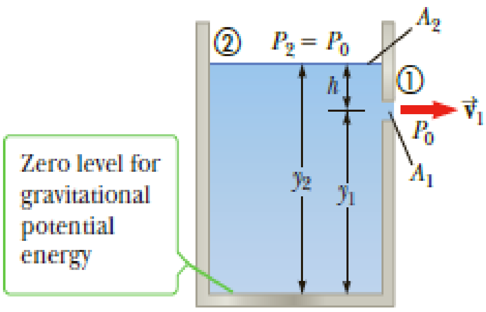
(c) As time passes, what happens to the speed of the water leaving the hole?

(d) Suppose, in a similar situation, the water hits the ground 4.20 m from the hole in the tank. If the hole is 2.00 m above the ground, how far above the hole is the water level

---

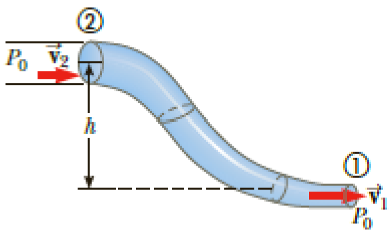
Solution:

(a) Assume the tank's cross-sectional area is large compared to the hole's ($A_2 >> A_1$), so the water level drops very slowly and $v_2 \approx 0$. Apply Bernoulli's equation to points ① and ② in the figure, noting that P_1 equals atmospheric pressure P_0 at the hole and is approximately the same at the top of the water tank. Part (b) can be solved with kinematics, just as if the water were a ball thrown horizontally.



Example 8.8: Fluid flow in a pipe

GOAL Solve a problem combining Bernoulli's equation and the equation of continuity.



A large pipe with a cross-sectional area of 1.00 m^2 descends 5.00 m and narrows to 0.500 m^2, where it terminates in a valve at point ① (Figure). If the pressure at point ② is atmospheric pressure, and the valve is opened wide and water allowed to flow freely, find the speed of the water leaving the pipe.

Solution

The equation of continuity, together with Bernoulli's equation, constitute two equations in two unknowns: the speeds v_1 and v_2. Eliminate v_2 from Bernoulli's equation with the equation of continuity, and solve for v_1.

Example 8.9: Lift on airfoil

GOAL Use Bernoulli's equation to calculate the lift on an airplane wing.

An airplane has wings, each with area 4.00 m^2, designed so that air flows over the top of the wing at 245 m/s and underneath the wing at 222 m/s. (a) Find the mass of the airplane such that the lift on the plane will support its weight, assuming the force from the pressure difference across the wings is directed straight upward. (b) Why is the maximum lift affected by increasing altitude? (c) Approximately what size wings would an aircraft need on Mars if its engine generates the same differences in speed as in the example and the total mass of the craft is 400 kg? The density of air on the surface of Mars is approximately one percent Earth's density at sea level, and the acceleration of gravity on the surface of Mars is about 3.8 m/s^2.

Solution:

This problem can be solved by substituting values into Bernoulli's equation to find the pressure difference between the air under the wing and the air over the wing, followed by applying Newton's second law to find the mass the airplane can lift.

8.4 Properties of Bulk Matter/Stress, Strain/

Learning Outcomes

After completing this section, students are expected to:

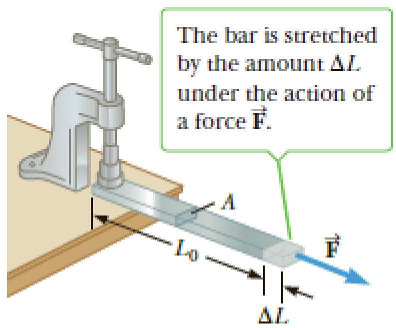
- Define stress and strain and their relationship
- Define Young's modulus, shear modulus and bulk modulus

The elastic properties of solids are discussed in terms of stress and strain. Stress is the force per unit area causing a deformation; strain is a measure of the amount of the deformation. For sufficiently small stresses, stress is proportional to strain, with the constant of proportionality depending on the material being deformed and on the nature of the deformation. We call this proportionality constant the elastic modulus:

$$\text{stress} = \text{elastic modulus} \times \text{strain} \qquad 8.11$$

8.4.1 Young's Modulus: elasticity in Length

Consider a long bar of cross-sectional area A and length L_0, clamped at one end. When an external force F is applied along the bar, perpendicular to the cross section, internal forces in the bar resist the distortion ("stretching") that F tends to produce. Nevertheless, the bar attains an equilibrium in which (1) its length is greater than L_0 and (2) the external force is balanced by internal forces.

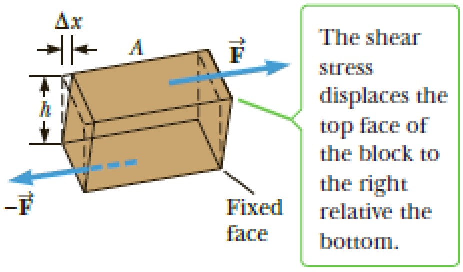


We define the tensile stress as the ratio of the magnitude of the external force F to the cross-sectional area A. The SI unit of stress is the newton per square meter (N/m^2), called the pascal (Pa). The tensile strain in this case is defined as the ratio of the change in length ΔL to the original length L_0 and is therefore a dimensionless quantity. Using Equation 8.11, we can write an equation relating tensile stress to tensile strain:

$$\frac{F}{A} = Y\frac{\Delta L}{L_0} \qquad 8.12$$

Y is the constant of proportionality, called Young's modulus.

8.4.2 Shear Modulus: elasticity of Shape



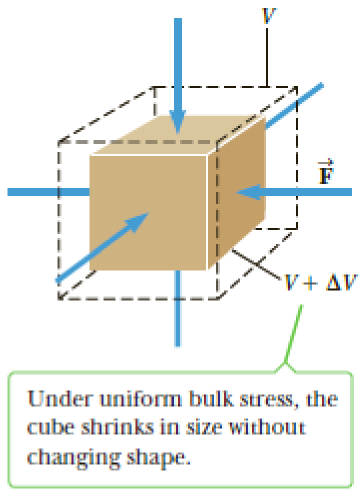
Another type of deformation occurs when an object is subjected to a force F *parallel* to one of its faces while the opposite face is held fixed by a second force. If the object is originally a rectangular block, such a parallel force results in a shape with the cross section of a parallelogram. This kind of stress is called a shear stress. There is no change in volume with this kind of deformation.

We define the shear stress as *F/A,* the ratio of the magnitude of the parallel force to the area *A* of the face being sheared. The shear strain is the ratio $\Delta x/h$, where Δx is the horizontal distance the sheared face moves and *h* is the height of the object. The shear stress is related to the shear strain according to

$$\frac{F}{A} = S\frac{\Delta x}{h} \qquad 8.13$$

where *S* is the shear modulus of the material, with units of Pascal.

8.4.3 8.4.3 Bulk Modulus: volume elasticity



The bulk modulus characterizes the response of a substance to uniform squeezing. Suppose the external forces acting on an object are all perpendicular to the surface on which the force acts and are distributed uniformly over the surface of the object. This occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. The volume stress ΔP is defined as the ratio of the change in the magnitude of the applied force ΔF to the surface area *A.* From the definition of pressure in Section 8.2, ΔP is also simply a change in pressure. The volume strain is equal to the change in volume ΔV divided by the original volume *V.* Again using Equation 8.11, we can relate a volume stress to a volume strain by the formula

$$\Delta P = -B\frac{\Delta V}{V} \qquad 8.14$$

Note that a negative sign is included in this defining equation so that *B* is always positive. An increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa.

Example 8.10: Built to last

GOAL Calculate a compression due to tensile stress and maximum load.

A vertical steel beam in a building supports a load of 6.0×10^4 N. (a) If the length of the beam is 4.0 m and its cross-sectional area is 8.0×10^{-3} m^2, find the distance the beam is compressed along its length. (b) What maximum load in newton could the steel beam support before failing?

Solution:

Equation 8.11 pertains to compressive stress and strain and can be solved for ?*L*, followed by substitution of known values. For part (b), set the compressive stress equal to the ultimate strength of steel from the Table. Solve for the magnitude of the force, which is the total weight the structure can support.

Example 8.11: American football injuries

GOAL Obtain an estimate of shear stress.

A defensive lineman of mass M = 125 kg makes a flying tackle at v_i = 4.00 m/s on a stationary quarterback of mass m = 85.0 kg, and the lineman's helmet makes solid contact with the quarterback's femur. (a) What is the speed v_f of the two athletes immediately after contact? Assume a linear perfectly inelastic collision. (b) If the collision lasts for 0.100 s, estimate the average force exerted on the quarterback's femur. (c) If the cross-sectional area of the quarterback's femur is equal to 5.00×10^{-4} m^2, calculate the shear stress exerted on the bone in the collision.

Solution:

The solution proceeds in three well-defined steps. In part (a), use conservation of linear momentum to calculate the final speed of the system consisting of the quarterback and the lineman. Second, the speed found in part (a) can be used in the impulse-momentum theorem to obtain an estimate of the average force exerted on the femur. Third, dividing the average force by the cross-sectional area of the femur gives the desired estimate of the shear stress.

Example 8.12: Lead ballast overboard

GOAL Apply the concepts of bulk stress and strain.

Ships and sailing vessels often carry lead ballast in various forms, such as bricks, to keep the ship properly oriented and upright in the water. Suppose a ship takes on cargo and the crew jettisons a total of 0.500 m^3 of lead ballast into water 2.00 km deep. Calculate (a) the change in the pressure at that depth and (b) the change in volume of the lead upon reaching the bottom. Take the density of sea water to be 1.025×10^3 kg/m^3, and take the bulk modulus of lead to be 4.2×10^{10} Pa.

Solution:

The pressure difference between the surface and a depth of 2.00 km is due to the weight of the water column. Calculate the weight of water in a column with cross section of 1.00 m^2. That number in newton will be the same magnitude as the pressure difference in pascal. Substitute the pressure change into the bulk stress and strain equation to obtain the change in volume of the lead.

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)
Iron	1.7×10^8	5.5×10^8
Steel	5.0×10^8	5.0×10^8
Aluminum	2.0×10^8	2.0×10^8
Bone	1.2×10^8	1.5×10^8
Marble	—	8.0×10^7
Brick	1×10^6	3.5×10^7
Concrete	2×10^6	2×10^7

8.5 Chapter Summary

Density and Pressure of Static Fluids

The density ρ of a substance of uniform composition is its mass per unit volume—kilograms per cubic meter (kg/m^3) in the SI system:

$$\rho \equiv \frac{M}{V}$$

The pressure *P* in a fluid, measured in Pascal (Pa), is the force per unit area that the fluid exerts on an object immersed in it:

$$P \equiv \frac{F}{A}$$

The pressure in an incompressible fluid varies with depth *h* according to the expression

$$P = P_0 + \rho g h$$

where P_0 is atmospheric pressure (1.013×10^5 Pa) and r is the density of the fluid.

Pascal's principle states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point of the fluid and to the walls of the containing vessel.

Buoyant Forces and Archimedes' Principle

When an object is partially or fully submerged in a fluid, the fluid exerts an upward force, called the buoyant force, on the object. This force is, in fact, due to the net difference in pressure between the top and bottom of the object. It can be shown that the magnitude of the buoyant force *B* is equal to the weight of the fluid displaced by the object, or

$$B = \rho_{fluid} V_{fluid} g$$

The above equation is known as Archimedes' principle.

Moving Fluids and Bernoulli's Equation

Certain aspects of a fluid in motion can be understood by assuming the fluid is non-viscous and incompressible and that its motion is in a steady state with no turbulence:

1. The flow rate through the pipe is a constant, which is equivalent to stating that the product of the cross-sectional area *A* and the speed *v* at any point is constant.

At any two points, therefore, we have

$$A_1 v_1 = A_2 v_2$$

This relation is referred to as the equation of continuity.

2. The sum of the pressure, the kinetic energy per unit volume, and the potential energy per unit volume is the same at any two points along a streamline:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This equation is known as Bernoulli's equation. Solving problems with Bernoulli's equation is similar to solving problems with the work–energy theorem, whereby two points are chosen, one point where a quantity is unknown and another where all quantities are known. The above equation is then solved for the unknown quantity.

Properties of Bulk Matter/Stress, Strain/

The elastic properties of a solid can be described using the concepts of stress and strain. Stress is related to the force per unit area producing a deformation; strain is a measure of the amount of deformation. Stress is proportional to strain, and the constant of proportionality is the elastic modulus:

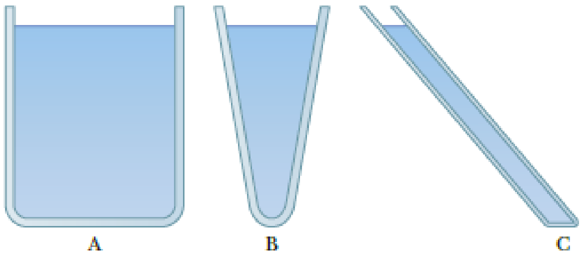
$$Stress = Elastic\ Modulys \times Strain$$

Three common types of deformation are (1) the resistance of a solid to elongation or compression, characterized by Young's modulus *Y*; (2) the resistance to displacement of the faces of a solid sliding past each other, characterized by the shear modulus *S*; and (3) the resistance of a solid or liquid to a change in volume, characterized by the bulk modulus *B.*

All three types of deformation obey laws similar to Hooke's law for springs. Solving problems is usually a matter of identifying the given physical variables and solving for the unknown variable.

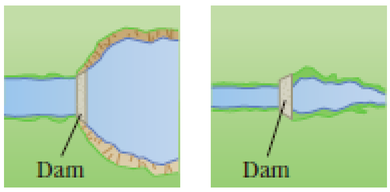
8.6 Conceptual Questions

1. The three containers are filled with water to the same level. Rank the pressures at the bottom of the containers (choose one): (a) $P_A > P_B > P_C$ (b) $P_A > P_B = P_C$ (c) $P_A = P_B > P_C$ (d) $P_A < P_B < P_C$ (e) $P_A = P_B = P_C$

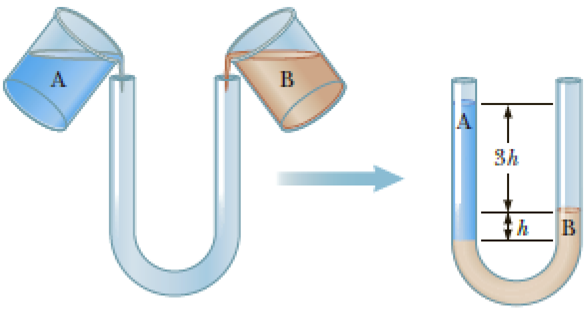


2. The density of air is 1.3 kg/m^3 at sea level. From your knowledge of air pressure at ground level, estimate the height of the atmosphere. As a simplifying assumption, take the atmosphere to be of uniform density up to some height, after which the density rapidly falls to zero. (In reality, the density of the atmosphere decreases as we go up.)

3. Figure shows aerial views from directly above two dams. Both dams are equally long (the vertical dimension in the diagram) and equally deep (into the page in the diagram). The dam on the left holds back a very large lake, while the dam on the right holds back a narrow river. Which dam has to be built more strongly?



4. Equal volumes of two fluids are added to the U-shaped pipe as shown in Figure. The pipe is open at



both ends and the fluids come to equilibrium without mixing. (a) Which fluid has the higher density, fluid A or fluid B? (b) What is the ratio ρ_B/ρ_A of the fluid densities?

5. Water flows along a streamline down a river of constant width. Over a short distance the water slows from speed *v* to *v*/3. Which of the following can you correctly conclude about the river's depth? (a) It became deeper by a factor of 3. (b) It became shallower by a factor of 3. (c) It became deeper by a factor of 3^2. (d) It became shallower by a factor of 3^2.
6. The water supply for a city is often provided from reservoirs built on high ground. Water flows from the reservoir, through pipes, and into your home when you turn the tap on your faucet. Why is the water flow more rapid out of a faucet on the first floor of a building than in an apartment on a higher floor?
7. An ice cube is placed in a glass of water. What happens to the level of the water as the ice melts?
8. Tornadoes and hurricanes often lift the roofs of houses. Use the Bernoulli Effect to explain why. Why should you keep your windows open under these conditions?
9. A person in a boat floating in a small pond throws an anchor overboard. What happens to the level of the pond? (a) It rises. (b) It falls. (c) It remains the same.

8.7 Exercises

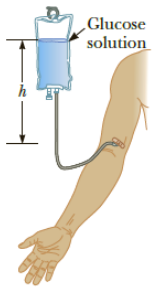
1. A giant oil storage facility contains oil to a depth of 40.0 m. How does the pressure at the bottom of the tank compare to the pressure at a depth of 40.0 m in water? Explain.
2. A large rectangular tub is filled to a depth of 2.60 m with olive oil, which has density 915 kg/m^3. If the tub has length 5.00 m and width 3.00 m, calculate (a) the weight of the olive oil, (b) the force of air pressure on the surface of the oil, and (c) the pressure exerted upward by the bottom of the tub.
3. An airplane takes off at sea level and climbs to a height of 425 m. Estimate the net outward force on a passenger's eardrum assuming the density of air is approximately constant at 1.3 kg/m^3 and that the inner ear pressure hasn't been equalized.
4. A hydraulic lift has pistons with diameters 8.00 cm and 36.0 cm, respectively. If a force of 825 N is exerted at the input piston, what maximum mass can be lifted by the output piston?
5. Blood pressure is normally measured with the cuff of the sphygmomanometer around the arm. Suppose the blood pressure is measured with the cuff around the calf of the leg of a standing person. Would the reading of the blood pressure be (a) the same here as it is for the arm, (b) greater than it is for the arm, or (c) less than it is for the arm?
6. Atmospheric pressure varies from day to day. The level of a floating ship on a high-pressure day is (a) higher, (b) lower, or (c) no different than on a low-pressure day.
7. The density of lead is greater than iron, and both metals are denser than water. Is the buoyant force on a solid lead object (a) greater than, (b) equal to, or (c) less than the buoyant force acting on a solid iron object of the same dimensions?
8. Calculate how much of an iceberg is beneath the surface of the ocean, given that the density of ice is 917 kg/m^3 and salt water has density 1 025 kg/m^3.

9. Water flowing in a horizontal pipe is at a pressure of 1.40×10^5 Pa at a point where its cross-sectional area is 1.00 m^2. When the pipe narrows to 0.400 m^2, the pressure drops to 1.16×10^5 Pa. Find the water's speed (a) in the wider pipe and (b) in the narrower pipe.
10. Rank by the amount of fractional increase in length under increasing tensile stress, from smallest to largest: rubber, tungsten, steel, aluminum.
11. A cable used to lift heavy materials like steel I-beams must be strong enough to resist breaking even under a load of 1.0×10^6 N. For safety, the cable must support twice that load. (a) What cross-sectional area should the cable have if it's to be made of steel? (b) By how much will an 8.0-m length of this cable stretch when subject to the 1.0×10^6 N load?
12. Rank the following substances in order of the fractional change in volume in response to increasing pressure, from smallest to largest: copper, steel, water, mercury.
13. (a) By what percentage does the volume of a ball of water shrink at that same depth? (b) What is the ratio of the new radius to the initial radius?

8.8 Problems

Density and Pressure in Static Fluids

1. The weight of Earth's atmosphere exerts an average pressure of 1.01×10^5 Pa on the ground at sea level. Use the definition of pressure to estimate the weight of Earth's atmosphere by approximating Earth as a sphere of radius $R_E = 6.38 \times 10^6$ m and surface area $A = 4\pi R_E^2$.
2. The four tires of an automobile are inflated to a gauge pressure of 2.0×10^5 Pa. Each tire has an area of 0.024 m^2 in contact with the ground. Determine the weight of the automobile.
3. A normal blood pressure reading is less than 120/80 where both numbers are gauge pressures measured in millimeters of mercury (mmHg). What are the (a) absolute and (b) gauge pressures in pascals at the base of a 0.120 m column of mercury?
4. A collapsible plastic bag (see figure) contains a glucose solution. If the average gauge pressure in the vein is 1.33×10^3 Pa, what must be the minimum height h of the bag to infuse glucose into the vein? Assume the specific gravity of the solution is 1.02.

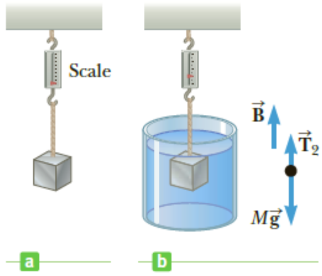


5. A hydraulic jack has an input piston of area 0.050 m^2 and an output piston of area 0.70 m^2. How much force on the input piston is required to lift a car weighing 1.2×10^4 N?
6. A container is filled to a depth of 20.0 cm with water. On top of the water floats a 30.0-cm-thick layer of oil with specific gravity 0.700. What is the absolute pressure at the bottom of the container?

Buoyant Forces and Archimedes' Principle

1. A table-tennis ball has a diameter of 3.80 cm and average density of 0.084 g/cm^3. What force is required to hold it completely submerged under water?

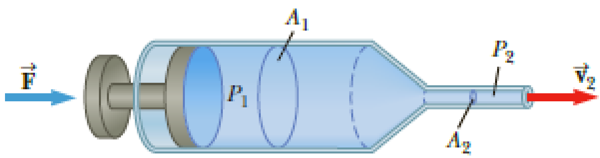
2. A 62.0-kg survivor of a cruise line disaster rests atop a block of Styrofoam insulation, using it as a raft. The Styrofoam has dimensions 2.00 m x 2.00 m x 0.090 m. The bottom 0.024 m of the raft is submerged. (a) Draw a force diagram of the system consisting of the survivor and raft. (b) Write Newton's second law for the system in one dimension, using B for buoyancy, w for the weight of the survivor, and w_r for the weight of the raft. (Set $a = 0$.) (c) Calculate the numeric value for the buoyancy, B. (Seawater has density 1 025 kg/m^3.) (d) Using the value of B and the weight w of the survivor, calculate the weight w_r of the Styrofoam. (e) What is the density of the Styrofoam? (f) What is the maximum buoyant force, corresponding to the raft being submerged up to its top surface? (g) What total mass of survivors can the raft support?
3. A hot-air balloon consists of a basket hanging beneath a large envelope filled with hot air. A typical hot-air balloon has a total mass of 545 kg, including passengers in its basket, and holds 2.55×10^3 m^3 of hot air in its envelope. If the ambient air density is 1.25 kg/m^3, determine the density of hot air inside the envelope when the balloon is neutrally buoyant. Neglect the volume of air displaced by the basket and passengers.
4. A cube of wood having an edge dimension of 20.0 cm and a density of 650. kg/m^3 floats on water. (a) What is the distance from the horizontal top surface of the cube to the water level? (b) What mass of lead should be placed on the cube so that the top of the cube will be just level with the water surface?



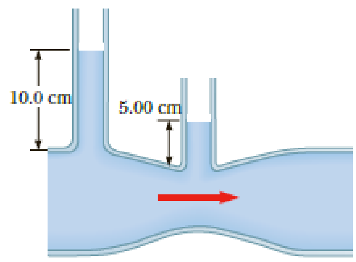
5. The gravitational force exerted on a solid object is 5.00 N as measured when the object is suspended from a spring scale as in the figure. When the suspended object is submerged in water, the scale reads 3.50 N (see figure). Find the density of the object.
6. A sample of an unknown material appears to weigh 300. N in air and 200. N when immersed in alcohol of specific gravity 0.700. What are (a) the volume and (b) the density of the material?

Moving Fluids and Bernoulli's Equation

1. A horizontal pipe narrows from a radius of 0.250 m to 0.100 m. If the speed of the water in the pipe is 1.00 m/s in the larger radius pipe, what is the speed in the smaller pipe?
2. A hypodermic syringe contains a medicine with the density of water. The barrel of the syringe has a cross-sectional area of 2.50×10^{-5} m^2. In the absence of a force on the plunger, the pressure everywhere is 1.00 atm. A force F of magnitude 2.00 N is exerted on the plunger, making medicine squirt from the needle. Determine the medicine's flow speed through the needle. Assume the pressure in the needle remains equal to 1.00 atm and that the syringe is horizontal.

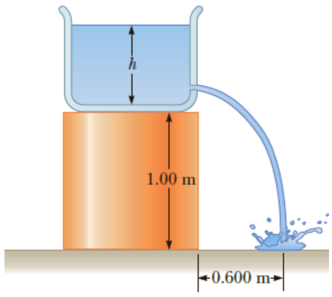


3. A jet airplane in level flight has a mass of 8.66×10^4 kg, and the two wings have an estimated total area of 90.0 m^2. (a) What is the pressure difference between the lower and upper surfaces of the wings? (b) If the speed



of air under the wings is 225 m/s, what is the speed of the air over the wings? Assume air has a density of 1.29 kg/m^3. (c) Explain why all aircraft have a "ceiling," a maximum operational altitude.

4. The inside diameters of the larger portions of the horizontal pipe depicted in the figure are 2.50 cm. Water flows to the right at a rate of 1.80 × 10^{-4} m^3/s. Determine the inside diameter of the constriction.
5. A man attaches a divider to an outdoor faucet so that water flows through a single pipe of radius 9.00 mm into two pipes, each with a radius of 6.00 mm. If water flows through the single pipe at 1.25 m/s, calculate the speed of the water in the narrower pipes.
6. A jet of water squirts out horizontally from a hole near the bottom of the tank shown in the figure. If the hole has a diameter of 3.50 mm, what is the height *h* of the water level in the tank?



Properties of Bulk Matter/Stress, Strain/

1. A 200.- kg load is hung on a wire of length 4.00 m, cross-sectional area 0.200 × 10^{-4} m^2, and Young's modulus 8.00 × 10^{10} N/m^2. What is its increase in length?
2. Artificial diamonds can be made using high-pressure, high temperature presses. Suppose an artificial diamond of volume 1.00 × 10^{-6} m^3 is formed under a pressure of 5.00 GPa. Find the change in its volume when it is released from the press and brought to atmospheric pressure. Take the diamond's bulk modulus to be B = 194 GPa.
3. Bone has a Young's modulus of 18 × 10^9 Pa. Under compression, it can withstand a stress of about 160 × 10^6 Pa before breaking. Assume that a femur (thigh bone) is 0.50 m long, and calculate the amount of compression this bone can withstand before breaking.

9 Heat and Thermodynamics

Learning Outcomes

After reading this chapter, students will be able to:

- explain the difference between heat and temperature.
- identify the lowest temperature as zero on the Kelvin scale (absolute zero).
- explain the zeroth and first law of thermodynamics.
- understand that heat is the amount of transferred energy (either to or from an object's thermal energy) due to a temperature difference between the object and its environment.
- convert a temperature between any two (linear) temperature scales, including the Celsius, Fahrenheit, and Kelvin scales.
- apply the first law of thermodynamics to relate the change in the internal energy of a system, the energy transferred as heat to or from the system, and the work done on or by the system.

Introduction

The terms temperature and heat are often used interchangeably in everyday language. In physics, however, these two terms have very different meanings. A quantitative

description of thermal phenomena requires careful definitions of such important terms as temperature, heat, and internal energy. Heat leads to changes in internal energy and thus to changes in temperature, which cause the expansion or contraction of matter. In this chapter we will define temperature in terms of how it is measured and see how temperature changes affect the dimensions of objects. We will see that heat refers to energy transfer caused by temperature differences only and learn how to calculate and control such energy transfers. Our emphasis in this chapter is on the concepts of

temperature and heat as they relate to macroscopic objects such as cylinders of gas, ice cubes, and the human body. We will also look into the concept of internal energy, the first law of thermodynamics, and some important applications of the first law. The first law of thermodynamics describes systems in which the only energy change is that of

internal energy, and the transfers of energy are by heat and work. A major difference in our discussion of work in this chapter from that in most of the chapters on mechanics is that we will consider work done on deformable systems.

9.1 The concept of Temperature: Zeroth Law of Thermodynamics

Learning Outcomes

After reading this section, students will be able to:

- understand temperature as the property that determines whether an object is in thermal equilibrium with other objects.
- explain that two objects in thermal equilibrium with each other are at the same temperature.
- explain the zeroth law of thermodynamics.
- understand the difference between heat and temperature.

We often associate the concept of temperature with how hot or cold an object feels when we touch it. In this way, our senses provide us with a qualitative indication of temperature. Our senses, however, are unreliable and often mislead us. For example, if you stand in bare feet with one foot on carpet and the other on an adjacent tile floor, the tile feels colder than the carpet even though both are at the same temperature. The two objects feel different because tile transfers energy by heat at a higher rate than carpet does. Your skin "measures" the rate of energy transfer by heat rather than the actual temperature. What we need is a reliable and reproducible method for measuring the relative hotness or coldness of objects or a method related solely to temperature measurement rather than the rate of energy transfer. Scientists have developed a variety of thermometers for making such quantitative measurements.

Two objects at different initial temperatures eventually reach some intermediate temperature when placed in contact with each other. For example, when hot water and cold water are mixed in a bathtub, energy is transferred from the hot water to the cold water and the final temperature of the mixture is somewhere between the initial hot and cold temperatures. Imagine that two objects are placed in an insulated container such that they interact with each other but not with the environment. If the objects are at different temperatures, energy is transferred between them, even if they are initially not in physical contact with each other. The energy-transfer mechanisms are heat and electromagnetic radiation. For purposes of this discussion, assume that two objects are in thermal contact with each other if energy can be exchanged between them by these processes due to a temperature difference. Thermal equilibrium is a situation in which two objects would not exchange energy by heat or electromagnetic radiation if they were placed in thermal contact.

Let's consider two objects A and B, which are not in thermal contact, and a third object C, which is our thermometer. We wish to determine whether A and B are in thermal equilibrium with each other. The thermometer (object C) is first placed in thermal contact with object A until thermal equilibrium is reached. From that moment on, the thermometer's reading remains constant and we record this reading. The thermometer is then removed from object A and placed in thermal contact with object B. The reading is again recorded after thermal equilibrium is reached. If the two readings are the same, we can conclude that object A and object B are in thermal equilibrium with each other. If they are placed in contact with each other, there is no exchange of energy between them. We can summarize these results in a statement known as ***the zeroth law of thermodynamics*** (the law of equilibrium):

If objects A and B are separately in thermal equilibrium with a third object C, then A and B are in thermal equilibrium with each other.

This statement can easily be proved experimentally and is very important because it enables us to define temperature. We can think of temperature as the property that determines whether an object is in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature. Conversely, if two objects have different temperatures, they are not in thermal equilibrium with each other. We now know that temperature is something that determines whether or not energy will transfer between two objects in thermal contact.

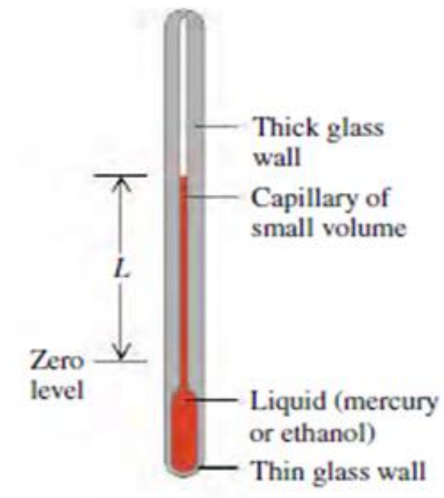


Figure 9.1 Changes in temperature cause the liquid's volume to change.

Example:

Does the temperature of a body depend on its size?

Solution:

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

Question:

Two objects, with different sizes, masses, and temperatures, are placed in thermal contact. In which direction does the heat energy travel? (a) From the larger object to the smaller object. (b) From the object with more mass to the one with less mass. (c) From the object at higher temperature to the object at lower temperature.

9.2 Temperature scales and Absolute Temperature

Learning Outcomes

After reading this section, students will be able to:

- explain the conditions for measuring a temperature with a constant-volume gas thermometer.
- relate the pressure and temperature of a gas in some given state to the pressure and temperature at the triple point for a constant-volume gas thermometer.
- convert a temperature between any two (linear) temperature scales, including the Celsius, Fahrenheit, and Kelvin scales.
- identify that a change of one degree is the same on the Celsius and Kelvin scales.
- Understand how to relate different temperature scales.

Thermometers are devices used to measure the temperature of a system. All thermometers are based on the principle that some physical property of a system changes as the system's temperature changes. Some physical properties that change with temperature are the volume of a liquid, the dimensions of a solid, the pressure of a gas at constant volume, the volume of a gas at constant pressure, the electric resistance of a conductor, and the color of an object.

A common thermometer in everyday use consists of a mass of liquid—usually mercury or alcohol that expands into a glass capillary tube when heated as shown in Figure 9.1. In this case, the physical property that changes is the volume of a liquid. Any temperature change in the range of the thermometer can be defined as being proportional to the change in length of the liquid column. In order to measure temperature quantitatively, some sort of numerical scale must be defined. The most common scale today is the Celsius scale, sometimes called the centigrade scale. In some countries, the Fahrenheit scale is also common. The most important scale in scientific work is the absolute or Kelvin scale, and it will be discussed later in this subsection. One way to define a temperature scale is to assign arbitrary values to two readily reproducible temperatures. For both the Celsius and Fahrenheit scales these two fixed points are chosen to be the freezing point and the boiling point of water, both taken at standard atmospheric pressure. On the Celsius scale, the freezing point of water is chosen to be 0°C ("zero degrees Celsius") and the boiling point 100°C. On the Fahrenheit scale, the freezing point is defined as 32°F and the boiling point 212°F. A practical thermometer is calibrated by placing it in carefully prepared environments at each of the two temperatures and marking the position of the liquid or pointer. For a Celsius scale, the odistance between the two marks is divided into one hundred equal intervals representing each degree between 0°C and 100°C (hence the name "centigrade scale" meaning "hundred steps"). For a Fahrenheit scale, the two points are labeled 32°F and 212°F and the distance between them is divided into 180 equal intervals. For temperatures below the freezing point of water and above the boiling point of water, the scales may be extended using the same equally spaced intervals. However, thermometers can be used only over a limited temperature range because of their own limitations; for example, the liquid mercury in a mercury-in-glass thermometer solidifies at some point, below which the thermometer will be useless. It is also rendered useless above temperatures where the fluid, such as alcohol, vaporizes. For very low or very high temperatures, specialized thermometers are required, some of which we will mention later. Every temperature on the Celsius scale corresponds to a particular temperature on the Fahrenheit scale. It is easy to convert from one to the other if you remember that 0°C corresponds to 32°F and that a range of 100° on the Celsius scale corresponds to a range of 180° on the Fahrenheit scale. Thus, one Fahrenheit degree (1F°) corresponds to $\frac{100}{180}$ (or $\frac{5}{9}$) of a Celsius degree (1C°). That is, $1\text{F}^{\text{o}} = \frac{9}{5}\text{C}^{\text{o}}$. (Notice that when we refer to a specific temperature, we say "degrees Celsius," as in 20°C; but when we refer to a change in temperature or a temperature interval, we say "Celsius degrees," as in "2C°.")

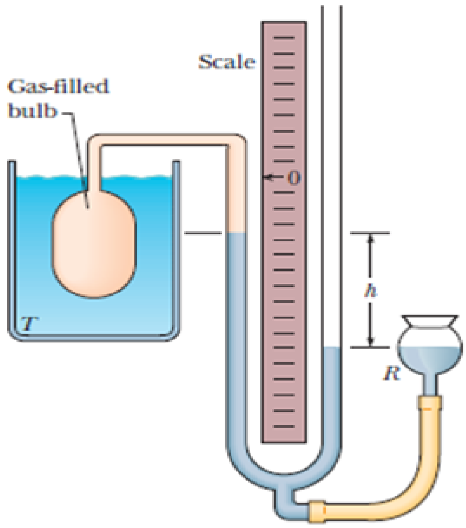


Figure 9.2 A constant-volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

Different materials, because of how we calibrate them, will agree at 0°C and at 100°C. But because of different expansion properties, they may not agree precisely at intermediate temperatures (remember we arbitrarily divided the thermometer scale into 100 equal divisions between 0°C and 100°C). Thus, a carefully calibrated mercury-in-glass thermometer might register 52.0°C, whereas a carefully calibrated thermometer of another type might read 52.6°C. Discrepancies below 0°C and above 100°C can also be significant. Because of such discrepancies, it is important to have a very precisely defined temperature scale so that measurements of temperature made at different laboratories around the world can be accurately compared.

As shown in the simplified diagram shown in Figure 9.2, this thermometer consists of a bulb filled with a dilute gas connected by a thin tube to a mercury manometer. The volume of the gas is kept constant by raising or lowering the right-hand tube of the manometer so that the mercury in the left-hand tube coincides with the reference mark. An increase in temperature causes a proportional increase in pressure in the bulb. Thus, the tube must be lifted higher to keep the gas volume constant. The height of the mercury in the right-hand column is then a measure of the temperature. This thermometer gives the same results for all gases in the limit of reducing the gas pressure in the bulb toward zero. The standard thermometer for this scale is the constant-volume gas thermometer. The scale itself is called the ideal gas temperature scale, since it is based on the property of an ideal gas that the pressure is directly proportional to the absolute temperature (Gay-Lussac's law).

A real gas, which would need to be used in any real constant-volume gas thermometer, approaches this ideal at low density. In other words, the temperature at any point in space is defined as being proportional to the pressure in the (nearly) ideal gas used in the thermometer. The scale that is set is called absolute temperature scale (also called the Kelvin scale) which uses the unit kelvin (abbreviated as K). To set a scale we need two fixed points. The first point is absolute zero (or T = 0 K) where pressure is zero. The second fixed point is chosen to be the triple point of water, which is the point where water in the solid, liquid, and gas states can coexist in equilibrium. This occurs only at a unique temperature of 0.01°C and pressure of 4.58 torr (mm of mercury). On the Kelvin scale the temperature of water at the triple point was set at 273.16 K. Therefore, the SI unit of temperature, the kelvin, is defined as 1/273.16 of the temperature of the triple point of water. The absolute or Kelvin temperature T at any point is then defined, using a constant-volume gas thermometer for an ideal gas, as

$$T = (273.16\,K)\left(\frac{P}{P_{tp}}\right). \qquad (9.1)$$

In this relation, P_{tp} is the pressure of the gas in the thermometer at the triple point temperature of water and P is the pressure in the thermometer when it is at the point where T is being determined. Note that if we let P = P_{tp} in this relation, then T = 273.16 K. The definition of temperature, Eq. (9.1), with a constant volume gas thermometer filled with a real gas is only approximate because we find that we get different results for the temperature depending on the type of gas that is used in the thermometer. Temperatures determined in this way also vary depending on the amount of gas in the bulb of the thermometer. However, as we use smaller and smaller amounts of gas to fill the bulb, the readings converge nicely to a single temperature, no matter what gas we use. Thus, the temperature T at any point in space, determined using a constant-volume gas thermometer containing a real gas, is defined using this limiting process:

$$T = (273.16\,K)\lim_{P_{tp}\to 0}\left(\frac{P}{P_{tp}}\right). \qquad (9.2)$$

This defines the ideal gas temperature scale. One of the great advantages of this scale is that the value for T does not depend on the kind of gas used. But the scale does depend on the properties of gases in general. Helium has the lowest condensation point of all gases; at very low pressures it liquefies at about 1 K, so temperatures below this cannot be defined on this scale. Absolute zero is used as the basis for the Kelvin temperature scale, which sets -273.15°C as its zero point (0 K). The size of a "degree" on the Kelvin scale is chosen to be identical to the size of a degree on the Celsius scale. If T_C represents a Celsius temperature and T a Kelvin temperature, then the relationship between these two temperature scales is

$$T_C = T - 273.15^o. \qquad Eq. (9.3)$$

In expressing temperatures on the Celsius scale, the degree symbol is commonly used. Thus, we write 20.00°C for a Celsius reading but 293.15 K for a Kelvin reading.

Example 1

On a day when the temperature reaches 50^{o}F, what is the temperature in degrees Celsius and in kelvins?

Solution

Celsius temperature: $T_C = \frac{5}{9}(T_C - 32) = \frac{5}{9}(50 - 32) = 10°C$

Kelvin temperature: $T = T_C + 273.15 = 10°C + 273.15 = 283$ K.

Example 2

Suppose you come across old scientific notes that describe a temperature scale called Z on which the boiling point of water is 75.0^{o}Z and the freezing point is -15.0^{o}Z. To what temperature on the Fahrenheit scale would a temperature of *T* = 45.0^{o}Z correspond? Assume that the Z scale is linear; that is, the size of a Z degree is the same everywhere on the Z scale.

Solution

To find the corresponding temperature on the Fahrenheit scale we set up a conversion factor between the Z and Fahrenheit scales using both known temperatures on the Z scale and the corresponding temperatures on the Fahrenheit scale. On the Z scale, the difference between the boiling and freezing points is 75.0^{o}Z - (-15.0^{o}Z) = 90.0 Z^o. On the Fahrenheit scale, it is 212^{o}F - 32.0^{o}F = 180 F^o. Thus, a temperature difference of 90.0 Z^o is equivalent to a temperature difference of 180 F^o, and we can use the relation $\frac{180\ F^o}{90.0\ Z^o} = \frac{212^oF - T}{75^oZ - 45^oZ}$. This means that T = 152^{o}F.

Question 1

Rank the following temperatures from highest to lowest: (i) 0.00^{o}C; (ii) 0.00^{o}F; (iii) 260.00 K; (iv) 77.00 K; (v) -180.00^{o}C.

Question 2

Suppose the temperature of a gas is 373.15 K when it is at the boiling point of water. What then is the limiting value of the ratio of the pressure of the gas at that boiling point to its pressure at the triple point of water? (Assume the volume of the gas is the same at both temperatures.)

Question 3

Which represents a larger temperature change, a Celsius degree or a Fahrenheit degree?

9.3 Thermal Expansion

Learning Outcomes

After reading this section, students will be able to:

- apply for one-dimensional thermal expansion the relationship between the temperature change, the length change, the initial length, and the coefficient of linear expansion.
- use one-dimensional thermal expansion to find the change in area for two-dimensional thermal expansion.

- apply for three-dimensional thermal expansion, the relationship between the
- temperature change, the volume change, the initial volume, and the coefficient of volume expansion.

Most substances expand when heated and contract when cooled. However, the amount of expansion or contraction varies, depending on the material.

9.3.1 Linear Expansion

Experiments indicate that the change in length Δl of almost all solids is, to a good approximation, directly proportional to the change in temperature ΔT, as long as ΔT is not too large. The change in length is also proportional to the original length of the object, l_0. That is, for the same temperature increase, a 4 m long iron rod will increase in length twice as much as a 2m long iron rod. We can write this proportionality as an equation:

$$\Delta l = \alpha l_0 \Delta T, \qquad (9.4)$$

where α, the proportionality constant, is called the coefficient of linear expansion for the particular material and has units of $(°C)^{-1}$. We write $l = l_0 + \Delta l$, and rewrite this equation as

$$l = l_0 + \Delta l = l_0 + \alpha l_0 \Delta T, \text{ or } l = l_0(1 + \alpha \Delta T), \qquad (9.5)$$

where l_0 is the initial length, at temperature T_0, and l is the length after heating or cooling to a temperature T. If the temperature change $\Delta T = T - T_0$ is negative, then $\Delta l = l - l_0$ is also negative; the length shortens as the temperature decreases. The values of α for various materials vary slightly with temperature. However, if the temperature range is not too great, the variation can usually be ignored.

9.3.2 Volume Expansion

The change in volume of a material which undergoes a temperature change is given by a relation similar to Eq. (9.4), namely,

$$\Delta V = \beta \Delta V_0 T, \qquad (9.6)$$

where ΔT is the change in temperature, V_0 is the original volume, ΔV is the change in volume, and β is the coefficient of volume expansion. The units of β are $(°C)^{-1}$. Notice that for solids, β is normally equal to approximately 3α. To see why, consider a rectangular solid of length l_0, width w_0, and height h_0. When its temperature is changed by ΔT, its volume changes from $V_0 = l_0 h_0 w_0$ to

$$V = l_0 (1 + \alpha \Delta T) h_0 (1 + \alpha \Delta T) w_0 (1 + \alpha \Delta T), \qquad (9.7)$$

using Eq. (4) and assuming α is the same in all directions. Thus,

$$\Delta V = V - V_0 = V_0(1 + \alpha \Delta T)^3 - V_0 = V_0[1 + 3\alpha \Delta T + 3(\alpha \Delta T)^3 + (\alpha \Delta T)^3].$$

If the amount of expansion is much smaller than the original size of the object, then $\alpha \Delta T << 1$ and we can ignore all but the first term and obtain $\Delta V \approx (3\alpha) V_0 \Delta T$. This is the same as Eq. (9.6) with $\beta \approx (3\alpha)$. For solids that are not isotropic (or that do not have the same properties in all directions), however, the relation $\beta \approx (3\alpha)$ is not valid. Note also that linear expansion has no meaning for liquids and gases since they do not have fixed shapes. In a similar way, you can show that the change in area of a rectangular plate is given by $\Delta A \approx (2\alpha) A_0 \Delta T$, where A_0 is the original area, and ΔA is the change in area. Equations (9.4) and (9.6) are accurate only if Δl (or ΔV) is small compared to l_0 (or V_0). This is of particular concern for liquids and even more so for gases because of the large values of β. Furthermore, β itself varies substantially with temperature for gases.

Example

A segment of steel railroad track has a length of 30.0 m when the temperature is 0.0°C. What is its length when the temperature is 40.0°C?

Solution

Using the value of the coefficient of linear expansion for steel $\alpha = 11 \times 10^{-6}(°C)^{-1}$ in Equation (9.4) we get

$\Delta l = \alpha l_0 \Delta T = [11 \times 10^{-6}(°C)^{-1}](30.000\ m)(40.0°C) = 0.013\ m$

Question 1

Assuming all have the same initial volume, rank the following substances by the amount of volume expansion due to an increase in temperature, from least to most: glass, mercury, aluminum, ethyl alcohol.

Question 2

If you are asked to make a very sensitive glass thermometer, which of the following working fluids would you choose? (a) mercury (b) alcohol (c) gasoline (d) glycerin

9.4 The Concept of Heat and Energy

Learning Outcomes

After completing this section, students will be able to:

- understand that heat is the amount of energy that is transferred either to or from an object when there is a temperature difference between the object and its environment.
- explain that internal energy is associated with the energies of its microscopic components, atoms and/or molecules, when viewed from a reference frame at rest with respect to the center of mass of the system.
- identify that energy is a state variable while heat is a parameter that occurs during a transformation of a system from one equilibrium state to another.

Internal energy is all the energy of a system that is associated with its microscopic components, atoms and/or molecules, when viewed from a reference frame at rest with respect to the center of mass of the system. Internal energy includes kinetic energy of random translational, rotational, and vibrational motion of molecules; vibrational potential energy associated with forces between atoms in molecules; and electric potential energy associated with forces between molecules. The bulk kinetic energy of the system due to its motion through space is not included in internal energy. Heat is defined as a process of transferring energy across the boundary of a system because of a temperature difference between the system and its surroundings. It is also the amount of energy Q transferred by this process. When you heat a substance, you are transferring energy into it by placing it in contact with surroundings that have a higher temperature. Such is the case, for example, when you place a pan of cold water on a stove burner. The burner is at a higher temperature than the water, and so the water gains energy by heat. It makes no sense to talk about the heat of a system; one can refer to heat only when energy has been transferred as a result of a temperature difference.

A common unit for heat, still in use today, is named after caloric. It is called the calorie (cal) and is defined as the amount of heat necessary to raise the temperature of 1 gram of water by 1 Celsius degree. To be precise, the particular temperature range from 14.5℃ to 15.5℃ is specified because the heat required is very slightly different at different temperatures. The difference is less than 1 percent over the range 0 to 100℃, and we will ignore it for most purposes. More often used than the calorie is the kilocalorie (kcal), which is 1000 calories. Thus 1 kcal is the heat needed to raise the temperature of 1 kg of water by 1 C°. Often a kilocalorie is called a Calorie (with a capital C), and this Calorie (or the kilo joule) is used to specify the energy value of food. In the British system of units, heat is measured in British thermal units (Btu). One Btu is defined as the heat needed to raise the temperature of 1 lb of water by 1 F° where 1 Btu = 0.252 kcal = 1056 J, where 4.186 J = 1 cal.

When different parts of an isolated system are at different temperatures, heat will flow (energy is transferred) from the part at higher temperature to the part at lower temperature—that is, within the system. If the system is truly isolated, no energy is transferred into or out of it, and the heat lost by one part of the system is equal to the heat gained by the other part:

heat lost = heat gained or energy out of one part = energy into another part.

These simple relations are very useful, but depend on the (often very good) approximation that the whole system is isolated (no other energy transfers occur).

9.5 Specific Heat and Latent Heat

Learning Outcomes

After completing this section, students will be able to:

- understand that a heat transfer changes either the temperature or phase of a substance.
- relate the temperature change of a substance to the heat transfer and the substance's heat capacity.
- relate the temperature change of a substance to the heat transfer and the substance's specific heat and mass.
- relate the heat transfer, the heat of transformation, and the amount of mass transformed for a phase change of a substance.

When heat is added to an object and there is no change in the kinetic or potential energy of the object, the temperature of the object rises (assuming no phase change). If the system consists of a sample of a substance, the quantity of heat required to raise the temperature of a given mass of the substance by some amount varies from one substance to another. The amount of heat needed to raise the temperature of a sample by 1°C is defined as the heat capacity C of the particular sample. From this definition, we see that if energy Q produces a change ΔT in the temperature of a sample, then

$$Q = C\Delta T. \quad (9.8)$$

The amount of heat Q required to change the temperature of a given material is proportional to the mass m of the material present and to the temperature change ΔT. This relation can be expressed in the equation

$$Q = mc\Delta T, \quad (9.9)$$

where c is a quantity characteristic of the material called its specific heat which is specified in units of J/kg. °C . The values of c for solids and liquids depend to some extent on temperature (as well as slightly on pressure), but for temperature changes that are not too great, c can often be considered constant.

When a material changes phase from solid to liquid, or from liquid to gas, a certain amount of heat is involved in this change of phase. The heat required to change 1.0 kg of a substance from the solid to the liquid state is called the heat of fusion; it is denoted by L_F. The heat required to change a substance from the liquid to the vapor phase is called the heat of vaporization, L_V. The melting-point and boiling-point temperatures and the corresponding heats of fusion and vaporization are different for different substances. Heats of fusion and vaporization are also called the latent heats. The heats of vaporization and fusion also refer to the amount of heat released by a substance when it changes from a gas to a liquid, or from a liquid to a solid. The heat involved in a change of phase depends not only on the latent heat but also on the total mass of the substance. That is,

$$Q = mL, \quad (9.10)$$

where L is the latent heat of the particular process and substance, m is the mass of the substance, and Q is the heat added or released during the phase change. The latent heat is specifies in units of J/kg.

Example

Explain why a cup of water (or soda) with ice cubes stays at 0ºC , even on a hot summer day.

Solution

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

Example

If 25 kJ is necessary to raise the temperature of a block from 25ºC to 30ºC , how much heat is necessary to heat the block from 45ºC to 50ºC ?

Solution

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

Example

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Solution

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0ºC . The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

Example

A 0.050 kg ingot of metal is heated to 200.0℃ and then dropped into a calorimeter containing 0.400 kg of water initially at 20.0℃. The final equilibrium temperature of the mixed system is 22.4℃. Find the specific heat of the metal.

Solution

Energy leaves the hot ingot and goes into the cold water, so the ingot cools off and the water warms up. Once both are at the same temperature T_f, the energy transfer stops. This can be written in equation form as

$$m_w c_w (T_f - T_w) = -m_x c_x (T_f - T_x)$$

Solve for c_x

$$c_x = \frac{m_w c_w (T_f - T_w)}{m_x (T_x - T_f)}$$

$$c_x = [(0.4)(4.186)(22.4 - 20.0)]/[0.5)(200) - 22.4] \text{ J/kg.℃}$$

$$c_x = 453 \text{ J/kg ℃}$$

Question 1

A person fires a silver bullet with a muzzle speed of 200 m/s into the pine wall of a saloon. Assume all the internal energy generated by the impact remains with the bullet. What is the temperature change of the bullet? The specific heat of silver is 234 J/kg .℃.

Question 2

A certain amount of heat *Q* will warm 1 g of material *A* by 3 C^o and 1 g of material *B* by 4 C^o. Which material has the greater specific heat?

9.6 Heat Transfer Mechanisms

Learning Outcomes

After completing this section, students will be able to:

- understand a thermal conduction through a layer, and apply the relationship between the energy transfer rate and the layer's area, thermal conductivity, thickness, and temperature difference between its two sides for thermal conduction through a layer.
- apply the relationship between thermal resistance, thickness, and thermal conductivity for thermal conduction through a layer.
- identify that thermal energy can be transferred by convection, in which a warmer fluid (gas or liquid) tends to rise in a cooler fluid.

- apply the relationship between the energy-transfer rate and an object's surface area, emissivity, and surface temperature in the emission of thermal radiation by the object.
- calculate the net energy-transfer rate of an object emitting radiation to its environment and absorbing radiation from that environment.

Heat is transferred from one place or body to another in three different ways: by conduction, convection, and radiation.

9.6.1 Conduction

When a long metal bar is put in a hot fire the exposed end of the bar soon becomes hot as well, even though it is not directly in contact with the source of heat. We say that heat has reached the cold end by means of conduction. Heat conduction in many materials can be visualized as being carried out via molecular collisions. As one end of an object is heated, the molecules there move faster and faster. As they collide with slower-moving neighbors, they transfer some of their kinetic energy to these other molecules, which in turn transfer energy by collision with molecules still farther along the object. In metals, collisions of free electrons are mainly responsible for conduction. Heat conduction from one point to another takes place only if there is a difference in temperature between the two points. Indeed, it is found experimentally that the rate of heat flow through a substance is proportional to the difference in temperature between its ends. The rate of heat flow also depends on the size and shape of the object. To investigate this quantitatively, let us consider the heat flow through a uniform cylindrical object. It is found experimentally that the heat flow ΔQ over a time interval Δt is given by the relation

$$\frac{\Delta Q}{\Delta t} = kA(T_1 - T_2)l, \qquad (9.11)$$

where A is the cross-sectional area of the object, l is the distance between the two ends, which are at temperatures T_1 and T_2, and k is a proportionality constant called the thermal conductivity which is characteristic of the material. From this equation, we see that the rate of heat flow (units of J/s) is directly proportional to the cross-sectional area and to the temperature gradient $\frac{T_1 - T_2}{l}$.

Substances for which k is large conduct heat rapidly and are said to be good thermal conductors. Most metals fall in this category, although there is a wide range even among them. Substances for which k is small are poor conductors of heat and are therefore good thermal insulators. The relative magnitudes of k can explain simple phenomena such as why a tile floor is much colder on the feet than a rug-covered floor at the same temperature. Tile is a better conductor of heat than the rug. Heat that flows from your foot to the rug is not conducted away rapidly, so the rug's surface quickly warms up to the temperature of your foot and feels good. But the tile conducts the heat away rapidly and thus can take more heat from your foot quickly, so your foot's surface temperature drops. For practical purposes the thermal properties of building materials, particularly when considered as insulation, are usually specified by R-values (or "thermal resistance"), defined for a given thickness l of material as

$R = l/k$. The R-value of a given piece of material combines the thickness l and the thermal conductivity k in one number.

9.6.2 Convection

Convection is the process whereby heat flows by the mass movement of molecules from one place to another. Whereas conduction involves molecules (and/or electrons) moving only over small distances and colliding, convection involves the movement of large numbers of molecules over large distances. A forced-air furnace, in which air is heated and then blown by a fan into a room, is an example of forced convection. Natural convection occurs as well, and one familiar example is that hot air rises. For instance, the air above a radiator (or other type of heater) expands as it is heated, and hence its density decreases. Because its density is less than that of the surrounding cooler air, it rises. Wind is another example of convection, and weather in general is strongly influenced by convective air currents. When a pot of water is heated, convection currents are set up as the heated water at the bottom of the pot rises because of its reduced density. That heated water is replaced by cooler water from above. The air throughout the room becomes heated as a result of convection. The air heated by the radiators rises and is replaced by cooler air, resulting in convective air currents. Other types of furnaces also depend on convection. Hot-air furnaces with openings near the floor often do not have fans but depend on natural convection, which can be appreciable. In other systems, a fan is used. In either case, it is important that cold air can return to the furnace so that convective currents circulate throughout the room if the room is to be uniformly heated.

9.6.3 Radiation

Unlike convection and conduction which require the presence of matter as a medium to carry the heat from the hotter to the colder region a heat transfer through radiation occurs without any medium at all. Radiation consists essentially of electromagnetic waves. The Sun consists of visible light plus many other wavelengths that the eye is not sensitive to, including infrared (IR) radiation. All life on the Earth depends on the transfer of energy from the Sun, and this energy is transferred to the Earth over empty (or nearly empty) space. This form of energy transfer is heat—since the Sun's surface temperature is much higher than Earth's—and is referred to as radiation. The warmth we receive from a fire is mainly radiant energy. All objects radiate energy continuously in the form of electromagnetic waves produced by thermal vibrations of the molecules. The rate at which the surface of an object radiates energy is proportional to the fourth power of the absolute temperature of the surface. This behavior is expressed in equation form as

$$P = \sigma A \epsilon T^4, \qquad (9.12)$$

where P is the power in watts of electromagnetic waves radiated from the surface of the object, σ is a constant equal to $5.6696 \times 10^{-8}\,\mathrm{Wm^{-2}.K^{-4}}$, A is the surface area of the object in square meters, ϵ is the emissivity, and T is the surface temperature in kelvins. The value of ϵ can vary between zero and unity depending on the properties of the surface of the object. The emissivity is equal to the absorptivity, which is the fraction of the incoming radiation that the surface absorbs. A mirror has very low absorptivity because it reflects almost all incident light. Therefore, a mirror surface also has a very low emissivity. At the other extreme, a black surface has high absorptivity and high emissivity. An ideal absorber is defined as an object that absorbs all the energy incident on it, and for such an object, $\epsilon = 1$. An object for which $\epsilon = 1$ is often referred to as a black body.

As an object radiates energy at a rate given by Equation (9.12), it also absorbs electromagnetic radiation from the surroundings, which consist of other objects that radiate energy. If the latter process did not occur, an object would eventually radiate all its energy and its temperature would reach absolute zero. If an object is at a temperature T and its surroundings are at an average temperature T_0, the net rate of radiant heat flow from the object is given by

$$P = \sigma\epsilon A(T^4 - T_0^4). \quad (9.13)$$

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and its temperature decreases.

Example

Name an example from daily life for each mechanism of heat transfer.

Solution

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make it hot.

Radiation: Reheating a cold cup of coffee in a microwave oven.

Example

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Solution

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled. The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two.

Example

Explain why using a fan in the summer feels refreshing!

Solution

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air “feels” cooler than still air.

Example

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^oC$ compared to when the body is at the temperature $T_2 = 40^oC$?

Solution

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\ K$ and $T_2 = 313\ K$, the rate of heat transfer increases by about 30 percent of the original rate.

9.7 Energy Conservation: The First Law of Thermodynamics

Learning Outcomes

After completing this section, students will be able to:

- apply the first law of thermodynamics to relate the change in the internal energy of a system, the energy transferred as heat to or from the system, and the work done on or by the gas.

- identify the algebraic sign of work when it is done by a system or on the system.
- identify the algebraic sign of a heat transfer that is associated with a transfer to a system and a transfer from the system.
- identify that the internal energy of a system tends to increase if the heat transfer is to the system, and it tends to decrease if the system does work on its environment.

Work is done when energy is transferred from one object to another by mechanical means. We saw that heat is a transfer of energy from one object to a second one at a lower temperature. Thus, heat is much like work. To distinguish them, heat is defined as a transfer of energy due to a difference in temperature, whereas work is a transfer of energy that is not due to a temperature difference. We also saw that the internal energy of a system as the sum total of all the energy of the molecules within the system. We would expect that the internal energy of a system would be increased if work was done on the system, or if heat were added to it. Similarly the internal energy would be decreased if heat flowed out of the system or if work were done by the system on something in the surroundings. Thus it is reasonable to extend conservation of energy and propose an important law: the change in internal energy of a closed system, ΔE, will be equal to the energy added to the system by heating minus the work done by the system on the surroundings. In equation form we write

$$\Delta E = Q - W, \qquad (9.14)$$

where Q is the net heat added to the system and W is the net work done by the system. In this case the work W in Eq. (9.14) is negative because the work done is by the system, but if the work is done on the system, W will be positive. Similarly, Q is positive for heat added to the system, but if heat leaves the system, Q is negative. Equation (9.14) is known as the first law of thermodynamics. Since Q and W represent energy transferred into or out of the system, the internal energy changes accordingly. Thus, the first law of thermodynamics is a great and broad statement of the law of conservation of energy. Equation (9.14) applies to a closed system. It also applies to an open system if we take into account the change in internal energy due to the increase or decrease in the amount of matter. For an isolated system, no work is done and no heat enters or leaves the system, so $W = Q = 0$, and hence $\Delta E = 0$. A given system at any moment is in a particular state and can be said to have a certain amount of internal energy, E. But a system does not "have" a certain amount of heat or work. Rather, when work is done on a system (such as compressing a gas), or when heat is added or removed from a system, the state of the system changes. Thus, work and heat are involved in thermodynamic processes that can change the system from one state to another; they are not characteristic of the state itself or not state variables. Quantities which describe the state of a system, such as internal energy E, pressure P, volume V , temperature T, and mass m or number of moles n, are called state variables. Because E is a state variable, which depends only on the state of the system and not on how the system arrived in that state, we can write $\Delta E = E_2 - E_1 = Q - W$ where E_1 and E_2 represent the internal energy of the system in states 1 and 2, and Q and W are the heat added to the system and work done by the system in going from state 1 to state 2.

Example

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Solution

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

Example

A 1.0 kg bar of copper is heated at atmospheric pressure so that its temperature increases from 20°C to 50°C. a) How much energy is transferred to the copper bar by heat? b) What is the increase in internal energy of the copper bar?

Solution

The energy transferred to the copper bar by the heat is found from the relation $Q = mc\Delta T = 1.2\text{x}10^4$ J, where m is 1 kg, c = 387 J/kg. °C is the specific heat capacity of copper, and ΔT = 50°C - 20°C = 30°C.

9.8 Summary

Two objects are in thermal equilibrium with each other if they do not exchange energy when in thermal contact.

Temperature is the property that determines whether an object is in thermal equilibrium with other objects. Two objects in thermal equilibrium with each other are at the same temperature. The SI unit of absolute temperature is the kelvin, which is defined to be 1/273.16 of the difference between absolute zero and the temperature of the triple point of water.

The zeroth law of thermodynamics states that if objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

Internal energy is a system's energy associated with its temperature and its physical state (solid, liquid, gas). Internal energy includes kinetic energy of random translation, rotation, and vibration of molecules; vibrational potential energy within molecules; and potential energy between molecules.

Heat is the process of energy transfer across the boundary of a system resulting from a temperature difference between the system and its surroundings.

A calorie is the amount of energy necessary to raise the temperature of 1 g of water from 14.5°C to 15.5°C.

The heat capacity C of any sample is the amount of energy needed to raise the temperature of the sample by 1°C.

Conduction can be viewed as an exchange of kinetic energy between colliding molecules or electrons. Convection occurs when temperature differences cause an energy transfer by motion within a fluid. Radiation is an energy transfer via the emission of electromagnetic energy.

9.9 Conceptual Questions

1. What does it mean to say that two systems are in thermal equilibrium?
2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.
4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics
5. Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.
6. How is heat transfer related to temperature?
7. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?
8. When heat transfers into a system, is the energy stored as heat? Explain briefly.
9. What three factors affect the heat transfer that is necessary to change an object's temperature?
10. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v . How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.
11. What is the temperature of ice right after it is formed by freezing water?
12. If you place 0ºC ice into 0ºC water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?
13. Is it possible for two objects to be in thermal equilibrium if they are not in contact with each other? Explain.
14. Suppose you empty a tray of ice cubes into a bowl partly full of water and cover the bowl. After one-half hour, the contents of the bowl come to thermal equilibrium, with more liquid water and less ice than you started with. Which of the following is true? (a) The temperature of the liquid water is higher than the temperature of the remaining ice. (b) The temperature of the liquid water is the same as that of the ice. (c) The temperature of the liquid water is less than that of the ice. (d) The comparative temperatures of the liquid water and ice depend on the amounts present.
15. A piece of copper is dropped into a beaker of water. (a) If the water's temperature rises, what happens to the temperature of the copper? (b) Under what conditions are the water and copper in thermal equilibrium?

9.10 problems

1. Markings to indicate length are placed on a steel tape in a room that is at a temperature of 22°C. Measurements are then made with the same tape on a day when the temperature is 27°C. Assume the objects you are measuring have a smaller coefficient of linear expansion than steel. Are the measurements (a) too long, (b) too short, or (c) accurate?
2. A temperature of 162^{o}F is equivalent to what temperature in kelvins?
3. Suppose you come across old scientific notes that describe a temperature scale called Z on which the boiling point of water is 65.0^{o}Z and the freezing point is -14.0^{o}Z. To what
4. temperature on the Fahrenheit scale would a temperature of T=-98.0^{o}Z correspond? Assume that the Z scale is linear; that is, the size of a Z degree is the same everywhere on the Z scale.

5. A pair of eyeglass frames is made of epoxy plastic. At room temperature (20.0°C), the frames have circular lens holes 2.20 cm in radius. To what temperature must the frames be heated if lenses 2.21 cm in radius are to be inserted in them? The average coefficient of linear expansion for epoxy is 1.30×10^{-4} $(°C)^{-1}$.
6. A poker is a stiff, nonflammable rod used to push burning logs around in a fireplace. For safety and comfort of use, should the poker be made from a material with (a) high specific heat and high thermal conductivity, (b) low specific heat and low thermal conductivity, (c) low specific heat and high thermal conductivity, or (d) high specific heat and low thermal conductivity?
7. An amount of energy is added to ice, raising its temperature from -10°C to -5°C. A larger amount of energy is added to the same mass of water, raising its temperature from 15°C to 20°C. From these results, what would you conclude?
8. The specific heat of substance A is greater than that of substance B. Both A and B are at the same initial temperature when equal amounts of energy are added to them. Assuming no melting or vaporization occurs, which of the following can be concluded about the final temperature T_A of substance A and the final temperature T_B of substance B? (a) $T_A > T_B$ (b) $T_A < T_B$ (c) $T_A = T_B$
9. A 100 g piece of copper, initially at 95.0°C, is dropped into 200 g of water contained in a 280 g copper can; the water and can are initially at 15.0°C. What is the final temperature of the system? (Specific heats of A glass windowpane in a home is 0.620 cm thick and has dimensions of 1.00 m x 2.00 m. On a certain day, the temperature of the interior surface of the glass is 25.0°C and the exterior surface temperature is 0°C. (a) What is the rate at which energy is transferred by heat through the glass? (b) How much energy is transferred through the window in one day, assuming the temperatures on the surfaces remain constant?
10. Rub the palm of your hand on a metal surface for about 30 seconds. Place the palm of your other hand on an unrubbed portion of the surface and then on the rubbed portion. The rubbed portion will feel warmer. Now repeat this process on a wood surface. Why does the temperature difference between the rubbed and unrubbed portions of the wood surface seem larger than for the metal surface?
11. What mass of water at 25.0°C must be allowed to come to thermal equilibrium with a 1.85 kg cube of copper initially at 150°C to lower the temperature of the copper to 65.0°C? Assume any water turned to steam subsequently condenses.
12. A thermodynamic system undergoes a process in which its internal energy decreases by 500 J. Over the same time interval, 220 J of work is done on the system. Find the energy transferred from it by heat.
13. The surface of the Sun has a temperature of about 5 800 K. The radius of the Sun is 6.96×10^8 m. Calculate the total energy radiated by the Sun each second. Assume the emissivity of the Sun is 0.986.

10 Oscillations and Waves

Learning Outcome

After completing this Chapter, students are expected to:

- Describe ...
- .

Introduction

Periodic motion or oscillatory motion is motion of an object that regularly returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motion in everyday life. In periodic motion, a body repeats a certain motion indefinitely, always returning to its starting point after a constant time interval and then starting a new cycle. Examples of periodic motion are:

- A mass attached to a spring executes periodic motion when the spring is pulled out and released.
- simple pendulum
- a bungee jumper hangs from a bungee cord and oscillates up and down
- a guitar string vibrates back and forth in a standing wave, with each element of the string moving in simple harmonic motion
- a piston in a gasoline engine oscillates up and down within the cylinder of the engine
- an atom in a diatomic molecule vibrates back and forth as if it is connected by a spring to the other atom in the molecule

10.1 Simple Harmonic Motion

In mechanics and physics, simple harmonic motion is a special type of periodic motion or oscillation where (a) motion is about an equilibrium position at which point no net force acts on the system,(b) the restoring force is directly proportional to the displacement x from the equilibrium position and (c) acts in the direction opposite to that of displacement.

Simple harmonic motion can serve as a mathematical model for a variety of motions, such as the oscillation of a spring. In addition, other phenomena can be approximated by simple harmonic motion, including the motion of a simple pendulum as well as molecular vibration. Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's law. The motion is sinusoidal in time and demonstrates a single resonant frequency. For simple harmonic motion to be an accurate model for a pendulum, the net force on the object at the end of the pendulum must be proportional to the displacement. This is a good approximation when the angle of the swing is small. We consider two important model systems that exhibit simple harmonic motion that is mass on a spring and simple pendulum.

10.1.1 Motion of an Object Attached to a Spring

Consider a block of mass *m* attached to the end of a spring, with the block free to move on a frictionless, horizontal surface (Figure10.1). When the spring is neither stretched nor compressed, the block is at rest at the position called the equilibrium position of the system, which we identify as *x* = 0 (Figure10.1b). We know from experience that such a system oscillates back and forth if disturbed from its equilibrium position. We can understand the oscillating motion by first recalling that when the block is displaced to a position *x,* the spring exerts on the block a force that is proportional to the position and given by Hooke's law:

$$F = -kx \qquad [10.1]$$

Her k is called the spring constant and F is called a restoring force because it is always directed toward the equilibrium position and therefore *opposite* the displacement of the block from equilibrium. That is, when the block is displaced to the right of *x* = 0 in Figure 10.1c, the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $x = 0$ as in Figure 10.1a, the position is negative and the restoring force is directed to the right. When the block is displaced from the equilibrium point and released, the particle is under a net force and consequently undergoes an acceleration. Applying Newton's second law for the net force acting on the particle in motion, the acceleration $a$of the particle in the *x* direction becomes

$$F = -kx = ma \qquad [10.2a]$$

$$a = -\frac{k}{m}x \qquad [10.2b]$$

where$k$ is the spring constant or force constant of the spring.

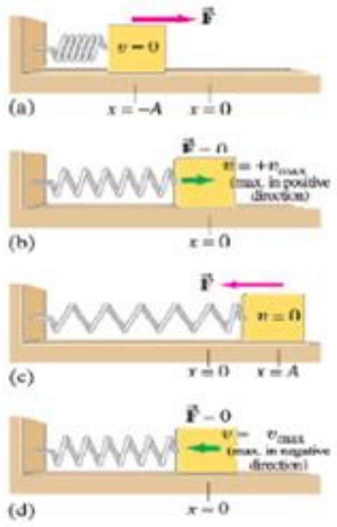


Figure 10.1

To discuss oscillatory motion, we need to define a few terms. The distance x of the mass from the equilibrium point at any moment is called the displacement. The amplitude A of a body undergoing simple harmonic motion is the maximum value of its displacement on either side of the equilibrium position. One cycle refers to the complete to-and-fro motion from some initial point back to that same point, say, from x = -A to x = A and back to x = -A.

The period T of a body undergoing simple harmonic motion is the time needed for one complete cycle. The frequency f of a body undergoing simple harmonic motion is the number of complete cycles per second it executes. It is easy to see, from their definitions, that frequency and period are inversely related. That is

$$\text{frequency} = 1/\text{period} \tag{10.3a}$$

$$f = 1/T \tag{10.3b}$$

The unit of frequency is hertz (Hz), where 1 Hz = 1 cycle per second (s^{-1}). Note that the frequency and period do not depend on the amplitude. Changing the amplitude of a simple harmonic oscillator does not affect its frequency.

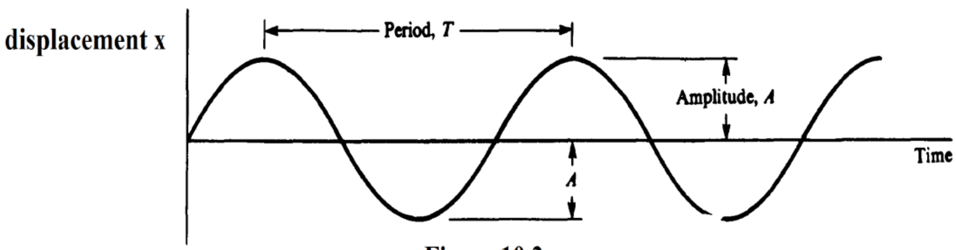


Figure 10.2

The angular frequency ω of a simple harmonic motion is defined by

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{10.4}$$

It has units of radians per second. It is a measure of how rapidly the oscillations are occurring; the more oscillations per unit time, the higher the value of ω. The angular frequency ω of the resulting simple harmonic motion is defined as

$$\omega = \sqrt{k/m} \tag{10.5}$$

From equation [10.5], we get

$$k = m\omega^2 \tag{10.6}$$

Using equations [10.4] and [10.5], the period $T$and frequency f of the motion for the particle in simple harmonic motion can be expressed in terms of the mass *m* and spring constant *k* of the system as:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{m/k} \tag{10.7a}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m} \tag{10.7b}$$

Equation [10.7] tells us that the greater the mass, the lower the frequency; and the stiffer the spring, the higher the frequency. This makes sense since a greater mass means more inertia and therefore a slower response (or acceleration); and larger k means greater force and therefore quicker response. The frequency f given in equation [10.7] at which a simple harmonic oscillator oscillates naturally is called its natural frequency (to distinguish it from a frequency at which it might be forced to oscillate by an outside force.

Upon substituting equation [10.6] into equation [10.2], we get the expressions of the restoring force, *F*, and acceleration, *a*:

$$F = -kx = -m\omega^2 x \quad [10.8a]$$

$$a = -\frac{k}{m}x = -\omega^2 x \quad [10.8b]$$

That is, the acceleration of the block is proportional to its position, and the direction of the acceleration is opposite the direction of the displacement of the block from equilibrium position. Systems that behave in this way are said to exhibit simple harmonic motion. An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

Example 10.1

Two springs, one of force constant k_1 and the other of force constant k_2, are connected end-to-end to a block of mass 0.33 kg that is set oscillating over a frictionless floor as shown in Figure 10.3. (a) Find the force constant k of the combination, (b) If $k_1 = 5\ N/m$ and $k_2 = 10\ N/m$, find k. (c) What is the frequency of the oscillations?

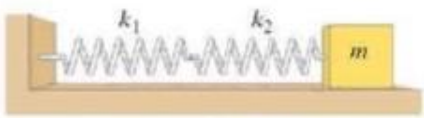


Figure 10.3

Solution

(a) When a force F is applied to the combination, each spring is acted on by this force. Hence the respective elongations of the springs are: $x_1 = F/k_1$ and $x_2 = F/k_2$ and the total elongation of the combination is

$$x = x_1 + x_2 = \frac{F}{x_1} + \frac{F}{x_2} = \frac{F(k_1+k_2)}{k_1 k_2}.$$

Since $F = \frac{k_1 k_2}{(k_1+k_2)}x = kx$ for the combination, where k is the effective force constant of the combination and is given by: $k = F/x = k_1 k_2/(k_1 + k_2)$.

(b) $k = \frac{(5\ N/m)(10\ N/m)}{5\ N/m+10N/m} = 3.3\ N/m$; (c)$f = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{(2)(3.14)}\sqrt{3.3/0.33} = 0.5033\ Hz$

10.1.2 Energy of Simple Harmonic Oscillator

Let us examine the mechanical energy of a system in which a particle undergoes simple harmonic motion, such as the block–spring system illustrated in Figure 10.4. Because the surface is frictionless, the system is isolated and we expect the total mechanical energy of the system to be constant. We assume a massless spring, so the kinetic energy of the system corresponds only to that of the block.

If the block is displaced to its maximum position $x = A$, and released from rest, its *initial* acceleration, $a_{max} = -\frac{k}{m}A = -\omega^2 A$, is also maximum and its velocity is zero. When the block passes through the equilibrium position $x = 0$, its acceleration is zero. At this instant, its speed is a maximum because the acceleration changes sign and is given by $v_{max} = wA = A\sqrt{k/m}$. Therefore, the maximum kinetic energy of the block occurs at $x = 0$, as shown in Figure 10.5, and is given by:

$$K = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m\omega^2 A^2 \quad [10.9]$$

The block then continues to travel to the left of the equilibrium position with a positive acceleration and finally reaches $x = -A$, at which time its acceleration is $a_{max} = +\frac{kA}{m} = +\omega^2 A$ and its speed is again zero as discussed above. The block completes a full cycle of its motion by returning to the original position, again passing through *x* = 0 with maximum speed. Therefore, the block oscillates between the turning points $x = \pm A$. In the absence of friction, this idealized motion will continue forever because the force exerted by the spring is conservative. Real systems are generally subject to friction, so they do not oscillate forever.

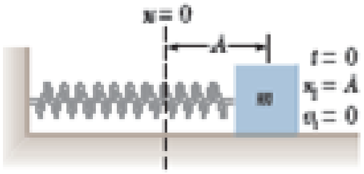


Figure 10.4

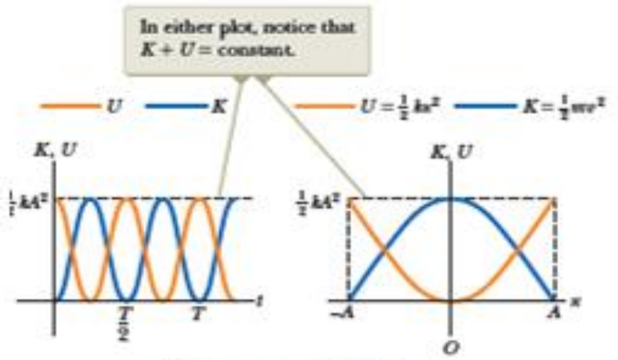


Figure 10.5

The elastic potential energy stored in the spring for any elongation *x* and the kinetic energy of the particle as a function of time and position is shown in Figure 10.5. The maximum elastic potential energy stored in the spring occurs at $x = \pm A$ and is given by:

$$U = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 \quad [10.10]$$

We see that *K* and *U* are *always* positive quantities or zero. The total mechanical energy of the simple harmonic oscillator is the sum of the kinetic and potential energies,

$$E = K + U \quad [10.11a]$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad [10.11b]$$

That is, the total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.

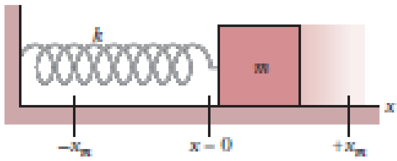
$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = constant \quad [10.12]$$

The total mechanical energy is equal to the maximum potential energy stored in the spring when $x = A$, that is $E = \frac{1}{2}kA^2$, because *v* = 0 at these points and there is no kinetic energy as shown in Figure 10.5. At the equilibrium position, where *U = 0* because *x* = 0, the total energy is all in the form of kinetic energy, that is, $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m\omega^2 A^2$. From the first expression of equation [10.12], the instantaneous velocity of the bob as a function of position becomes:

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{(A^2 - x^2)} \quad [10.13]$$

Example 10.2

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m as shown in the Figure below. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.



(a) What are the angular frequency, the frequency, and the period of the resulting motion?

Solution

The block–spring system forms a linear simple harmonic oscillator, with the block undergoing simple harmonic motion. The angular frequency is given by equation [10.5]:
$\omega = \sqrt{k/m} = \sqrt{(65\ N/m)/(0.68\ kg)}$ = 9.78 rad/s. The frequency is $f = \frac{\omega}{2\pi} = \frac{9.78\ rad/s}{2\pi} =$ 1.56 Hz. The period is $T = \frac{1}{f} = \frac{1}{1.56} = 0.64\ s = 640\ ms$.

(b) What is the amplitude of the oscillation?

With no friction involved, the mechanical energy of the spring–block system is conserved. The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system is a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm, that is, $x_m = A = 11\ cm$. (Answer)

(c) What is the maximum speed v_{max} of the oscillating block, and where is the block when it has this speed?

The maximum speed is the velocity amplitude $v_{max} = A\omega = (9.78\ \text{rad/s})(0.11\ \text{m}) = 1.1\ \text{m/s}$. This maximum speed occurs when the oscillating block is rushing through the origin.

(d) What is the magnitude a_{max} of the maximum acceleration of the block?

The magnitude a_{max} of the maximum acceleration is the acceleration amplitude $a_{max} = \omega^2 A = (9.78\ rad/s)^2(0.11\ m) = 11\ m/s^2$. This maximum acceleration occurs when the block is at the ends of its path, where the block has been slowed to a stop so that its motion can be reversed. At those extreme points, the force acting on the block has its maximum magnitude.

10.1.3 Comparing Simple Harmonic Motion with Uniform Circular Motion

We can obtain an expression for the position of an object moving with simple harmonic motion as a function of time by considering the relationship between simple harmonic motion and uniform circular motion. Consider a particle located at point P on the circumference of a circle of radius A as in Figure 10.6a, with the line OP making an angle $\theta = 0$ with the x axis at $t = 0$. We call this circle a reference circle for comparing simple harmonic motion with uniform circular motion, and we choose the position of P at $t = 0$ as our reference position. If the particle moves along the circle with constant angular speed ω until OP makes an angle θ with the x axis as in Figure 10.6a, at some time $t > 0$ the angle between OP and the x axis is $\theta = \omega t$. As the particle moves along the circle, the projection of P on the x axis, labelled point Q, moves back and forth along the x axis between the limits $x = \pm A$.

Notice that points P and Q always have the same x coordinate. From the right triangle OPQ, we see that this x coordinate is

$$x = A\,cos(\omega t) \qquad [10.14]$$

This expression shows that the point Q moves with simple harmonic motion along the x axis. Therefore, the motion of an object described by the analysis model of a particle in simple harmonic motion along a straight line can be represented by the projection of an object that can be modelled as a particle in uniform circular motion along a diameter of a reference circle.

Therefore, the angular speed ω of P is the same as the angular frequency ω of simple harmonic motion of Q along the x-axis. This geometric interpretation shows that the time interval for one complete revolution of point P on the reference circle is equal to the period of motion T for simple harmonic motion between $x = \pm A$ of Q along the x-axis. In one complete revolution, point P rotates through an angle of 2π rad in a time equal to the period T. In other words, the motion repeats itself every T seconds. Note that the position x(t) of the particle must (by definition) return to its initial value at the end of a period. That is, if x(t) is the position at some chosen time t, then the particle must return to that same position at time $t + T$. Using equation [10.14] to express this condition, returning to the same position can be written as $A\,cos(\omega t) = A\,cos(\omega(t + T))$. The cosine function first repeats itself when its argument (the phase, remember) has increased by 2π rad, so that, $\omega(t + T) = \omega t + 2\pi$, and this means that $\omega T = 2\pi$. Therefore, the angular speed ω of P is given by

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f \qquad [10.4]$$

which is similar to equation [10.4].

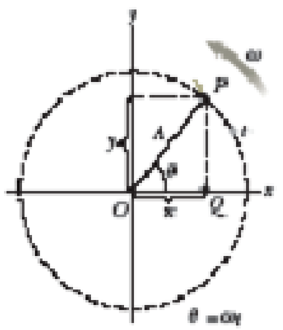


Figure 10.6a

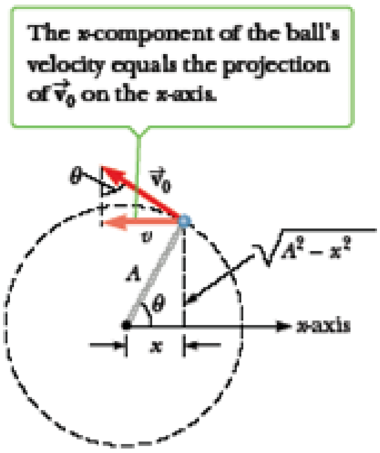


Figure 10.6b

From the geometry of Figure 10.6b, we get

$$v = -v_0 \sin(\theta) = -v_0 \sin(\omega t) \qquad [10.15]$$

Because the relationship between linear and angular speed for circular motion is $v = r\omega$, where r is radius of circle, the particle moving on the reference circle of radius A has a velocity of magnitude $v = \omega A$. Using $v_0 = \omega A$ into equation [10.15] or using equation [10.14] into equation [10.13], we get the expression of the velocity (v) and acceleration (a) as a function of time:

$$v(t) = -A\omega sin(wt) = -v_{max}\sin(\omega t) \quad [10.16a]$$

$$a(t) = -A\omega^2 cos(wt) = -a_{max}cos(\omega t) \quad [10.16b]$$

$$a(t) = -\omega^2 x(t) \quad [10.16c]$$

where $v_{max} = A\omega$, $a_{max} = A\omega^2 A$, A, called the amplitude, is simply the maximum value of the position of the particle in either the positive or negative *x* direction and is determined uniquely by the position and velocity of the particle at $t = 0$. The position, x(t), velocity,$v(t)$, and acceleration, a(t), of a particle undergoing simple harmonic motion as a function of time are plotted in Figure 10.7.

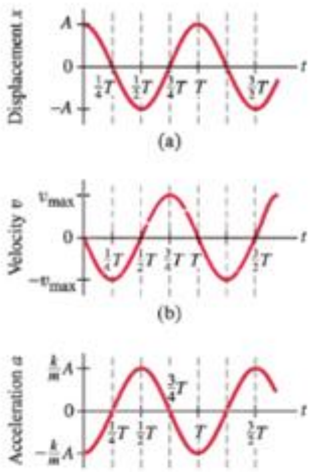


Figure 10.7

Example 10.3

Consider the spring–block system of example 10.2.

(a) What is the velocity function $v(t)$?

Solution

Using the value of v_{max} in example 10.2 and the expression of velocity function $v(t)$ given by equation [10.16a]: $v(t) = -v_{max}\sin(\omega t) = (1.1\ m/s)\sin[(9.8\ rad/s)t] = 1.1\sin(9.8t)$.

(b) What is the acceleration function $a(t)$?

Solution

Using the value of a_{max} in example 10.2 and the expression of acceleration $a(t)$ given by equation [10.16b]: $a(t) = -a_{max}\cos(\omega t) = (11\ m/s^2)\cos[(9.8\ rad/s)t] = 11\ cos(9.8t)$.

10.1.4 The Simple Pendulum

The simple pendulum is another mechanical system that exhibits periodic motion. It consists of a particle-like bob of mass m suspended by a light string of length L that is fixed at the upper end as shown in Figure 10.8. The motion occurs in the vertical plane and is driven by the gravitational force. We shall show that, provided the angle θ is small (less than about 10^0), the motion is very close to that of a simple harmonic oscillator. The forces acting on the bob are the tension force T exerted by the string and the gravitational force mg. The tangential component $mgsin(\theta)$ of the gravitational force always acts toward $\theta = 0$, opposite the displacement of the bob from the lowest position.

Therefore, the tangential component is a restoring force, and we can apply Newton's second law for motion in the tangential direction:

$$F_t = ma_t = -mgsin(\theta) \quad [10.17]$$

where the negative sign indicates that the tangential force acts toward the equilibrium (vertical) position and $x$is the bob's position measured along the arc.

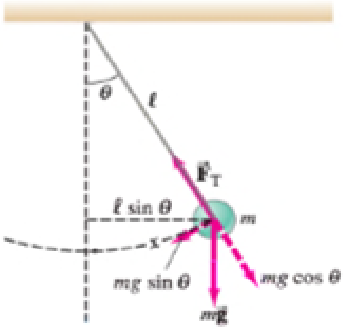


Figure 10.8

The angular frequency ω of a simple pendulum is defined as:

$$\omega = \sqrt{g/L} \quad [10.18]$$

The period T of the motion of simple pendulum is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{L/g} \quad [10.19a]$$

$$g = \frac{4\pi^2 L}{T^2} \quad [10.19b]$$

In other words, the period and frequency of a simple pendulum depend only on the length L of the string and the acceleration due to gravity g. Because the period is independent of the mass, we conclude that all simple pendula that are of equal length and are at the same location (so that g is constant) oscillate with the same period. The simple pendulum can be used as a time keeper because its period depends only on its length and the local value of g by equation [10.19b]. It is also a convenient device for making precise measurements of the free-fall acceleration. Such measurements are important because variations in local values of g can provide information on the location of oil and other valuable underground resources.

Example 10.4

(a) A pendulum clock is in an elevator that descends at a constant velocity. Does it keep correct time?

(b) The same clock is in an elevator in free fall. Does it keep correct time?

Solution

(a) The motion of the pendulum bob is not affected by motion of its support at constant velocity, so the clock keeps correct time.

(b) In free fall the pendulum's support has the same downward acceleration of g as the bob, so no oscillations occur and the clock does not operate at all.

Example 10.5

Using a small pendulum of length 0.171 m, a geophysicist counts 72.0 complete swings in a time of 60.0 s. What is the value of g in this location?

Solution

First calculate the period by dividing the total elapsed time by the number of complete oscillations: That is $T = \frac{time}{number\ of\ oscillations} = \frac{60\ s}{72.0} = 0.833\ \mathrm{s}$.

Now using equation [10.19b]: $g = \frac{4\pi^2 L}{T^2} = \frac{(39.5)(\,0.171\ \mathrm{m})}{(0.833\ \mathrm{s})^2} = 9.73\ \mathrm{m/m^2}$.

10.2 Resonance

The oscillations of a macroscopic oscillator decay over time because the energy leaks out into the surroundings. That is, the energy of a damped oscillator, an oscillator subjected to friction, decreases overtime because of energy loss due work done against friction. For an oscillation to be sustained, this energy loss must be balanced by the energy added to the oscillator. In other words, it's possible to compensate for this energy loss by applying an external force that does positive work on the system.

For example, suppose an object–spring system having some natural frequency of vibration f_0 is pushed back and forth by a periodic force with frequency f, given by $F = F_0 cos(2\pi f t)$ as shown in Figure 10.9. The system vibrates at the frequency f of the driving force. This type of motion is referred to as a forced vibration. It's amplitude reaches a maximum when the frequency of the driving force f equals the natural frequency of the system f_0, called the resonant frequency of the system. Under this condition, the system is said to be in resonance.

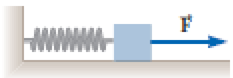


Figure 10.9

For example, we might pull the mass on the spring of Figure 10.1 back and forth at a frequency f. The mass then oscillates at the frequency f of the external force, even if this frequency is different from the natural frequency of the spring, which we will now denote by f_0, where $\omega_0 = 2\pi f_0 = \sqrt{k/m}$. In a forced oscillation the amplitude of oscillation, and hence the energy transferred to the oscillating system, is found to depend on the difference between f and f_0 aswell as on the amount of damping, reaching a maximum when the frequency of the external force equals the natural frequency of the system, that is, when $f = f_0$. The amplitude is plotted in Figure 10.10 as a function of the external frequency f. The amplitude can become large when the driving frequency f is near the natural frequency, $f \approx f_0$, as long as the damping is not too large. When the damping is small, the increase in amplitude near $f = f_0$ is very large (and often dramatic). This effect is known as resonance. The natural frequency f_0 of a system is called it's resonant frequency.

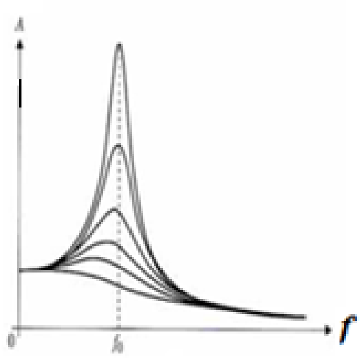


Figure 10.10

But we learned that a stretched string can vibrate in one or more of its natural modes. Here again, if a periodic force is applied to the string, the amplitude of vibration increases as the frequency of the applied force approaches one of the string's natural frequencies of vibration.

A simple example of resonance is a child being pushed on a swing, which is essentially a pendulum with a natural frequency that depends on its length. The swing is kept in motion by a series of appropriately timed pushes. For its amplitude to increase, the swing must be pushed each time it returns to the person's hands. This corresponds to a frequency equal to the natural frequency of the swing. If the energy put into the system per cycle of motion equals the energy lost due to friction, the amplitude remains constant.

10.3 Mechanical Waves

10.3.1 Wave Types and Propagation of a Disturbance

A wave is, in general, a disturbance that moves through a medium. A wave carries energy, but there is no transport of matter. A wave is thus described as the transfer of energy through space without the accompanying transfer of matter. All waves carry energy and momentum. The amount of energy transmitted through a medium and the mechanism responsible for the transport of energy differ from case to case. In a periodic wave, pulses of the same kind follow one another in regular succession. The world is full of waves: sound waves, waves on a string, seismic waves, and electromagnetic waves, such as visible light, radio waves, television signals, and x-rays. All these waves have as their source a vibrating object, so we can apply the concepts of simple harmonic motion in describing them.

In the list of energy transfer, two mechanisms, mechanical waves and electromagnetic waves or radiation, depend on waves. By contrast, in another mechanism, matter transfer, the energy transfer is accompanied by a movement of matter through space with no wave character in the process. Electromagnetic waves can travel through a vacuum. Examples are light and radio waves. All mechanical waves require (1) some source of disturbance, (2) a medium containing elements that can be disturbed, and (3) some physical connection or mechanism through which adjacent portions or elements of the medium can influence each other. Mechanical waves can be transverse waves or longitudinal waves.

One way to demonstrate wave motion is to flick one end of a long string that is under tension and has its opposite end fixed as shown in Figure 10.11. In this manner, a single bump (called a *pulse*) is formed and travels along the string with a definite speed. Figure 10.11(a) the creation and propagation of the traveling pulse. The pulse has a definite height and a definite speed of propagation along the medium. As the pulse travels, each disturbed element of the string moves in a direction (y axis) that is perpendicular to the direction of propagation (x-axis). Notice that no part of the string ever moves in the direction of the propagation.

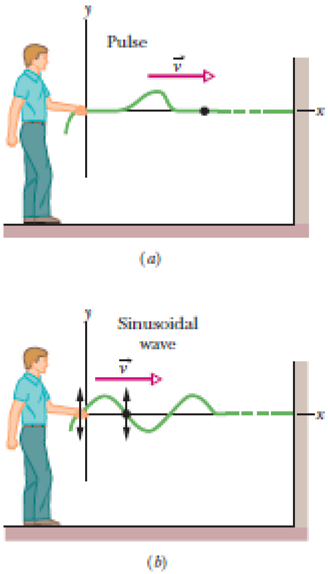


Figure 10.11

A wave is a periodic disturbance traveling through a medium. If we were to move the end of the string up and down repeatedly, we would create a traveling wave, which has characteristics a pulse does not have as shown in Figure 10.11 (b). A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a transverse wave. Therefore, in a transverse wave, the particles of the medium move back and forth perpendicular to the direction of the wave. Waves that travel down a stretched string when one end is shaken are transverse as shown in Figure 10.11.

Another type of wave or pulse, one moving down a long, stretched spring as shown in Figure 10.12. The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring (to the right in Figure 10.12).

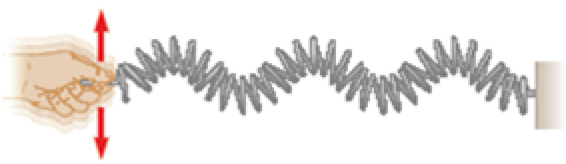


Figure 10.12

Notice that the direction of the displacement of the coils is parallel to the direction of propagation of the compressed region. A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a longitudinal wave. Sound waves are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travel through air.

A sinusoidal wave could be established on the rope in Figure 10.13 by shaking the end of the rope up and down in simple harmonic motion. This movement is the motion of the wave. If we focus on one element of the medium, such as the element at x = 0, we see that each element moves up and down along the y-axis in simple harmonic motion. This movement is the motion of the elements of the medium. It is important to differentiate between the motion of the wave and the motion of the elements of the medium.

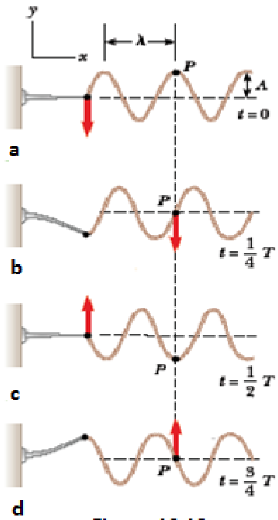


Figure 10.13

A point at which the displacement of the element from its normal position is highest is called the crest of the wave. The lowest point is called the trough. The distance from one crest to the next is called the wavelength λ as shown in Figure 10.14b. More generally, the wavelength is the minimum distance between any two identical points on adjacent waves as shown in Figure 10.13 and in Figure 10.14b. If you count the number of seconds between the arrivals of two adjacent crests at a given point in space, you measure the period T of the waves. In general, the period is the time interval required for two identical points of adjacent waves to pass by a point as shown in Figure 10.14a.

The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium. The same information is more often given by the inverse of the period, which is called the frequency f.

In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval. The frequency of a sinusoidal wave is related to the period by the expression $f = 1/T$. The maximum position of an element of the medium relative to its equilibrium position is called the amplitude A of the wave as indicated in Figure 10.14.

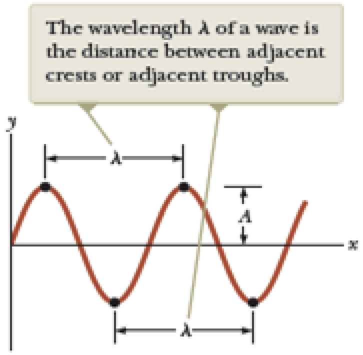


Figure 10.14b

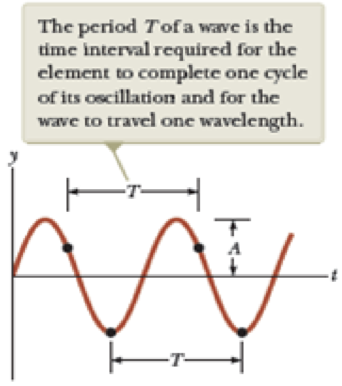


Figure 10.14a

10.3.2 A Travelling Wave

Waves travel with a specific speed, and this speed depends on the properties of the medium being disturbed. Imagine a source vibrating such that it influences the medium that is in contact with the source. Such a source creates a disturbance that propagates through the medium. If the source vibrates in simple harmonic motion with period T, sinusoidal waves propagate through the medium at a speed v as shown in Figure 10.15. By definition, the wave travels through a displacement Δx equal to one wavelength λ in a time interval Δt of one period T.

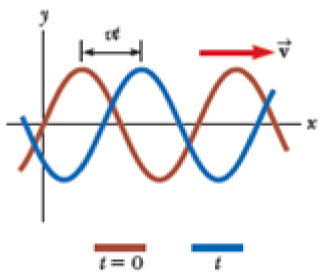


Figure 10.15

Therefore, the wave speed, wavelength, and period are related by the expression

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f \qquad [10.20]$$

We can express the wave function in a convenient form by defining two other quantities, the angular wave number k (usually called simply the wave number) and the frequency f:

$$k = 2\pi/\lambda \qquad [10.21]$$

$$\omega = \frac{2\pi}{T} = 2\pi f \qquad [10.22]$$

The mathematical representation of the traveling wave is given by the sinusoidal wave function y(x,t) given by:

$$y(x,t) = A\sin(kx \pm \omega t + \phi) \qquad [10.23]$$

where ϕ is the phase constant. This constant can be determined from the initial conditions. For a wave traveling to the right, we chose the (-) sign and for a wave traveling to the left, we chose the (+) sign. Using equations [10.20], [10.21] and [10.22], the wave speed v can also be expressed in the following alternative forms

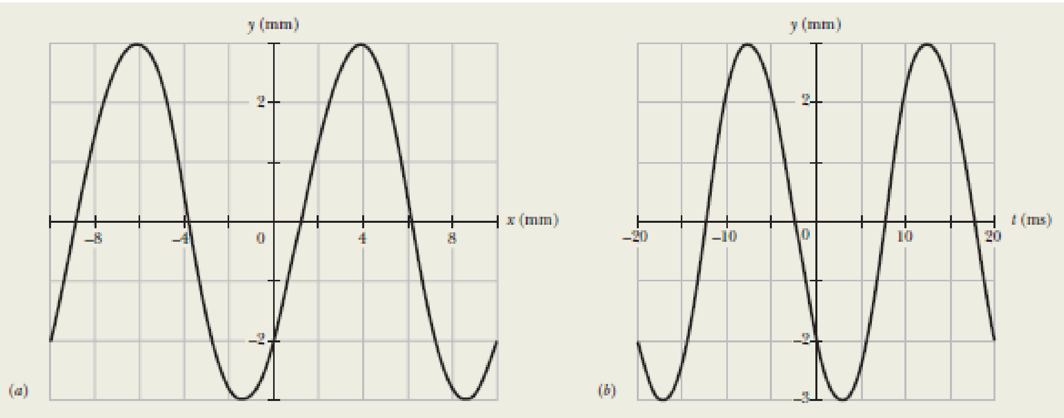
$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \lambda f \quad [10.24]$$

Examples:

- a vibrating blade sends a sinusoidal wave down a string attached to the blade
- a loudspeaker vibrates back and forth, emitting sound waves into the air
- a guitar body vibrates, emitting sound waves into the air
- a vibrating electric charge creates an electromagnetic wave that propagates into space at the speed of light

Example 10.6

A transverse wave traveling along an x axis has the form given in the Figure below. In (a) it gives the displacements of string elements (y) as a function of (x), all at time $t = 0$. In (b) it gives the displacements of the element at $x = 0$ as a function of t. Find the values of the quantities in equation [10.23], that is, the (a) amplitude (y_{max}), (b) period (T), (c) wavelength (λ), (d) angular wave number k, (e) the angular frequency ω, (f) phase constant (ϕ), (g) direction of propagation of the wave and (h) wave function y(x, t).



Solution

From the graph, we can determine

(a) $y_{max} = 3.0\ mm$

(b) $T = 20\ ms$

(c) $\lambda = 10\ mm$. Now we can calculate the quantities

(d) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.010\ m} = 200\pi\ \text{rad/m}$

(e) $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.020\ s} = 100\pi\ rad/s$

(f) The value of ϕ is set by the conditions at $x = 0$ at the instant $t = 0$. From either figure we see that at that location and time, $y = 2.0\ mm$. Substituting these three values and also $y_{max} = 3\ mm$ into equation [10.23], gives us $-2\ mm = (3.0\ mm)\sin(0 + 0 + \phi)$ and thus $\phi = sin^{-}\left(-\frac{2}{3}\right) =$

$-0.73\ rad$. Note that this is consistent with the rule that on a plot of y versus x, a negative phase constant shifts the normal sine function rightward, which is what we see in Figure (a).

(g) To find the direction, we apply a bit of reasoning to the figures. In the snapshot at $t = 0$ given in Figure (a), note that if the wave is moving rightward, then just after the snapshot, the depth of the wave at $x = 0$ should increase (mentally slide the curve slightly rightward). If, instead, the wave is moving leftward, then just after the snapshot, the depth at$x = 0$ should decrease. Now let's check the graph in Figure (b). It tells us that just after $t = 0$, the depth increases. Thus, the wave is moving rightward, in the positive direction of x, and we choose the minus sign in the wave equation.

(h) The wave function for the wave is thus: $y(x,t) = (3.0\ mm)\sin(200\pi x - 100\pi t - 0.73\ rad)$, with x in meters and t in seconds.

10.4 Standing Waves

Standing wave, also called stationary wave, combination of two waves moving in opposite directions, each having the same amplitude and frequency. The phenomenon is the result of interference, that is, when waves are superimposed, their energies are either added together or cancelled out. In the case of waves moving in the same direction, interference produces a travelling wave; for oppositely moving waves, interference produces an oscillating wave fixed in space. A vibrating rope tied at one end will produce a standing wave, as shown in the Figure 10.16; the wave train, after arriving at the fixed end of the rope, will be reflected back and superimposed on itself as another train of waves in the same plane.

Standing waves are waves which appear to be vibrating vertically without traveling horizontally. Created from waves with identical frequency and amplitude interfering with one another while traveling in opposite directions. At all times there are positions (N) along the rope, called nodes, at which there is no movement at all; there the two wave trains are always in opposition. On either side of a node is a vibrating antinode (A). The antinodes alternate in the direction of displacement so that the rope at any instant resembles a graph of the mathematical function called the sine. Nodes are thus positions on a standing wave where the wave stays in a fixed position over time because of destructive interference and antinodes are positions on a standing wave where the wave vibrates with maximum amplitude. Both longitudinal (e.g., sound) waves and transverse (e.g., water) waves can form standing waves.

10.4.1 Standing Waves in a String

Standing Wave harmonics: A wave that travels down a rope gets reflected at the rope's end. If the end of the rope is free, then the wave returns right side up. If the end of the rope is fixed, then the wave will be inverted as shown in Figure 10.16.

For a rope with two fixed ends, another wave travelling down the rope will interfere with the reflected wave. At certain frequencies, this produces standing waves where the nodes and antinodes stay at the same places over time. For all standing wave frequencies, the nodes and antinodes alternate with equal spacing. The lowest frequency (which corresponds with the longest wavelength) that will produce a standing wave has one "bump" along the string length L as shown in Figure 10.17. This standing wave is called the fundamental frequency. Thus, the fundamental frequency is the lowest frequency of a standing wave that has the fewest number of nodes and antinodes, where there are two nodes and one antinode.

$$L = \lambda/2 \quad [10.24a]$$

$$\lambda = 2L \quad [10.24b]$$

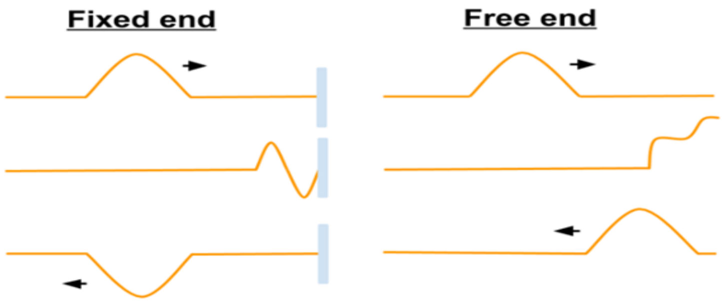


Figure 10.16

If v is the speed of the wave along the string, then the fundamental frequency is given by

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} \quad [10.25]$$

That is, for the fundamental frequency of a standing wave between two fixed ends, the wavelength is double the length of the string. Standing waves on a string or rope of length L with both fixed and free end are shown in Figure 10.17.

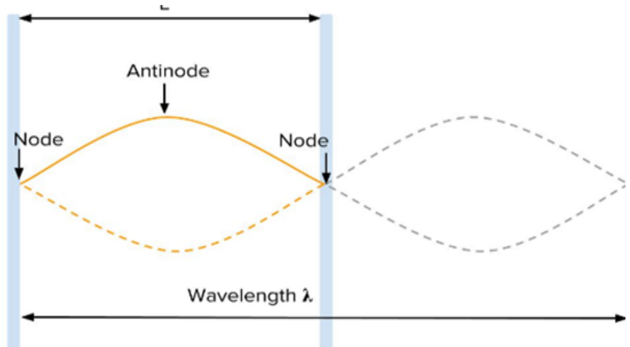


Figure 10.17

Each successive harmonic has an additional node and antinode. For the second harmonic, there are two "bumps", for the third, there are three, and so on. Examples of the second harmonics are shown in Figure 10.18. For the second harmonic of a standing wave between two fixed ends, the wavelength is the length of the string and its frequency is twice the fundamental frequency.

$$\mathrm{L} = \lambda \quad [10.26]$$

$$f_2 = \frac{v}{\lambda} = \frac{v}{L} = 2f_1 \quad [10.27]$$

For the third harmonic of a standing wave between two fixed ends, the wavelength is two-thirds the length of the string and its frequency is triple the fundamental frequency.

$$L = 3\lambda/2 \quad [10.28]$$

$$f_3 = \frac{v}{\lambda} = \frac{3v}{2L} = 3f_1 \quad [10.29]$$

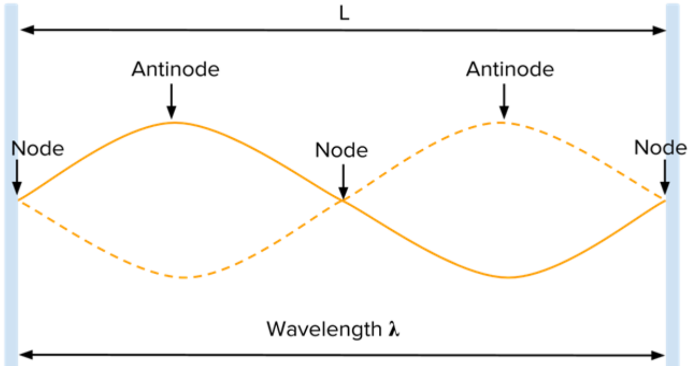


Figure 10.18

Examples of the third harmonics are shown in Figure 10.19.

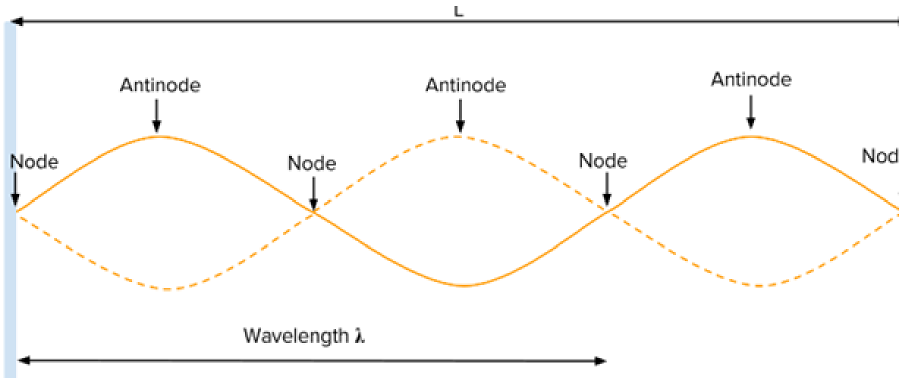


Figure 10.19

A string has an infinite number of resonant frequencies. The nth harmonics are a standing wave that is a positive integer multiple of the fundamental frequency

$$L = n\lambda/2 \qquad [10.30]$$

$$f_n = \frac{v}{\lambda} = \frac{nv}{2L} = nf_1 \qquad [10.31]$$

where is a positive integer, $n = 1,2,3,4, \ldots.$

10.4.2 Standing Waves in Air Columns

The waves under boundary conditions model can also be applied to sound waves in a column of air such as that inside an organ pipe or a clarinet. Standing waves in this case are the result of interference between longitudinal sound waves traveling in opposite directions.

In a pipe closed at one end, the closed end is a displacement node because the rigid barrier at this end does not allow longitudinal motion of the air. Because the pressure wave is 90° out of phase with the displacement wave, the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation). The open end of an air column is approximately a displacement antinode and a pressure node. We can understand why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; therefore, the pressure at this end must remain constant at atmospheric pressure.

The first three normal modes of oscillation of a pipe open at both ends (a) and a pipe open at one end ony (b) are shown in Figure 10.20. For a pipe open at both ends, both ends are displacement antinodes (approximately) and for a pipe open at one end only, the open end is antinode where as the closed end is a node.

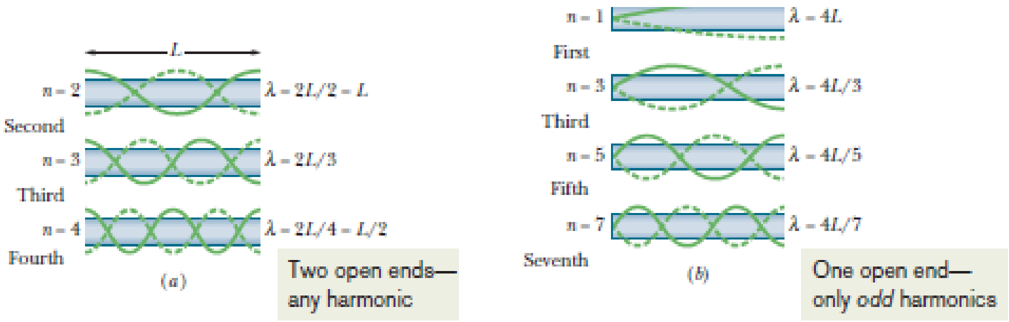


Figure 10.20

In the first normal mode of a pipe pipe open at both ends, the standing wave extends between two adjacent antinodes, which is a distance of half a wavelength. Therefore, the wavelength is twice the length of the pipe, and the fundamental frequency is

$$f_2 = v/L \qquad \text{(pipe open at both ends)} \qquad [10.32a]$$

$$f_1 = v/4L \qquad \text{(pipe open only at end)} \qquad [10.32b]$$

More generally, the resonant frequencies for a pipe of length *L* with two open ends correspond to the wavelengths

$$\lambda = 2L/n \qquad \text{for } n = 1,2,3, \ldots, \qquad [10.33]$$

where *n* is called the *harmonic number*. Letting *v* be the speed of sound, because all harmonics are present for pipe open at both ends and because the fundamental frequency is given by the same expression as that for a string, we can express the natural frequencies of oscillation as

$$f_n = \frac{v}{\lambda} = \frac{nv}{2L} = nf_1 \quad \text{for } n = 1,2,3,4,5, \ldots \text{ (pipe, two open ends)} \qquad [10.34]$$

In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.

More generally, the resonant frequencies for a pipe of length L with only one open end correspond to the wavelengths

$$\lambda = 4L/n \qquad \text{for } n = 1,3,5,7, \ldots, \qquad [10.35]$$

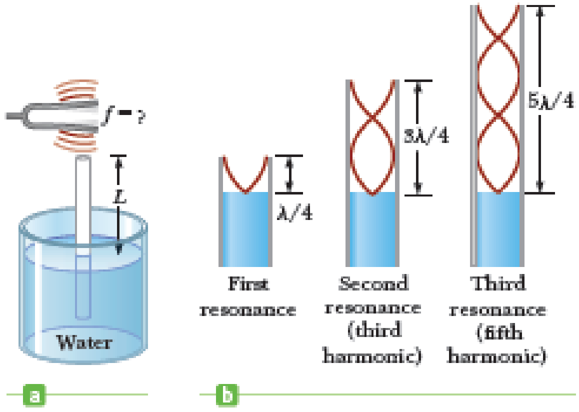
in which the harmonic number n must be an odd number. The resonant frequencies are then given by

$$f_n = \frac{v}{\lambda} = \frac{nv}{4L} = nf_1 \quad \text{for } n = 1,3,5,7, \ldots \text{ (pipe, one open end)} \qquad [10.36]$$

Note again that only odd harmonics can exist in a pipe with one open end. For example, the second harmonic, with $n = 2$, cannot be set up in such a pipe.

Example 10.7

The Figure below shows a simple apparatus for demonstrating resonance in a tube. A long tube open at both ends is partially submerged in a beaker of water, and a vibrating tuning fork of unknown frequency is placed near the top of the tube. The length of the air column, L, is adjusted by moving the tube vertically. The sound waves generated by the fork are reinforced when the length of the air column corresponds to one of the resonant frequencies of the tube. Suppose the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm. (a) With this measurement, determine the frequency of the tuning fork. (b) Find the wavelength and the next two air-column lengths giving resonance. Take the speed of sound to be 343 m/s.



Solution

Once the tube is in the water, the setup is the same as a pipe closed at one end. For (a), equation [10.36] can used to find the frequency of the tuning fork. (b) The next resonance maximum occurs when the water level is low enough to allow a second node (Figure b), which is another half-wavelength in distance. The third resonance occurs when the third node is reached, requiring yet another half-wavelength of distance. The frequency in each case is the same because it's generated by the tuning fork.

(a) Find the frequency of the tuning fork. Substitute $n = 1$, $v = 343\ m/s$, and $L_1 = 9.0 \times 10^{-2}\ m$ into equation [10.36]: $f_1 = \frac{nv}{4L} = \frac{v}{4L} = \frac{343\ m/s}{(4)(9.0\times10^{-2}\ m)} = 953\ \mathrm{Hz}$.

(b) Calculate the wavelength, using the fact that, for a tube open at one end, $\lambda = 4L$ for the fundamental. $\lambda = 4L_1 = 4(9.0 \times 10^{-2}\ m) = 0.360\ \mathrm{m}$. Add a half-wavelength of distance to L_1 to get the next resonance position: $L_2 = L_1 + \frac{\lambda}{2} = 0.090\ \mathrm{m} + 0.180\ \mathrm{m} = 0.270\ \mathrm{m}$. Add another half-wavelength to L_2 to obtain the third resonance position: $L_3 = L_2 + \frac{\lambda}{2} = 0.270\ \mathrm{m} + 0.180\ \mathrm{m} = 0.450\ \mathrm{m}$. This experimental arrangement is often used to measure the speed of sound, in which case the frequency of the tuning fork must be known in advance.

10.5 The Doppler Effect

If a car or truck is moving while its horn is blowing, the frequency of the sound you hear is higher as the vehicle approaches you and lower as it moves away from you. This phenomenon is one example of the Doppler effect. The same effect is heard if you're on a motor cycle and the horn is stationary: the frequency is higher as you approach the source and lower as you move away. Although the Doppler effect is most often associated with sound, it's common to all waves, including light. In deriving the Doppler effect, we assume the air is stationary and that all speed measurements are made relative to this stationary medium. In the general case, the speed of the observer v_O, the speed of the source, v_S, and the speed of sound *v* are all measured relative to the medium in which the sound is propagated.

Case 1: The Observer Is Moving Relative to a Stationary Source

In Figure 10.21, an observer is moving with a speed of v_O toward the source (considered a point source), which is at rest ($v_S = 0$). We take the frequency of the source to be f_S, the wavelength of the source to be λ_S, and the speed of sound in air to be v. If both observer and source are stationary, the observer detects f_S wave fronts per second. (That is, when $v_O = 0$ and $v_S = 0$, the observed frequency f_O equals the source frequency f_S.) An observer moving with a speed v_0 toward a stationary point source (S) hears a frequency f_0 that is greater than the source frequency f_S.

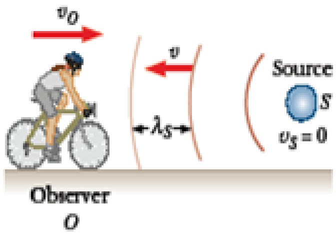


Figure 10.21

When moving toward the source, the observer moves a distance of $v_O t$ in t seconds. During this interval, the observer detects an additional number of wave fronts. The number of extra wave fronts is equal to the distance traveled, $v_O t$, divided by the wavelength λ_S:

$$additional\ wavefronts\ detected = \frac{v_0 t}{\lambda_S} \qquad [10.37]$$

Divide this equation by the time *t* to get the number of additional wave fronts detected *per second,* v_O/λ_S . Hence, the frequency heard by the observer is *increased* to

$$f_O = f_S + v_O/\lambda_S \qquad [10.38]$$

Substituting $\lambda_S = v/f_S$ into this expression for f_O, we obtain

$$f_O = f_S\left(\frac{v+v_O}{v}\right) \quad \text{(observer moving towards a stationary source)} \qquad [10.39]$$

Therefore, an observer moving with a speed of v_0 towards a stationary source hears a frequency f_O that is higher than the source frequency f_S .

When the observer is moving away from a stationary source (Figure 10.22), the observed frequency decreases. A derivation yields the same result as equation 10.39, but with $v - v_0$ in the numerator. Thus, when the observer is moving away from the source, the frequency heard by the observer is

$$f_O = f_S\left(\frac{v-v_O}{v}\right) \quad \text{(observer moving away fron a stationary source)} \qquad [10.40]$$

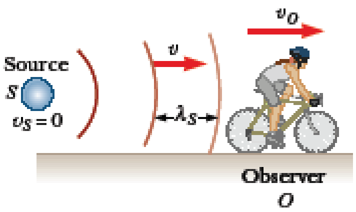


Figure 10.22

Case 2: The Source Is Moving Relative to a Stationary Observer

Now consider a source moving toward an observer at rest, as in Figure 10.23. Here, the wave fronts passing observer A are closer together because the source is moving in the direction of the outgoing wave. As a result, the wavelength λ_0 measured by observer A is shorter than the wavelength λ_S of the source at rest. During each vibration, which lasts for an interval T (the period), the source moves a distance

$$v_S T = \frac{v_S}{f_S} \qquad [10.41]$$

and the wavelength is shortened by that amount. The observed wavelength is therefore given by

$$\lambda_O = \lambda_S - v_S/f_S \qquad [10.42]$$

Because $\lambda_S = v/f_S$, the frequency observed by *A* is

$$f_O = \frac{v}{\lambda_O} = \frac{v}{\lambda_S - v_S/f_S} = \frac{v}{\frac{v}{f_S} - \frac{v_S}{f_S}}$$

$$f_O = f_S\left(\frac{v}{v - v_S}\right) \quad (\textit{source moving towards an observer at rest}) \qquad [10.43]$$

Therefore, when a source is moving toward an observer at rest with a speed of v_S, the observer hears a frequency f_O that is higher than the source frequency f_S .

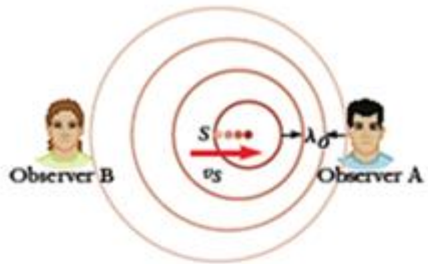


Figure 10.23

As expected, the observed frequency increases when the source is moving toward the observer. When the source is moving away from an observer at rest, the observed frequency decreases and hence minus sign in the denominator of equation [10.43] must be replaced with a plus sign, so the observed frequency becomes

$$f_O = f_S\left(\frac{v}{v + v_S}\right) \quad (\textit{source moving away from an observer at rest}) \qquad [10.44]$$

For a source S moving with speed v_S toward stationary observer A and away from stationary observer B, observer A hears an increased frequency, and observer B hears a decreased frequency.

Case 3: General Case

When both the source and the observer are in motion relative to Earth, equations [10.39] and [10.44] can be combined to give the observed frequency. For an observer moving towards a source and a source moving toward an observer or detector, the observed frequency is:

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right) \qquad [10.45a]$$

For an observer moving towards a source and a source moving away from an observer or detector, the observed frequency is:

$$f_O = f_S\left(\frac{v+v_O}{v+v_S}\right) \quad [10.45b]$$

For an observer moving away from a source and a source moving toward an observer or detector, the observed frequency is:

$$f_O = f_S\left(\frac{v-v_O}{v-v_S}\right) \quad [10.45c]$$

For an observer moving away from a source and a source moving way from an observer or detector, the observed frequency is:

$$f_O = f_S\left(\frac{v-v_O}{v+v_S}\right) \quad [10.45d]$$

The general Doppler-effect equation can be written as

$$f_O = f_S\left(\frac{v\pm v_O}{v\pm v_S}\right) \quad [10.46]$$

In this expression, the signs for the values substituted for v_O and v_S depend on the direction of the velocity. When the observer moves toward the source, a positive speed is substituted for v_O; when the observer moves away from the source, a negative speed is substituted for v_O. Similarly, a positive speed is substituted for v_S when the source moves toward the observer, a negative speed when the source moves away from the observer.

Example 10.8

Bats navigate and search out prey by emitting, and then detecting reflections of, ultrasonic waves, which are sound waves with frequencies greater than can be heard by a human. Suppose a bat emits ultrasound at frequency $f_{be} = 82.52\ kHz$ while flying with velocity $v_b = 9.0\ m/s$ as it chases a moth that flies with velocity $v_m = 8.0\ m/s$ both in the positive x-direction. (a) What frequency f_{md} does the moth detect? (b) What frequency f_{bd} does the bat detect in the returning echo from the moth?

Solution

The frequency is shifted by the relative motion of the bat and moth. Because they move along a single axis, the shifted frequency is given by the general Doppler equation [10.46]. Motion toward tends to shift the frequency up, and motion away tends to shift it down.

(a) Detection by moth: Here, the detected frequency f_0, that we want to find is the frequency f_{md} detected by the moth. On the right side, the emitted frequency f_S is the bat's emission frequency $f_{be} = 82.52\ kHz$, the speed of sound is $v = 343\ m/s$, the speed v_0 of the detector is the moth's speed $v_m = 8.0\ m/s$, and the speed v_S of the source is the bat's speed $v_b = 9.0\ m/s$. We have the speed of the moth (the detector) in the numerator of equation [10.46]. The moth moves away from the bat, which tends to lower the detected frequency and thus we use $v - v_0$ in the numerator of equation [10.46] to make the numerator smaller. The bat moves toward the moth, which tends to increase the detected frequency and thus we use $v - v_0$ in the denominator of equation [10.46] to make the denominator smaller. Therefore, $f_{md} = f_{be}\left(\frac{v-v_m}{v-v_b}\right) = (82.52\ \text{kHz})\frac{343\ \text{m/s} - 8.00\ \text{m/s}}{343\ \text{m/s} - 9.00\ \text{m/s}} =$ $82.767\ \text{kHz} = \ 82.8\ \text{kHz}.$

(b) Detection of echo by bat: In the echo back to the bat, the moth acts as a source of sound, emitting at the frequency f_{md} we just calculated. So now the moth is the source (moving away) and the bat is the detector (moving toward). To find the frequency f_{bd} detected by the bat, we write equation [10.46] as

$$f_{bd} = f_{md}\left(\frac{v+v_b}{v+v_m}\right) = (82.52 \text{ kHz})\frac{343 \text{ m/s}+9.00 \text{ m/s}}{343 \text{ m/s}+ 8.00 \text{ m/s}} = 83.00 \text{ kHz} = 83.0 \text{ kHz}.$$

Some moths evade bats by “jamming” the detection system with ultrasonic clicks.

10.6 Chapter Summary

10.7 Conceptual Questions

10.8 Problems

1. A spring is cut into three equal parts. If its original force constant was k, what is the force constant of each new spring?
2. A vertical spring 60 mm long resting on a table is compressed by 5.0 mm when a 200-g mass is placed on it. What is the force constant of the spring?
3. If it is pressed down and released, with what period does the mass of problem 10.2 oscillate up and down?
4. In Figure 10.24, two springs are joined and connected to a block of mass 0.245 kg that is set oscillating over a frictionless floor. The springs each have spring constant $k = 6430\ N/m$. (a) What is the effective spring constant of the combination? (b) What is the frequency of the oscillations?

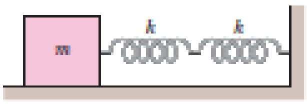


Figure 10.24

5. A simple harmonic oscillator consists of a block attached to a spring with $k = 200\ N/m$. The block slides on a frictionless surface, with equilibrium point $x = 0$ and amplitude $A = 0.20\ m$. A graph of the block’s velocity v as a function of time t is shown in Figure 10.25. The horizontal scale is set by $t_s = 0.20\ s$. What are (a) the period of the SHM, (b) the block’s mass, (c) its displacement at $t = 0$, (d) its acceleration at $t = 0.10\ s$, and (e) its maximum kinetic energy?

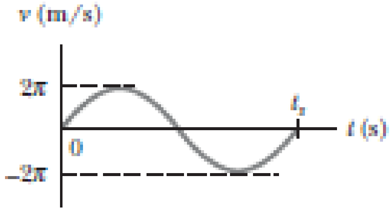


Figure 10.25

6. A block weighing 10.0 N is attached to the lower end of a vertical spring ($k = 200.0\ N/m$), the other end of which is attached to a ceiling. The block oscillates vertically and has a kinetic energy of 2.00 J as it passes through the point at which the spring is unstretched. (a) What is the period of the oscillation? (b) Use the law of conservation of energy to determine the maximum distance the block moves both above and below the point at which the spring is unstretched. (These are not necessarily the same.) (c) What is the amplitude of the oscillation? (d) What is the maximum kinetic energy of the block as it oscillates?
7. (a) What length pipe open at both ends has a fundamental frequency of $3.70 \times 10^2\ Hz$? Find the first overtone. (b) If the one end of this pipe is now closed, what is the new fundamental frequency? Find the first overtone. (c) If the pipe is open at one end only, how many harmonics are possible in the normal hearing range from 20 to 20 000 Hz?

11 Electromagnetism and Electronics

Learning Outcome

After completing this Chapter, students are expected to:

- apply Coulomb's law.
- Describe the concept of electric field and electric field lines
- Calculate the potential difference between two charged objects
- Define current
- Describe Ohm's law
- Calculate the electric power dissipated in a given resistor
- Sate the two Kirchhoff's rules
- Differentiate resistors combination in parallel and series

Introduction

In this chapter, we discuss Coulomb's law, which is the fundamental law of force between any two stationary charged particles. The concept of an electric field associated with charges is introduced and its effects on other charged particles described. Moreover, we define an electric potential — the potential energy per unit charge — corresponding to the electric field. We can define current and discuss some of the factors that contribute to the resistance to the flow of charge in conductors. The chapter presents the study and analyzes a number of simple direct-current circuits. The analysis is simplified by the use of two rules known as Kirchhoff's rules, which follow from the principle of conservation of energy and the law of conservation of charge. Most of the circuits are assumed to be in *steady state,* which means that the currents are constant in magnitude and direction.

11.1 Coulomb's Law and Electric Fields

11.1.1 Coulomb's Law

Learning outcome

After completing this section, students are expected to:

- Calculate the attractive or repulsive force between two point charges
- Explain the relation between the force between two point charges and the separation between the charges
- Recall the concept of inverse square law used earlier to use it in Coulomb's law.

In 1785, Charles Coulomb (1736–1806) experimentally established the fundamental law of electric force between two stationary charged particles, like the one shown in Fig.1.1.

An electric force has the following properties:

1. It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance *r*, between them.

2. It is proportional to the product of the magnitudes of the charges, $|q\ 1|$ and $|q\ 2|$, of the two particles.
3. It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

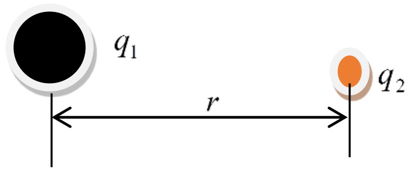


Fig.11.1: two charged particles separated by a distance r

From these observations, Coulomb proposed the following mathematical form for the electric force between two charges:

The magnitude of the electric force *F* between charges *q* 1 and *q* 2 separated by a distance *r* is given by

$$F = \frac{k_e |q_1||q_2|}{r^2} \qquad (11.1)$$

where k_e is a constant called the *Coulomb constant.*

Equation 11.1, known as **Coulomb's law**, applies exactly only to point charges and to spherical distributions of charges, in which case *r* is the distance between the two centers of charge. Electric forces between unmoving charges are called *electrostatic* forces. Moving charges, in addition, create magnetic forces.

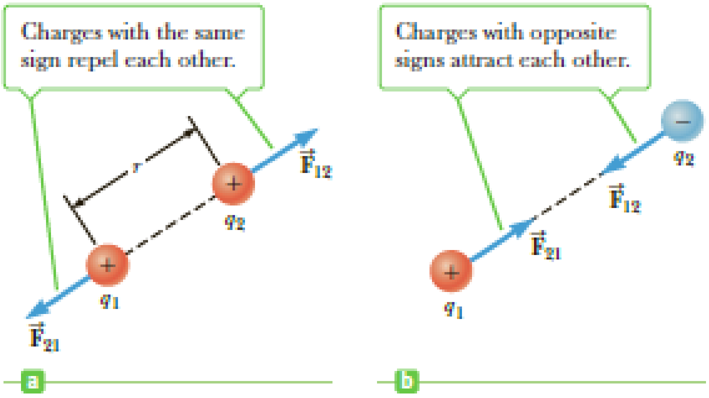
The value of the **Coulomb constant** in Equation 11.1 depends on the choice of units. The SI unit of charge is the **coulomb** (C). From experiment, we know that the Coulomb constant in SI units has the value, to five significant figures, of

$k_e = 8.9876 \times 10^9\ Nm^2C^{-2}$, this can be approximated as $k_e = 9.0 \times 10^9\ Nm^2C^{-2}$.

Figure 11.2 shows the electric force of repulsion between two positively - charged particles. Like other forces, electric forces obey Newton's third law; hence, the forces $\vec{F}_{12}$ and $\vec{F}_{21}$are equal in magnitude but opposite in direction. (The notation $\vec{F}_{12}$denotes the force exerted by particle 1 on particle 2; likewise, $\vec{F}_{21}$is the force exerted by particle 2 on particle 1.) From Newton's third law, $\vec{F}_{12}$ and $\vec{F}_{21}$are always equal regardless of whether q_1 and q_2 have the same magnitude.

The Coulomb force is similar to the gravitational force. Both act at a distance without direct contact. Both are inversely proportional to the distance squared, with the force directed along a line connecting the two bodies. The mathematical form is the same, with the masses m_1 and m_2 in Newton's law replaced by q_1 and q_2 in Coulomb's law and with Newton's constant *G* replaced by Coulomb's constant k_e. There are two important differences: (1) electric forces can be either attractive or repulsive, but gravitational forces are always attractive, and (2) the electric force between charged elementary particles is far stronger than the gravitational force between the same particles.

Figure 11.2 Two point charges separated by a distance *r* exert a force on each other given by Coulomb's law. The force on q_1 is equal in magnitude and opposite in direction to the force on q_2.



Examples:

1. A charged particle *A* exerts a force of 2.62 N to the right on charged particle *B* when the particles are 13.7 mm apart. Particle *B* moves straight away from *A* to make the distance between them 17.7 mm. What vector force does particle *B* then exert on *A*?

Solution:

By Colum's law the force the particles A and B exerted on each other is given by

$$F = \frac{k_e |q_B||q_A|}{r^2}$$

Hence, we can determine the product of the two charges as

$$|q_B||q_A| = \frac{Fr^2}{k_e} = \frac{2.62\,N \times (1.37\times10^{-2}\,m)^2}{9.0\times10^9\,Nm^2C^{-2}} = 5.44\times10^{-14}\,C^2$$

Now we can use this result to determine the force in new position.

$$F = \frac{k_e |q_B||q_A|}{r^2} = \frac{9.0\times10^9\ Nm^2C^{-2} \times 5.44\times10^{-14}\,C^2}{(1.77\times10^{-2}\,m)^2} = 1.56\ N$$

2. In figure 11.3 particle 3 lies on the x-axis between particle 1 ($q_1 = 1.6 \times 10^{-19}C$) and 2 ($q_2 = 3.2 \times 10^{-19}C$). Particle 3 has charge $q_3 = -3.20\times10^{-19}\,C$ and is at a distance $\frac{3}{4}R$ from particle 1 (R is the total distance between particle 1 and 2 and it is 20 cm). What is the net electrostatic force $\vec{F}_{1,net}$ on particle 1 due to particles 2 and 3?

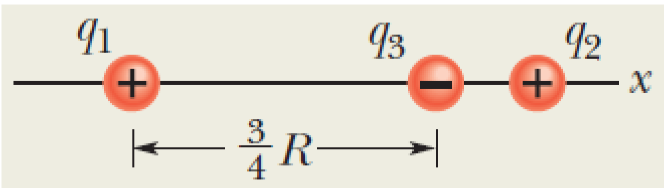


Fig. 11.3

Solution:

The presence of particle 3 does not alter the electrostatic force on particle 1 from particle 2. Thus, $\vec{F}_{12}$ force still acts on particle 1. Similarly, the force $\vec{F}_{13}$that acts on particle 1 due to particle 3

$\vec{F}_{12}$is not affected by the presence of particle 2. Because particles 1 and 3 have charge of opposite signs, particle 1 is attracted to particle 3. Thus, force is directed *toward* particle 3, as indicated in the free-body diagram Fig. 11.4.

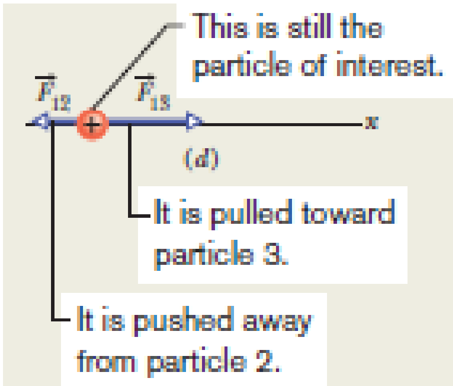


Fig.11.4

The magnitude of $\vec{F}_{12}$ can be calculated as

$$\vec{F}_{12} = \frac{k_e|q_1||q_2|}{R^2} = \frac{9.0\times10^9 Nm^2C^{-2}3.2\times1.6\times10^{-38}C^2}{(2\times10^{-2}m)^2}$$
$$= 1.15\times10^{-24}N$$

Similarly, the magnitude of $\vec{F}_{13}$ can be calculated as

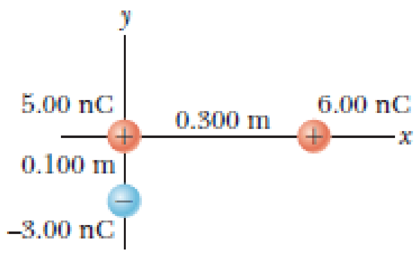
$$\vec{F}_{13} = \frac{k_e|q_1||q_3|}{R^2} = \frac{9.0\times10^9 Nm^2C^{-2}3.2\times1.6\times10^{-38}C^2}{(1.48\times10^{-2}m)^2}$$
$$= 2.05\times10^{-24}N$$

$$\vec{F}_{13} = \vec{F}_{12} + \vec{F}_{13} = (-1.15+2.05)\times10^{-24}N\hat{x}$$
$$= 9\times10^{-25}N\hat{x}$$

Exercises

1. A 7.50-nC point charge is located 1.80 m from a 4.20-nC point charge. (a) Find the magnitude of the electric force that one particle exerts on the other. (b) Is the force attractive or repulsive?
2. (a) Find the magnitude of the electric force between a Na^+Na+ ion and a Cl^- ion separated by 0.50 nm. (b) Would the answer change if the sodium ion were replaced by Li^+ and the chloride ion byBr^-? Explain.
3. (a) Two protons in a molecule are $3.8\times10^{-8}m$ apart. Find the magnitude of the electric force exerted by one proton on the other. (b) State how the magnitude of this force compares with the magnitude of the gravitational force exerted by one proton on the other. (c) What If? What must be a particle's charge-to-mass ratio if the magnitude of the gravitational force between two of these particles is equal to the magnitude of electric force between them?
4. Three point charges are arranged as shown in Fig. 11.5. Find (a) the magnitude and (b) the direction of the electric force on the particle at the origin.

FiG.11.5



11.1.2 Electric Fields

Learning Outcome

After completing this section, students are expected to:

- Specify the direction of the electric field from charged object.
- Identify the point where electric field strength equals zero between two charged particles
- Calculate the electric field strength for point charge

An electric field is a region around a charged object. Eelectric field exerts an electric force on any other charged object within the field. This differs from the Coulomb's law concept of a force exerted at a distance in that the force is now exerted by something — the field — that is in the same location as the charged object.

Figure 11.6 shows an object with a small positive charge q_0 placed near a second object with a much larger positive charge Q .

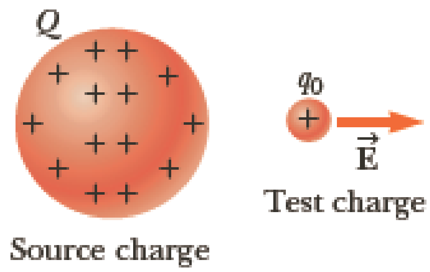


Fig. 11.6: A small object with a positive charge q_0 placed near an object with a larger positive charge Q is subject to an electric field $\vec{E}$ directed as shown. The magnitude of the electric field at the location of q_0 is defined as the electric force on q_0 divided by the charge *q*0.

The electric field $\vec{E}$ produced by a charge Q at the location of a small "test" charge q_0 is defined as the electric force $\vec{F}$ exerted by Q on q_0 divided by the test charge q_0

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad (11.2)$$

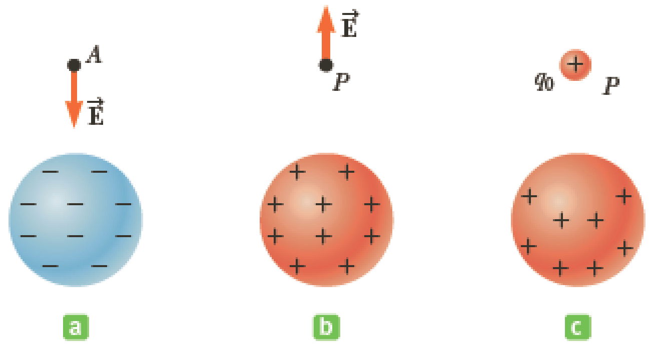
The SI unit of electric field strength is N/C.

When a positive test charge is used, the electric field always has the same direction as the electric force on the test charge, which follows from Equation 11.2. Hence, in Figure 11.6, the direction of the electric field is horizontal and to the right. The electric field at point *A* in Figure 11.7a is vertical and downward because at that point a positive test charge would be attracted toward the negatively - charged sphere.

Once the electric field due to a given arrangement of charges is known at some point, the force on *any* particle with charge q placed at that point can be calculated from a rearrangement of Equation 11.2:

$$\vec{F} = q_0\vec{E} \qquad (11.3)$$

Fig. 11.7: (a) The electric field at A due to the negatively – charged sphere is downward, toward the negative charge. (b) The electric field at P due to the positively – charged conducting sphere is upward, away from the positive charge. (c) A test charge q_0 placed at P will cause a rearrangement of charge on the sphere unless q_0 is negligibly small compared with the charge on the sphere.



As shown in Figure 11.8, the direction of $\vec{E}$ is the direction of the force that acts on a positive test charge q_0 placed in the field. We say that an electric field exists at a point if a test charge at that point is subject to an electric force.

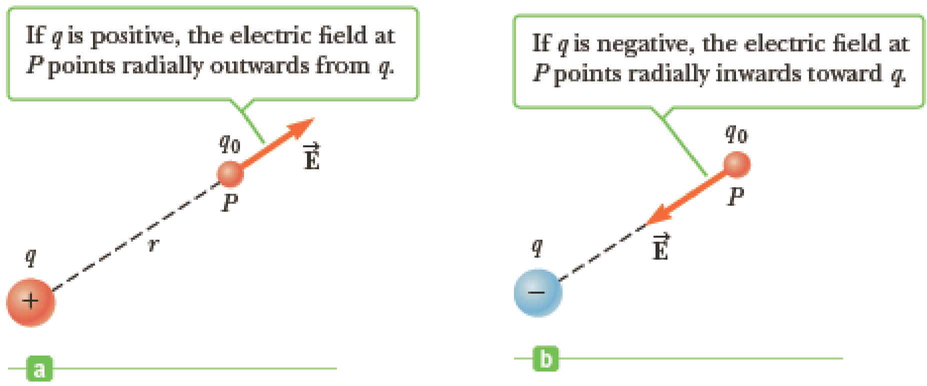


Fig. 11.8 A test charge q_0 at *P* is a distance r from a point charge q .

Consider a point charge q located a distance r from a test charge q_0 . According to Coulomb's law, the *magnitude* of the electric force of the charge q on the test charge q_0 is

$$\vec{F} = k_e \frac{|q_0||q|}{r^2} \qquad (11.4)$$

Because the magnitude of the electric field at the position of the test charge is defined as $\vec{E} = \vec{F}/q_0$, we see that the *magnitude* of the electric field due to the charge q at the position of q_0 is

$$\vec{E} = k_e \frac{|q|}{r^2} \qquad (11.5)$$

Examples

1. A small object of mass $3.80\,g$ and charge $-18.0\ \mu C$ is suspended motionless above the ground when immersed in a uniform electric field perpendicular to the ground. What is the magnitude and direction of the electric field?

Solution:

Since the object is motionless and suspended above the ground, the gravitational force mg of the object must be balanced by the electrostatic force on the object.

$$\vec{F}_e = -\vec{F}_g = mg\ \hat{y}$$

$$|\vec{F}_e| =, g = 3.8\times10^{-3}\ kg\times9.8\,ms^{-2} = 3.724\times10^{-3}\ N$$

Hence

$$\vec{E} = \frac{\vec{F}}{q} = \frac{3.724\times10^{-3}\,N}{1.8\times10^{-5}\,C} = 2.1\times10^{2}\ N/C\,\hat{y}$$

2. Four point charges are located at the corners of a square. Each charge has magnitude $3.2\ nC$ and the square has sides of length 2.00 cm. Find the magnitude of the electric field at the center of the square if (a) all of the charges are positive and (b) three of the charges are positive and one is negative.

Solution:

a) Let represent the problem graphically as shown in Fig. 11.9 and point P is the center of the square. The distance from pint P to any chare at the corner of the square is $2\sqrt{2}cm$. As all charges are positive, the electric field due to any charge is away from the charge and this can be represented by the free body diagram as shown. All fields lie along the diagonal of the square that makes 45^0 to the horizontal.

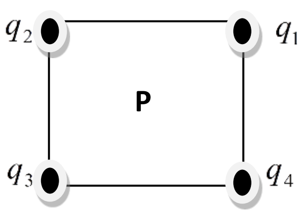
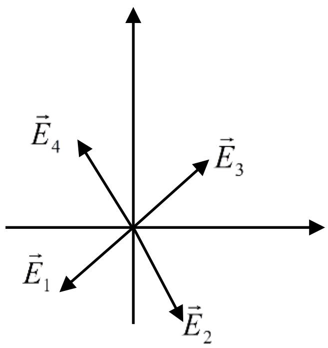


Fig.11.9



$$|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| = |\vec{E}_4| = \frac{k_e\,|q|}{r^2} = \frac{9.0\times10^{9}\ Nm^2C^{-2}\times3.2\times10^{-9}\ C}{\left(2\sqrt{2}\times10^{-2}\right)^{-2}} = 3.6\times10^{4}N/C$$

However, $\vec{E}_1$ and $\vec{E}_3$ are opposite in direction. Similarly, $\vec{E}_2$ and $\vec{E}_4$ are also opposite in direction. Hence, the net electric field at the center of a square due to the four charges located at the edge of the square is zero.

b) Now let in the above arrangement q_1 is negative and the rest three charges are positive. In this case the direction of the electric field due to q_1 ($\vec{E}_1$) is changed and directed along the direction $\vec{E}_3$. There is no change in magnitude of all four fields. With the same reasoning $\vec{E}_2$ and $\vec{E}_4$ cancel each other.

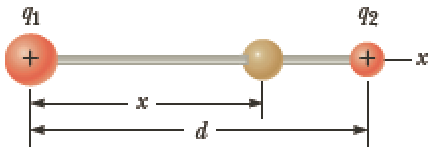
Therefore, the net field at point P for the present case is

$\vec{E} = \vec{E}_1 + \vec{E}_3 = 2\times3.6\times10^{4}\ N/C = 7.2\times10^{4}\ N/C$, that make 45^0 with positive x-axis.

Exercises

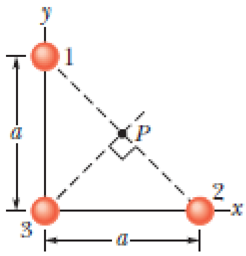
1. Two small beads having positive charges $q_1 = 3q$ and $q_2 = q$ are fixed at the opposite ends of a horizontal insulating rod of length $d = 1.50\ m$. The bead with charge q_1 is at the origin. As shown in Figure Fig.11.10, a third small charged bead is free to slide on the rod. At what position x is the third bead in equilibrium?

Fig. 11.10



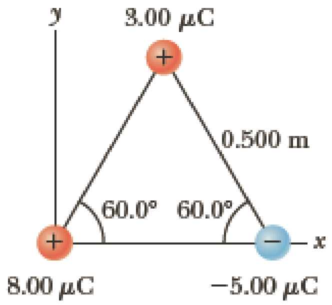
2. Figure 11.11 shows how the three particles are fixed. The particles have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6.00\ \mu m$. What are the (a) magnitude and (b) direction of the net electric field at point *P* due to the particles?

Fig. 11.11



3. Three charges are at the corners of an equilateral triangle, as shown in figure 11.12. Calculate the electric field at a point midway between the two charges on the *x* - axis.

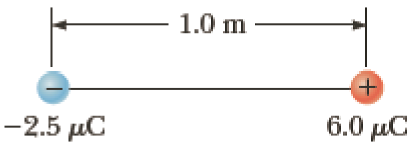
Fig. 11.12



4. A helium nucleus of mass $m = 6.64 \times 10^{-27}\ kg$ and charge $6.41 \times 10^{-19}\ C$ is in a constant electric field of magnitude $E = 2.0 \times 10^{-8}\ N/C$ pointing in the positive *x* - direction. Neglecting other forces, calculate (a) the nucleus' acceleration and (b) its displacement after 3.00 s if it starts from rest.

5. In figure 11.13, determine the point (other than infinity) at which the total electric field is zero.

Fig.11.13



11.2 Electric Potential and Electric potential Energy of point charge

Learning outcome

After completing this section, students are expected to:

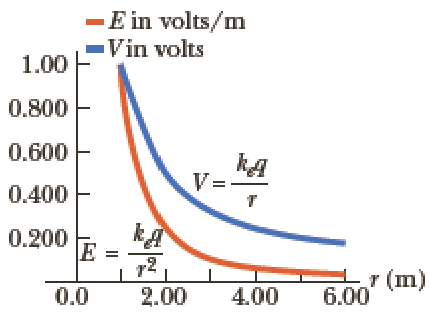
- Describe electric potential.
- Understand the relation between electric field and electric potential.
- Define electrical potential energy
- Differentiate between electric potential and the electric potential energy.

The electric potential created by a point charge *q* at any distance *r* from the charge is given by

$$V = \frac{k_e q}{r} \quad Or \quad V = \frac{-W_\infty}{q_0} = \frac{PE}{q_0} \qquad (11.6)$$

Equation 11.14 shows that the electric potential, or work per unit charge, required to move a positive test charge in from infinity to a distance r from a positive point charge *q* increases as the test charge moves closer to *q*. A plot of Equation 11.6 in Figure 11.14 shows that the potential associated with a point charge decreases as $1/r$ with increasing *r* , in contrast to the magnitude of the charge's electric field, which decreases as $1/r^2$.

Figure 11.14 Electric field and electric potential versus distance from a point charge of 1.11 3 10210 C. Note that *V* is proportional to 1/*r,* whereas *E* is proportional to 1/*r* 2.



The electric potential of two or more charges is obtained by applying the superposition principle: the total electric potential at some point *P* due to several point charges is the algebraic sum of the electric potentials due to the individual charges.

If V_1 is the electric potential due to charge q_1 at a point *P* (Fig. 11.15a), the work required to bring charge q_2 from infinity to *P* without acceleration is $q_2 V_1$. By definition, this work equals the potential energy PE of the two particle system when the particles are separated by a distance r (Fig. 11.15b).

We can therefore express the electrical potential energy of the *pair* of charges as

$$PE = V_1 q_2 = k_e q_1 q_2 / r \qquad (11.7)$$

If the charges are of the *same* sign, *PE* is positive. This is because like charges repel each other, positive work must be done on the system by an external agent to force the two charges near each other. Conversely, if the charges are of *opposite* sign, the force is attractive and *PE* is negative. This means that negative work must be done to prevent unlike charges from accelerating toward each other as they are brought close together.

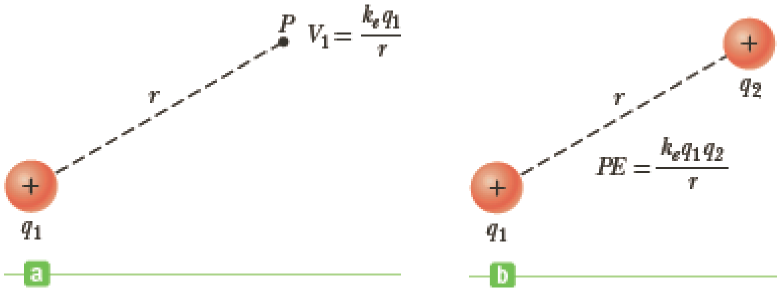
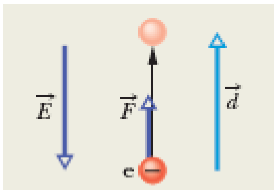


Figure 11.15 (a) The electric potential V_1 at P due to the point charge q_1 is $V_1 = k_e q_1/r$. (b) If a second charge, q_2, is brought from infinity to P, the potential energy of the pair is $PE = k_e q_1 q_2/r$.

Examples

1. Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electric force due to the electric field that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $\vec{E} = 150N/C$ and is directed down ward. What is the change ΔPE in the electric potential energy of a released electron when the electric force causes it to move vertically upward through a distance $d = 520m$ (Fig. 11.16)? Through what potential change does the electron move?

Fig. 11.16 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electric force $\vec{F}$ due to an electric field $\vec{E}$.



Solution:

$$W = \vec{F}.\vec{d}$$

The electrostatic force $\vec{F} = q\vec{E} = -e\vec{E}$

Where, the charge q in this case is the charge carried by electron e with negative sign. Moreover, negative charge.

$$W = q\vec{E}.\vec{d} = qEd\cos\theta = (-1.6\times10^{-19}\,C\times150N/C\times520m\times\cos180^0 = 1.2\times10^{-14}\,J.$$

The change in potential energy equals the negative of the work done on the charge.

That is $\Delta PE = -W = -1.2\times10^{-14}J$.

2. A particular 12 V car battery can send a total charge of $84.0 A.h$ (ampere-hours) through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

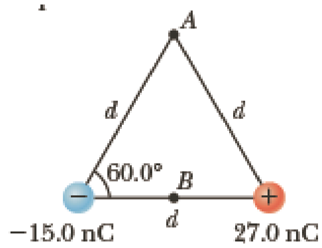
Solution:

a) $Q = I\,t = 84\,A \times 1.0\,h = 84\,A \times 3600s = 3.024 \times 10^{5}\ C \approx 3.0 \times 10^{5}\ C$

b) $PE = Q.V = 3.0 \times 10^{5}.C \times 12.0V = 3.6 \times 10^{6}\ J$

3. The two charges in Figure P11.17 are separated by $d = 2.0\,cm$. Find the electric potential at (a) point *A* and (b) point *B*, which is halfway between the charges.

Fig.11.17



Solution:

a) As the electric potential is a scalar quantity, we have not bother about the direction, but the sign of the charge matters.

Applying equation 911.6),

$$V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = \frac{9.0 \times 10^{9}\ Nm^2C^{-2}}{2.0 \times 10^{-2}\ m}(27 - 15) \times 10^{-9} C = 54.0\ V.$$

b) $V = \frac{k_e q_1}{d/2} + \frac{k_e q_2}{d/2} = \frac{9.0 \times 10^{9}\ Nm^2C^{-2}}{1.0 \times 10^{-2}\ m}(27 - 15) \times 10^{-9} C = 108.0\ V.$

4. The electric potential difference between the ground and a cloud in a particular thunderstorm is $1.2 \times 10^{9} V$. In the unit electron-volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud?

Solution

The change in the potential energy is

$\Delta PE = Ve = 1.0 \times 10^{9}.V \times 1.6 \times 10^{-19} C = 1.6 \times 10^{-10}\ J$

But, $1eV = 1.6 \times 10^{-19} J$

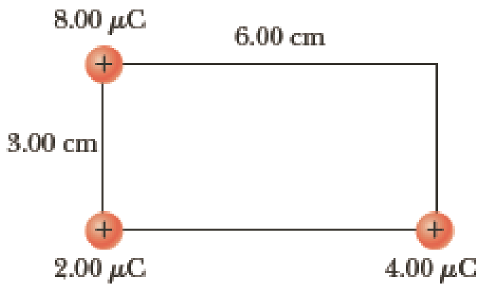
Hence, $\Delta PE = \frac{1.6 \times 10^{-10}\ J}{1.6 \times 10^{-19} J} \times 1eV = 1.0 \times 10^{9}\ eV$

Exercises

1. Consider a charge q_1 (+5.0μC) fixed at a site with another charge q_2 (charge +3.0μC, mass 6.0μg) moving in the neighboring space. (a) Evaluate the potential energy of q_2 when it is 4.0 cm fromq_1. (b) If q_2 starts from rest from a point 4.0 cm from q_1, what will be its speed when it is 8.0 cm from q_1? (*Note: Q*1 is held fixed in its place.)

2. Two charges q_1 (+2.00μC) and q_2 (+2.00μC) are placed symmetrically along the *x*-axis at *x*=±3.00*cm*. Consider a charge q_3 of charge +4.00μC and mass 10.0 mg moving along the *y*-axis. If *Q*3 starts from rest at *y*=2.00*cm*, what is its speed when it reaches *y*=4.00*cm?*
3. To form a hydrogen atom, a proton is fixed at a point and an electron is brought from far away to a distance of $0.529 \times 10^{-10}m$, the average distance between proton and electron in a hydrogen atom. How much work is done?
4. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. Non-relativistically, what would be the maximum speed of these electrons?
5. (a) Find the electric potential, taking zero at infinity, at the upper right corner (the corner without a charge) of the rectangle in Figure P11.18. (b) Repeat if the $q = 2.0\mu C$charge is replaced with a charge of $q = -2.0\mu C$.

Fig. 11.18



11.3 Current, resistance and Ohm's Law

11.3.1 Current

Learning outcome

After completing this section, students are expected to:

- Define conventional current.
- Apply the definition of current to solve related problems.
- Describe the motion of conduction electrons in a conductor.
- Draw simple circuit diagrams indicating current by arrows.

The current is the rate at which charge flows through a surface of conductor. Suppose ΔQ is the amount of charge that flows through an area *A* in a time interval Δt and that the direction of flow is perpendicular to the area. Then the average current I_{av} is equal to the amount of charge divided by the time interval:

$$I = \frac{\Delta Q}{\Delta t} \qquad (11.8)$$

The SI unit of current is Ampere (A). $1A = 1C/1s$

When charges flow through a surface of a conductor, they can be positive, negative, or both. The direction of conventional current is the direction positive charges flow. In a common conductor such as copper, the current is due to the motion of negatively charged electrons, so the direction of the current is opposite the direction of motion of the electrons. On the other hand, for a beam of positively charged protons in an accelerator, the current is in the same direction as the motion of the protons. Moving charges, whether positive or negative, are referred to as *charge carriers.*

11.3.2 Resistance and Ohm's Law

Learning outcome:

After completing this section, students are expected to

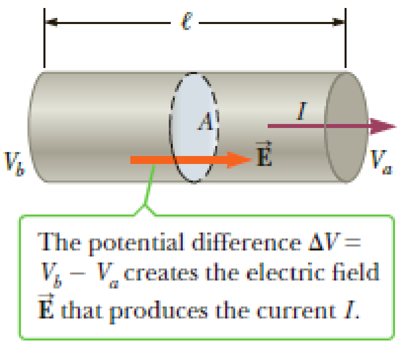
- Apply Ohm's law to calculate the current in a give.
- States Ohm's law

When a voltage (potential difference) ΔV is applied across the ends of a metallic conductor as in Figure 11.19, the current in the conductor is found to be proportional to the applied voltage. If the proportionality holds, we can write $V = IR$, where the proportionality constant *R* is called the *resistance* of the conductor. In fact, we define the resistance as the ratio of the voltage across the conductor to the current it carries:

$$R = \frac{\Delta V}{I} \qquad (11.9)$$

The unit of resistance is Ohm (Ω).

Fig. 11.19: A uniform conductor of length l and cores sectional area A . The current I is proportional to the potential difference



For many materials, including most metals, experiments show that the resistance remains constant over a wide range of applied voltages or currents. This statement is known as **Ohm's law**.

$$\Delta V = IR \qquad (11.10)$$

R is independent of the potential drop across the resistor and the current I flow through the resistor. A resistor is a conductor that provides a specified resistance in an electric circuit. Ohm's law is an empirical relationship valid only for certain materials. Materials that obey Ohm's law, and hence have a constant resistance over a wide range of voltages, are said to be **ohmic**. Materials having resistance that change with voltage or current are **nonohmic**. Ohmic materials have a linear current – voltage relationship over a large range of applied voltages.

Examples

1. During the 4.0 min a 5.0 A current is set up in a wire, how many (a) coulombs and (b) electrons pass through any cross section across the wire's width?

Solution:

a) bUsing equation (11.8), $Q = I\,t = 5.0\,A \times 4 \times 60\,s = 1200\,C$

b) The charge that one electron carries is $1.6 \times 10^{-19}\,C$. The number of electron in 1200 C can be determined by dividing this amount of charge with the number of charges in one electron.

Hence, $Q = Ne \Rightarrow N = Q/e = 1200C/1.6 \times 10^{-19}C = 7.5 \times 10^{21}$

2. A typical lightning bolt may last for 0.200 s and transfer 1.0×10^{20} electrons. Calculate the average current in the lightning bolt.

Solution:

First find the amount of charge contained in 1.0×10^{20} electrons.

That is $Q = Ne \Rightarrow Q = 1.0 \times 10^{20} \times 1.6 \times 10^{-19}\,C = 16C$.

$I = Q/t = 16C/0.2\,A = 80\ A.$

3. An electric heater carries a current of 13.5 A when operating at a voltage of$1.2 \times 10^{2}\,V$. What is the resistance of the heater?

Solution:

$$R = \frac{V}{I} = \frac{1.2 \times 10^{2}\,V}{13.5\,A} = 8.9\Omega$$

4. How long does it take electrons to get from a car battery to the starting motor? Assume the current is 300 A and the electrons travel through a copper wire with cross-sectional area $0.21\,cm^{2}$ and length $0.85m$ .The number of charge carriers per unit volume is $8.49 \times 10^{28}\,m^{-3}$.

Solution:

Let first calculate the number of charge carriers. The number of charge carries can be obtained by multiplying the number of charge carriers per unit volume with the volume of copper wire. The volume of the copper wire is equal to the product of cross-sectional area and length.

$V = 0.12cm^{2} \times 0.85\,m = 1.2 \times 10^{-5}\,m^{2} \times 8.5 \times 10^{-1}\,m = 1.02 \times 10^{-5}\,m^{3}.$

$N = nV = 8.49 \times 10^{28}\,m^{-3} \times 1.02 \times 10^{-5} = 8.7 \times 10^{23}$

Next determine the charged transferred Q.

$Q = Ne = 8.7 \times 10^{23} \times 1.6 \times 10^{-19} = 1.39 \times 10^{5}\,C.$

Hence, the time it take to travel through the sated length is
$t = Q/I = 1.39 \times 10^{5}\ C/300A = 4.62 \times 10^{3}\,s$

Exercises

1. A person notices a mild shock if the current along a path through the thumb and index finger exceeds $80.0 \mu A$. Compare the maximum possible voltage without shock across the thumb and index finger with a dry - skin resistance of $4.0 \times 10^5 \, \Omega$ and a wet - skin resistance of $2.0 \, k\Omega$.
2. Nichrome wire of cross - sectional radius 0.791 mm is to be used in winding a heating coil. If the coil must carry a current of 9.25 A when a voltage of $1.2 \times 10^2 \, V$ is applied across its ends, find the required resistance of the wire.
3. The current supplied by a battery in a portable device is typically 0.15 A. Find the number of electrons passing through the device in one hour.
4. A rectangular block of copper has sides of length 10.0 cm, 20.0 cm, and 40.0 cm. If the block is connected to a 6.0 V source across two of its opposite faces, what are (a) the maximum current and (b) the minimum current the block can carry?
5. The human body can exhibit a wide range of resistances to current depending on the path of the current, contact area, and sweatiness of the skin. Suppose the resistance across the chest from the left hand to the right hand is $1.0 \times 10^6 \, \Omega$ (a) how much voltage is required to cause possible heart fibrillation in a man, which corresponds to $0.5 \, A$ of direct current? (b) Why should rubber - soled shoes and rubber gloves be worn when working around electricity?
6. A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material to be used in a fuse melts when the current density rises to $440 \, A/cm^2$. What diameter of cylindrical wire should be used to make a fuse that will limit the current to 0.50 A?
7. A charged belt, 50 cm wide, travels at 30 m/s between a source of charge and a sphere.The belt carries charge into the sphere at a rate corresponding to $100 \mu A$. Compute the surface charge density on the belt.
8. A voltmeter connected across the terminals of a tungsten filament light bulb measures $115V$ when an ammeter in line with the bulb registers a current of $0.522 \, A$. Find the resistance of the light bulb.

11.4 Electrical Energy and Power

Learning outcome:

After completing this section, students are expected to:

- Explain how conduction electrons in a circuit lose energy in a resistive device.
- Define and apply the concept of electrical power.
- Understand the relationships between power, current, voltage, and resistance.
- Explain how conservation of energy is used in simple circuit analysis.

Figure 11.20 shows a circuit consisting of a battery B that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude V across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal b.

Because there is an external conducting path between the two terminals of the battery, and because the potential differences set up by the battery are maintained, a steady current I is produced in the circuit, directed from terminal a to terminal b. The amount of charge ΔQ that moves between those terminals in time interval Δt is equal to $I\Delta t$. This charge ΔQ moves through a decrease in potential of magnitude V, and thus its electric potential energy decreases in magnitude by the amount

$$\Delta PE = V\Delta Q = VI\Delta t \qquad (11.11)$$

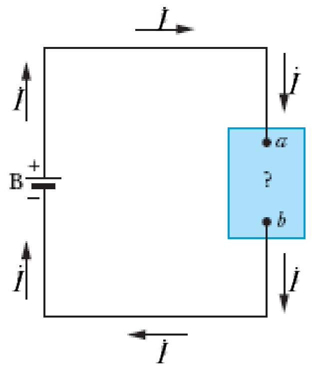


Fig. 11.20: A battery B sets up a current I in a circuit containing an unspecified conducting device.

The principle of conservation of energy tells us that the decrease in electric potential energy from terminal a to terminal b is accompanied by a transfer of energy to some other form. The power *P* associated with that transfer is the rate of transfer $\Delta PE/\Delta t$, which is given by Eq. (11.11) as

$$P = \Delta PE/\Delta t = VI\Delta t/\Delta t = VI \qquad (rate of electrical energy transfer) \qquad (11.12)$$

Moreover, this power P is also the rate at which energy is transferred from the battery to the unspecified device. If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load. If the device is a storage battery that is being charged, the energy is transferred to stored chemical energy in the storage battery. If the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature. The SI unit of power is Watt (W). $1W = 1J/1s$.

For a resistor or some other device with resistance *R*, we can combine the equation of resistance (Eq. 11.9) and Eq. (11.12) to obtain, for the rate of *electrical energy dissipation* due to a resistance, either

$$P = I^2R \quad or$$
$$P = V^2/R \qquad (resistive dissipation) \qquad (11.13)$$

Note: $P = IV$ applies to electrical energy transfers of all kinds; $P = I^2R$ and $P = V^2/R$ apply only to the transfer of electric potential energy to thermal energy in a device with resistance.

Examples:

1. In Fig. 11.21a, a 20.0Ω resistor is connected to a battery. Figure 11.21b shows the increase of thermal energy E_{th} in the resistor as a function of time *t*. The vertical scale is set by $E_{th,S} = 2.5mJ$, and the horizontal scale is set by $t_S = 4.0\,s$ .What is the electric potential across the battery?

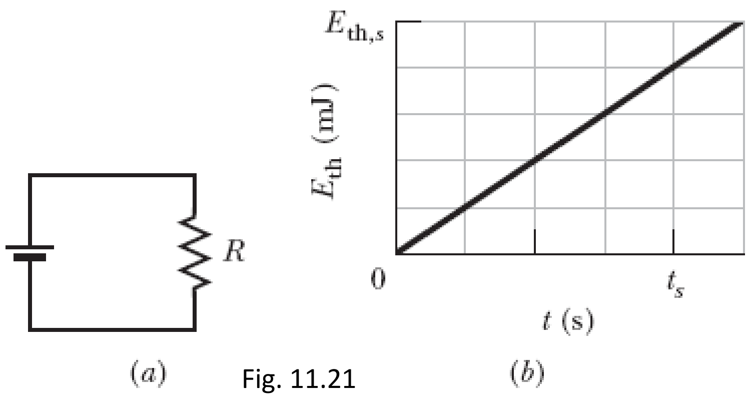


Fig. 11.21

Solution:

The slope of the graph provides the power lost in heating effect.

$$P = \frac{5\times 2.5mJ - 0mJ}{5\times 4.0\ s - 0s} = \frac{12.5\ mJ}{20s} = 6.25\times 10^{-4}\,W$$

Hence, power lost in heating the resistor is

$$P = V^2/R \Rightarrow V = \sqrt{PR} = \sqrt{6.25\times 20}\times 10^{-2}\ V = 0.112.V$$

2. Thermal energy is produced in a resistor at a rate of 100 W when the current is 3.00 A. What is the resistance?

Solution:

$$P = I^2 R \ \Rightarrow\ R = P/I^2 = 100.0W/(3.0\,A)^2 = 11.11\Omega$$

3. A $120.0V$ potential difference is applied to a space heater whose resistance is 14.0Ω when hot. (a) At what rate is electrical energy transferred to thermal energy? (b) What is the cost for 5.0 h at $1.5 cents/kW.h$?

Solution:

a) $P = V^2/R \ = (120V)^2/14.0\Omega = 128.57\ W \approx 1kW.$

b) $$Energycost = (rate/kW.h)\times total\ energyused\ in kW.h$$
$$= 1.5\ cents/kW.h \times 1kW.\times 5.0\,h = 5.5cents$$

4. A portable coffee heater supplies a potential difference of 12.0 V across a Nichrome heating element with a resistance of 2.0Ω (a) Calculate the power consumed by the heater. (b) How many minutes would it take to heat 1.00 kg of coffee from 20.0°C to 50.0°C with this heater? Coffee has a specific heat of 4 184 J/(kg . °C). Neglect any energy losses to the environment.

Solution:

a) The power consumed by the heater is $P = V^2/R \; = (120V)^2/2.0\Omega = 7.2\ kW$.

b) Neglecting the energy loss to the environment the energy lost by the heater was equals to energy gained by coffee.

Hence, energy lost by heater $E = P.t$

And energy gaining by the coffee is
$E = Specifieadheat(c) \times mas\ (M) \times changein\ temperature = cM\Delta T$

Equating the two equations will give us,

$$P.t = cM\Delta T \quad \Rightarrow t = CM\Delta T / P = \frac{1484\ J\,kg^{-1}C^{\circ} \times 1.0\,kg \times 30\ C^{0}}{7.2 \times 10^{3}\,W} = 17.43\ s.$$

5. An electric utility company supplies a customer's house from the main power lines (120.0 V) with two copper wires, each of which is 50.0 m long and has a resistance of 0.108Ω per 300.0 m (a) Find the potential difference at the customer's house for a load current of 110.0A. For this load current, find (b) the power delivered to the customer.

Solution:

a) Let first calculate the potential drop in a given length of copper wire.

In a single wire, the potential drop equals

$V_{drop} = (50m/300m) \times 0.108\Omega \times 110.0A = 1.98V.$

The potential drop in both wires (pair) is $2 \times 1.98V = 3.96V$

The potential difference$\Delta V = 120V - 3.96V = 116.04V \approx 116V$

b) $P = IV = 110.0V \times 116V = 12760W = 12.8kW.$

Exercise

1. A certain brand of hot-dog cooker works by applying a potential difference of 120 V across opposite ends of a hot dog and allowing it to cook by means of the thermal energy produced. The current is 10.0 A, and the energy required to cook one hot dog is 60.0 kJ. If the rate at which energy is supplied is unchanged, how long will it take to cook three hot dogs simultaneously?

2. In Fig. 11.22, a battery of potential difference $V = 12.0V$ is connected to a resistive strip of resistance$R = 6.0\Omega$. When an electron moves through the strip from one end to the other, (a) in which direction in the figure does the electron move, (b) how much work is done on the electron by the electric field in the strip, and (c) how much energy is transferred to the thermal energy of the strip by the electron?

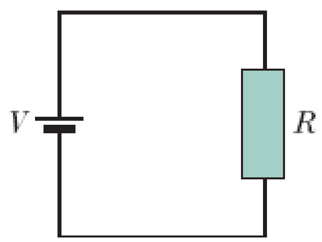


Fig. 11.22

3. The heating element of a coffeemaker operates at 120. V and carries a current of 2.00 A. Assuming the water absorbs all the energy converted by the resistor, calculate how long it takes to heat 0.500 kg of water from room temperature (23.0°C) to the boiling point.

4. Batteries are rated in terms of ampere - hours (A .h). For example, a battery that can deliver a current of 3.0 A for 5.0 h is rated at 15 A. h. (a) What is the total energy, in kilowatt - hours, stored in a 12 - V battery rated at 55 A . h? (b) At 3.6 cents per kilowatt-hour, what is the value of the electricity that can be produced by this battery?
5. The potential difference across a resting neuron in the human body is about 75.0 mV and carries a current of about 0.200 mA. How much power does the neuron release?
6. Two wires *A* and *B* made of the same material and having the same lengths are connected across the same voltage source. If the power supplied to wire *A* is three times the power supplied to wire *B*, what is the ratio of their diameters?
7. A 120 V potential difference is applied to a space heater that dissipates 500 W during operation. (a) What is its resistance during operation? (b) At what rate do electrons flow through any cross section of the heater element?
8. An 18.0 W device has 9.00 V across it. How much charge goes through the device in 4.00 h?
9. A resistor with a potential difference of 200 V across it transfers electrical energy to thermal energy at the rate of 3000W.What is the resistance of the resistor?
10. A 2.0 kW heater element from a dryer has a length of 80 cm. If a 10 cm section is removed, what power is used by the now shortened element at 120 V?

11.5 Equivalent Resistance and Kirchhoff's law

Learning outcome:

After completing this section, students are expected to:

- Apply Kirchhoff's laws to complex circuits to find current values and equivalent resistors.
- Apply the resistance and emf rules
- Calculate the equivalent of series resistors.
- Calculate the potential difference between any two points in a circuit.

11.5.1 Sources of electromotive forces (emf)

A source of emf can be thought of as a "charge pump" that forces electrons to move in a direction opposite the electrostatic field inside the source. The emf ε of a source is the work done per unit charge; hence, the SI unit of emf is the volt.

In Fig. 11.23b a positive charge moving through the battery from *a* to *b.* As the charge passes from the negative to the positive terminal of the battery, the potential of the charge increases by ε . As the charge moves through the resistance r , however, its potential decreases by the amount Ir , where I is the current in the circuit. The terminal voltage of the battery, $V = V_b - V_a$, is therefore given by

$$\Delta V = \varepsilon - Ir \qquad (11.13)$$

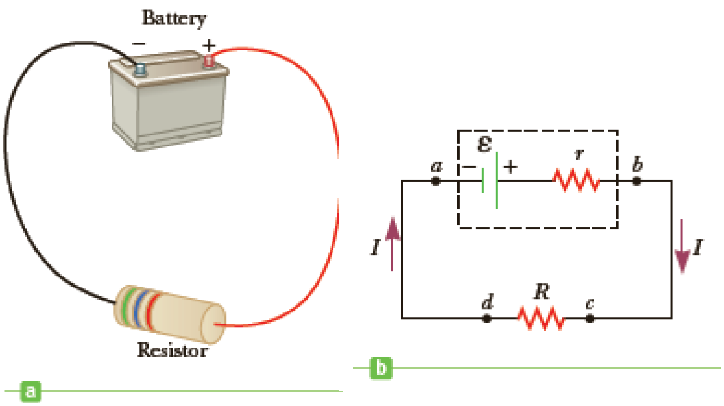


Fig. 11 23 (a) A circuit consisting of a resistor connected to the terminals of a battery. (b) A circuit diagram of a source of emf ε having internal resistance *r* connected to an external resistor *R.*

From this expression, we see that ε is equal to the terminal voltage when the current is zero, called the open - circuit voltage. By inspecting Figure 11.23b, we find that the terminal voltage ΔV must also equal the potential difference across the external resistance *R*, often called the load resistance; that is, $\Delta V = IR$. Combining this relationship with Equation 11.13, we arrive at

$$\varepsilon = IR + Ir = I(R + r) \qquad (11.14)$$

Solving for the current

$$I = \frac{\varepsilon}{(R + r)} \qquad (11.15)$$

Multiplying equation (11.14) by I we obtain

$$I\varepsilon = I^2R + I^2r \qquad (11.16)$$

This equation tells us that the total power output $I\varepsilon$ of the source of emf is converted at the rate I^2R at which energy is delivered to the load resistance, *plus* the rate I^2r at which energy is delivered to the internal resistance. Again, $r << R$, most of the power delivered by the battery is transferred to the load resistance.

Examples

1. A battery having an emf of 9.0 $\rightleftarrows$ V delivers $117mA$ when connected to a 72.0Ω load. Determine the internal resistance of the battery.

Solution:

$$\varepsilon = IR + Ir \quad \Rightarrow r = \frac{\varepsilon - IR}{I} = \frac{\varepsilon}{I} - R$$

$$r = \frac{9.0V}{1.17 \times 10^{-1}A} - 72.0\Omega = 4.92\Omega$$

2. A battery with a 0.10Ω internal resistance supplies $15.0W$ of total power with a $9.0V$ terminal voltage. Determine (a) the current I and (b) the power delivered to the load resistor.

Solution:

$$I\varepsilon = I^2R + I^2r \ \Rightarrow 15.0W = I(IR) + I^2 \times 0.10\Omega$$

Where $IR = 9.0V$

$$15.0W = 9I + I^2 \times 0.10\Omega \;\Rightarrow\; I^2 + 90I - 150$$

$$I = \frac{-90 + \sqrt{90^2 + 4\times 150}}{2} = 1.23\Omega$$

Exercise

1. (a) Find the current in an 8.0Ω resistor connected to a battery that has an internal resistance of $0.15\ \Omega$ if the voltage across the battery (the terminal voltage) is 9.00 V. (b) What is the emf of the battery?
2. A battery with an emf of $12.0V$ has a terminal voltage e of $11.5V$ when the current is 3.00 A. (a) Calculate the battery's internal resistance r. (b) Find the load resistance R .

11.5.2 Combinations of Resistor

11.5.2.1 Combinations of resistors in Series

When two or more resistors are connected end to end as in Figure 11.24, they are said to be in *series,* and for such connection the current is the same in the two resistors because any charge that flow though R_1 is the same as the charge flow through R_2. For series connection the equivalent resistance is the sum of the individual resistance.

$$R = R_1 + R_2$$

And for N resistors connected in series

$$R = \sum_i^N R_i \qquad (11.16)$$

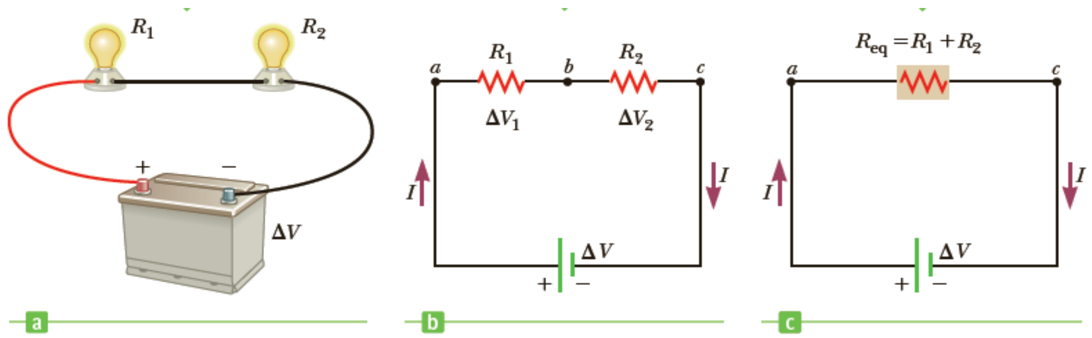


Fig. 11. 24: Two resistors, R_1 and R_2, in the form of incandescent light bulbs, in series with a battery. The currents in the resistors are the same, and the equivalent resistance of the combination is $R = R_1 + R_2$

11.5.2.2 Resistors in Parallel

Let consider two resistors connected in parallel, as in Figure 11.25. In this case the potential differences across the resistors are the same because each is connected directly across the battery terminals.

$$I = I_1 + I_2$$

This leads to

$$1/R_{eq} = 1/R_1 + 1/R_2$$

For N resistors connected in parallel

$$1/R_{eq} = \sum_{i=1}^{N} \frac{1}{R_i} \qquad (11.17)$$

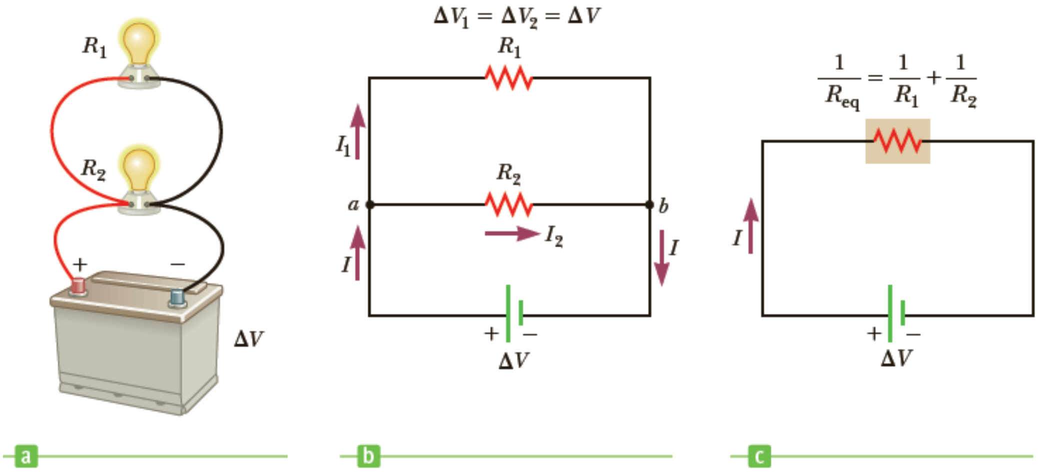


Fig. 11.25 Two resistors, R_1 and R_2, in the form of incandescent light bulbs, in parallel with a battery. The potential differences across R_1 and R_2 are the same. Currents in the resistors are different, and the equivalent resistance of the combination is given by $1/R_{eq} = 1/R_1 + 1/R_2$.

Examples

1. Three 9.0Ω resistors are connected in series with a $12.0V$ battery. Find (a) the equivalent resistance of the circuit and (b) the current in each resistor. (c) Repeat for the case in which all three resistors are connected in parallel across the battery.

Solution:

$$R = R_1 + R_2 + R_3 = 27\Omega$$

The total current in the circuit is the same as the current in each resistor in series connection.

$$I = I_1 = I_2 = I_3 = V/R = 12V/27\Omega = 0.44A$$

$$1/R_{eq} = \sum_{i=1}^{3} 1/R_i = \frac{1}{9\Omega} + \frac{1}{9\Omega} + \frac{1}{9\Omega} = 1/3\Omega \quad \Rightarrow R_{eq} = 3\Omega$$

The potential difference across each resistor is the same as the poetical of the source in this case. But the total current is the sum of the individual current through each resistor.

$$I = V/R_{eq} = 12V/3\Omega = 4.0A$$

$I_1 = V / R_1 = 12V / 9\Omega = 1.33A$
$I_2 = V / R_2 = 12V / 9\Omega = 1.33A$
$I_{31} = V / R_3 = 12V / 9\Omega = 1.33A$

2. (a) Find the equivalent resistance between points *a* and *b* in Figure11.26. (b) Calculate the current in each resistor if a potential difference of 34.0 V is applied between points *a* and *b.*

Solution

First determine the equivalent resistance for 7.0Ω and 10.0Ω resistors.

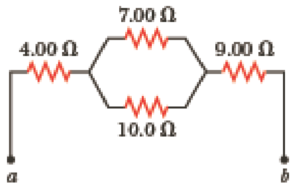


Fig. 11.26

That is $R_{7,10} = \dfrac{7\times10\ \Omega}{7+10} = 4.12\ \Omega$

Now this is connected in series to the 4.0Ω and the 9.0Ω resistors.

$R_{eq} = 4.0\Omega + 4.12\Omega + 10.0\Omega = 18.12\Omega$

3. A car battery with a$12.0V$ emf and an internal resistance of 0.04Ω is being charged with a current of 50 A. What are (a) the potential difference *V* across the terminals, (b) the rate which energy dissipated inside the battery, and (c) the rate at which energy converted to chemical form?

Solution:

$V = IR = \varepsilon - Ir = 12.0V - 0.04\Omega \times 50.0\,A = 10.0V$

$P_r = Ir^2 = (50.0A)^2 \times 0.04\Omega = 100W$

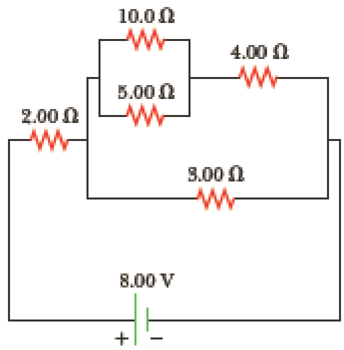
$P_\varepsilon = I\varepsilon = 50.0A \times 12.0V = 600.0W$

Exercises

1. A certain battery has a 12.0-V emf and an internal resistance of 0.100 Ω. (a) Calculate its terminal voltage when connected to a 10.0-Ω load. (b) What is the terminal voltage when connected to a 0.500-Ω load? (c) What power does the 0.500-Ω load dissipate? (d) If the internal resistance grows to**0.500 Ω**, find the current, terminal voltage, and power dissipated by a 0.500-Ω load.
2. A 5.0 A current is set up in a circuit for 6.0 min by a rechargeable battery with a 6.0 V emf. By how much is the chemical energy of the battery reduced?
3. Consider the circuit shown in Figure P11.27. (a) Calculate the equivalent resistance of the 10.0Ω and 5.0Ω resistors connected in parallel. (b) Using the result of part (a), calculate the combined resistance of the10.0Ω, 5.0Ω, and 4.0Ω resistors. (c) Calculate the equivalent resistance of the combined resistance found in part (b) and the parallel 3.0Ω resistor. (d) Combine the equivalent resistance found in part (c) with the 2.0Ω resistor. (e) Calculate the total current in the circuit. (f) What is the voltage drop across the 2.0Ω resistor? (g)

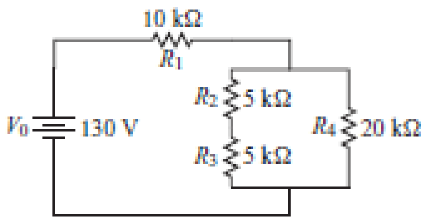
Subtracting the result of part (f) from the battery voltage, find the voltage across the $3.0\,\Omega$ resistor. (h) Calculate the current in the $3.0\,\Omega$ resistor.

Fig. 11.27



4. For the circuit shown in Fig. 11.28, how much current flows through the 20 k_ resistor? What must its power rating be?

Fig.11.28



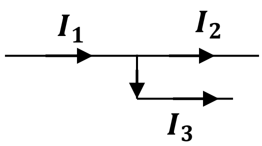
11.5.3 Kirchhoff's Law

In section 11.52 we discussed simply way of analyzing circuits. The procedure for analyzing more complex circuits can be facilitated by the use of two simple rules called Kirchhoff's rules:

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction. (This rule is often referred to as the junction rule or conservation of charge)

$$I_1 = I_2 + I_3$$

Fig.11.29



2. The sum of the potential differences across all the elements around any closed circuit loop must be zero. (This rule is usually called the loop or voltage rule.)

$$\sum \Delta V = 0$$

Examples

1. For the loop shown by Fig. 11.30 determine the values of all currents.

For loop 1:

$$-I_2R_5 + I_3R_9 = 0 \text{ or } I_2R_5 = I_3R_9$$

Using the given values $5I_2 = 9I_3 \Rightarrow I_3 = \frac{5}{9}I_2$

For loop 2:

$$6.0V - I_1R_4 - I_3R_9 = 0$$
$$6.0V = 4I_1R_4 + 9I_3$$

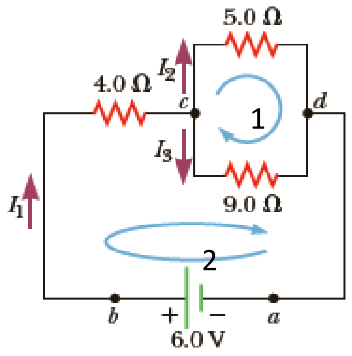


Fig.11.30

From junction rule: $I_1 = I_2 + I_3$

$$6.0V = 4(I_2 + I_3) + 9I_3 = 4I_2 + 4\frac{5}{9}I_2 + 5I_2 = \frac{101}{9}I_2$$

$$I_2 = \frac{6 \times 9}{101}A = 0.54A, \quad I_3 = \frac{5}{9}I_2 = \frac{0.54 \times 5}{9}A = 0.3A, \qquad I_1 = I_2 + I_3 = 0.54 + 0{,}30 = 0.84A$$

Exercises

1. For the circuit of Fig. 11.31 calculate (a) the current drawn from the source, (b) the potential difference across each resistor, (c) the current through each resistor, and (d) the power dissipated by the 5.0Ω resistor.
2. For the circuit of Fig. 11.32 determine the value and direction of the current in each branch, and the potential difference across the 10.0Ω resistor.
3. For the bridge network shown in Fig. 11.33 calculate the current through each resistor, and the current drawn from the supply.

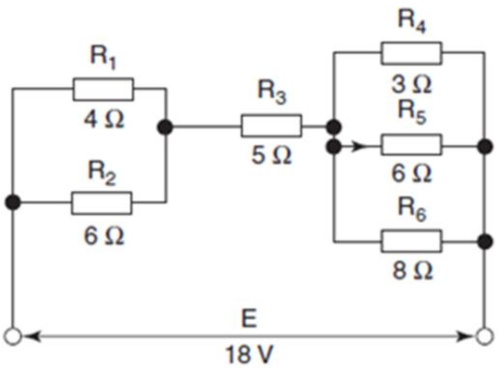


Fig. 11.31

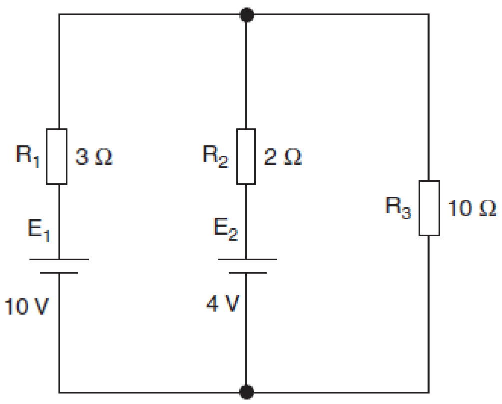


Fig. 11.32

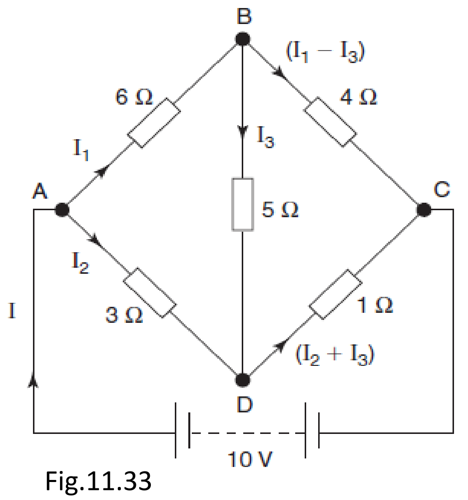


Fig.11.33

11.6 Magnetic field and Magnetic Flux

Learning Outcome

After completing this Chapter, students are expected to:

- Describe the relationship between electricity and magnetism.
- Define magnetic field and describe the magnetic field lines of various magnetic fields.
- Explain the effects of magnetic fields on moving charges and current-carrying conductors.
- Use the right-hand rule to determine directions of magnetic fields and forces on a moving charge.
- Explain the phenomenon of electromagnetic induction.
- Solve problems related to magnetic fields and electromagnetic induction.

Introduction

Magnets and their properties have been known for thousands of years. It is believed that magnetic rocks were first found in a place called Magnesia, now part of western Turkey. Interest to study magnetic properties gradually lead to practical applications for magnets such as using them as navigational compasses in long-distance sailing. Today magnetism plays many important roles in our lives. Mobile phones wouldn't have been possible without the applications of magnetism and electricity on a small scale. Large electromagnets are used to pick up heavy loads, levitate high-speed trains, generate electric power. Magnets are in audio/video recording devices to store computer data. Magnetic fields are used in MRI medical treatments, in particle accelerators to guide particles into targets at nearly the speed of light. The use of magnetism to explore brain activity is a subject of contemporary research and development. The Earth's magnetic field protects us by trapping charged particles from outer space by trapping them in the Van Allen belts. This chapter discusses that all these and other applications of the magnetism are based on a few underlying physical principles.

In this section we discuss we discuss the relation between magnetic fields and moving charges. Changing magnetic fluxes can create electric fields. These phenomena signify an underlying unity of electricity and magnetism. We will see that the ultimate source of any magnetic field is electric current.

11.6.1 Magnetic Field

11.6.1.1 Magnets

Magnets come in various shapes, sizes, and strengths as shown in Figure 11-1 (below). All magnets have two inseparable poles called north pole and a south pole. A single isolated pole (a monopole) has never been observed so far. The names come from the observation that a freely hanging bar magnet aligns itself in the north-south geographic direction; the north pole of the bar magnet is the one that points north. Observations also show that like poles repel each other and unlike poles attract each other (**Figure 11-2**, below).

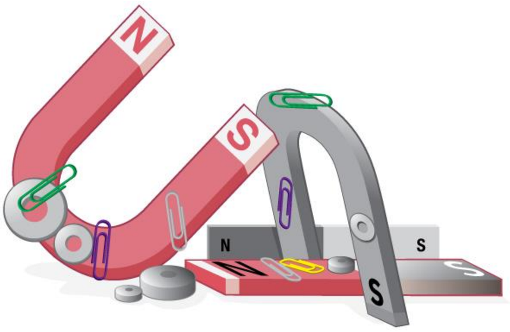


Figure 11-1: Magnets with different shapes and sizes

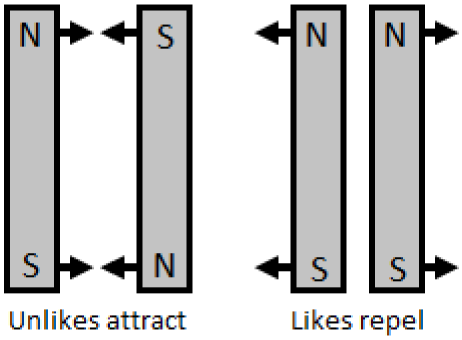


Figure 11-2: interaction between magnets

11.6.1.2 The Source of All Magnetism – current

A compass needle deflects when brought close to a current-carrying wire, indicating that electric currents produce magnetic effects. This phenomenon, first observed by the Danish scientist Hans Christian Oersted (1777–1851), shows the connection between currents and magnets. Electric current is used to make magnets called electromagnets. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines. Figure 11-3 (below) shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The similarity of the patterns indicate that electromagnets have the same basic characteristics as bar magnets (ferromagnets)—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.

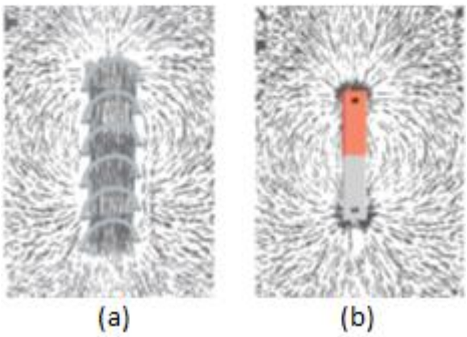


Figure 11-3: Iron filings sprayed near (a) a current-carrying coil and (b) a bar magnet produce very similar patterns, especially near the ends of the coil and the magnet. The current-carrying coil is an electromagnet with north and south poles similar to the bar magnet.

The magnetic properties of bar magnets are also due to current loops at the atomic and subatomic levels. These current loops are formed by the motion of charged particles such as electrons and protons in the same way as currents are formed in wires by the motion of electrons. Scientific observations so far confirm that electric currents (in wires or atoms) are sources of all magnetism.

11.6.1.3 Magnetic Fields and Magnetic Field Lines

It is common experience to observe that magnets interact at a distance, that is, without touching each other. In this sense magnetic forces are similar to electric and gravitational forces. We therefore define a ***magnetic field*** to represent ***magnetic forces***. To help us visualize magnetic fields we draw lines around magnets. These lines, called ***magnetic field lines***, represent both the strength and direction of the magnetic field. As shown in Figure 11-4 (below), the direction of magnetic field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B-field***.

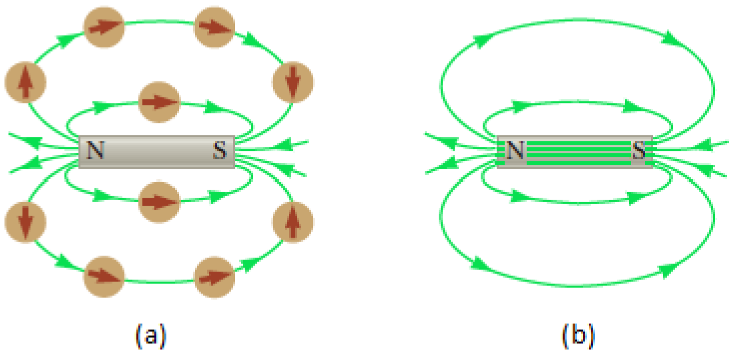


Figure 11-4: (a) Magnetic field lines traced out using a compass needle. (b) Each field line forms a continuous closed loop going through the interior of the magnet.

The strength of the **B**-field is proportional to the closeness of the lines. As shown in Figure 11-4 (above), the lines are very close to each other near the poles indicating that the magnetic field is stronger near the poles.

Similarly, small magnetic compasses can be used to show how the magnetic field appears for a current loop and a long straight wire (Figure 11-5, below). A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of **B**. We use the symbols ⊗ and ⊙ to indicate fields into and out of the paper, respectively.

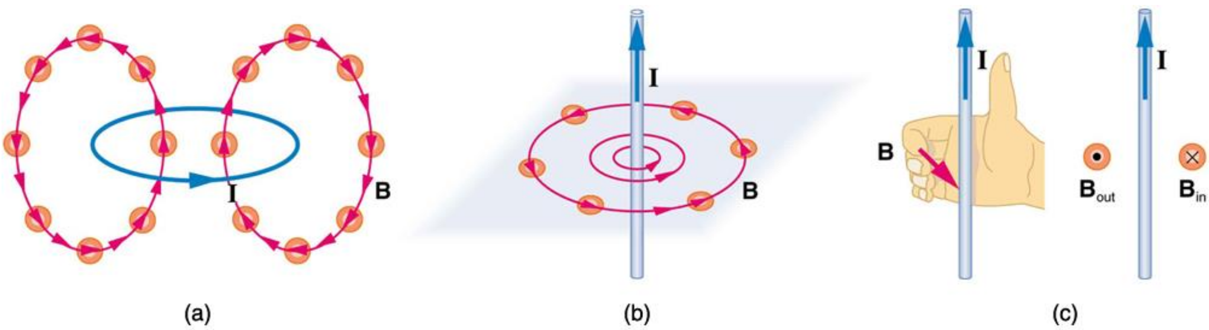


Figure 11-5: (a) The magnetic field of a circular current loop (blue) is similar to that of a bar magnet. The compass placed within the current loop points up indicating the top of the loop is the north pole and the bottom is the south pole. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) The right-hand rule is used to determine the direction of the **B**-field

around the wire. The 'dot' symbol means the field is pointing out and the 'cross' symbol means the field is into the page.

Studies of magnetic fields revealed the following properties:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines.
3. Magnetic field lines never cross, that is, the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end.

The last property is related to the fact that the north and south poles cannot be separated (no magnetic monopoles). It is a distinct difference from electric field lines, which begin and end on the positive and negative charges which exist separately.

11.6.1.4 Magnetic Force on a Moving Charge

Moving charges produce currents, which in turn produce magnetic fields. These magnetic fields of moving charges interact with other magnetic fields through which the moving charges pass.

The magnitude of the magnetic force $\boldsymbol{F}$ on a charge $\boldsymbol{q}$ moving at a speed $\boldsymbol{v}$ in a magnetic field of strength $\boldsymbol{B}$ is given by

$$\boldsymbol{F = qvB \sin\theta} \tag{11-1}$$

where $\boldsymbol{\theta}$ is the angle between the directions of $\boldsymbol{v}$ and $\boldsymbol{B}$. This force is often called the ***Lorentz force***. The SI unit for magnetic field strength $\boldsymbol{B}$ is called the tesla (T).

$$\mathbf{1T} = \frac{\mathbf{1N}}{\mathbf{C \cdot m/s}} = \frac{\mathbf{1N}}{\mathbf{A \cdot m}}$$

Another unit is the gauss (G) which is defined as $\mathbf{1G = 10^{-4}\ T}$. The strongest permanent magnets have fields near 2T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $\mathbf{5 \times 10^{-5}}$ T, or 0.5 G.

The direction of the magnetic force $\vec{\boldsymbol{F}}$ is perpendicular to the plane formed by $\vec{\boldsymbol{v}}$ and $\vec{\boldsymbol{B}}$, as determined by the right-hand rule, which is illustrated in Figure 11-6 (below). To employ right-hand rule:

1. Point the fingers of your right hand in the direction of the velocity vector, $\vec{\boldsymbol{v}}$.
2. Curl the fingers in the direction of the magnetic field $\vec{\boldsymbol{B}}$, moving through the smallest angle (as in Figure 11-6).
3. Your thumb is now pointing in the direction of the magnetic force $\vec{\boldsymbol{F}}$ exerted on a positive charge.

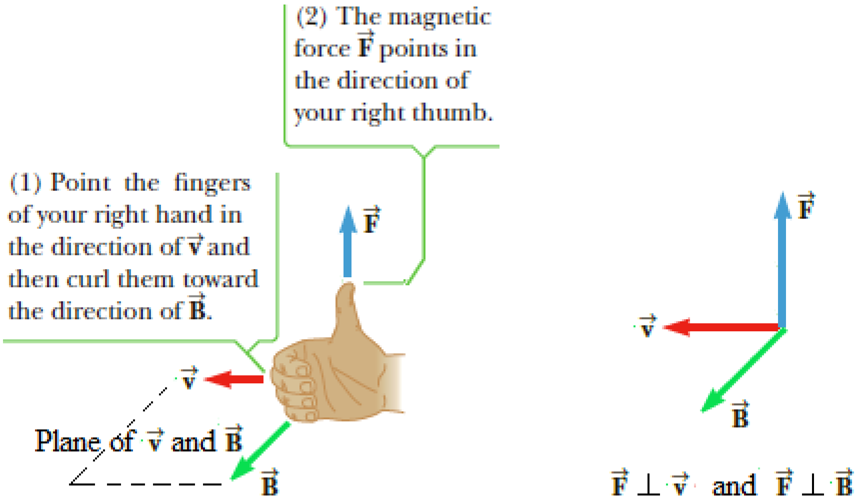
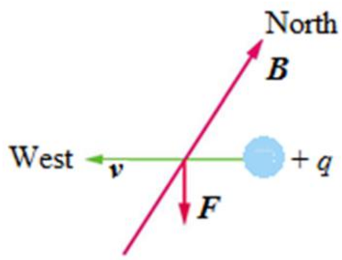


Figure 11-6: The right-hand rule for determining the direction of the magnetic force on a positive charge moving in a magnetic field.

The Lorentz force shows that if a charged particle moves in the direction of the magnetic field (θ = 0), it experiences no magnetic force; if the charged particle moves perpendicular to the magnetic field (θ = 90°), the magnetic force acting on it will be maximum.

Examples

1. In a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground.



Solution

The magnetic force is $F = qvB\sin\theta$

The angle between $\boldsymbol{v}$ and $\boldsymbol{B}$ is 90° so sin θ = 1. Substituting given values yields

$$F = (20 \times 10^{-9}\ C)(10\ m/s)(5 \times 10^{-5}\ T)$$
$$F = 1 \times 10^{-11}\ N$$

This force is completely negligible on any **macroscopic** object, consistent with experience. The Earth's magnetic field, however, does produce very important effects, particularly on sub-microscopic particles.

2. A proton moves with a speed of 1.00×10^5 m/s through Earth's magnetic field, which has a value of 55.0×10^{-3} T at a particular location. When the proton moves eastward, the magnetic force on it is upward, and when it moves northward, no magnetic force acts on it. What is the direction of the magnetic field and the strength of the magnetic force when the proton moves eastward?

Solution

Find the direction of the magnetic field

No magnetic force acts on the proton when it's going north because the angle the direction of the proton's velocity the direction of the magnetic field is either 0° or 180°. Therefore, the magnetic field ***B*** must point either north or south.

When the particle travels east, the magnetic force is upward. Now employ the right-hand rule. Point your thumb in the direction of the force (upward) and your fingers in the direction of the velocity eastward. When you curl your fingers, they point north, which must therefore be the direction of the magnetic field.

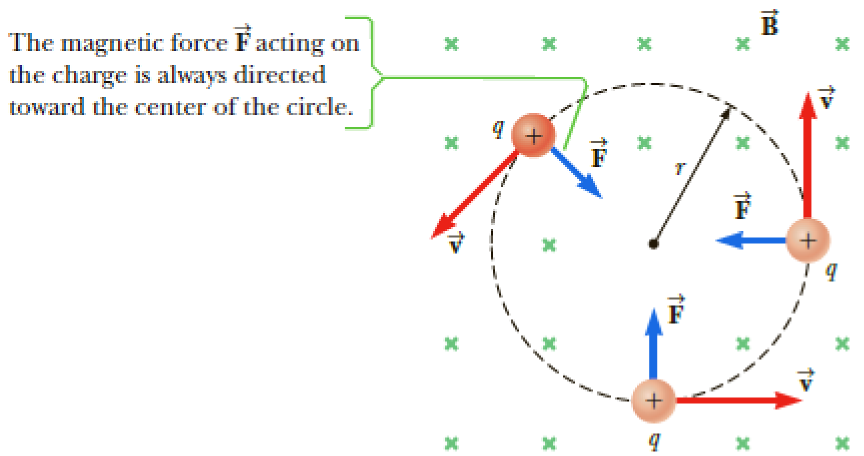
Find the magnitude of the magnetic force

Substitute the given values into the Lorentz Equation. From part (a), the angle between the velocity $\boldsymbol{v}$ of the proton and the magnetic field ***B*** is 90.0°.

$$
\begin{aligned}
F &= qvB \sin\theta \\
&= (1.60 \times 10^{-19}\ C)\left(1.00 \times 10^{5} \frac{m}{s}\right)(55.0 \times 10^{-6}\ T) \sin 90.0° \\
&= 8.80 \times 10^{19}\ N
\end{aligned}
$$

Exercise

1. A charged particle shot perpendicular to a uniform B -field as shown in the figure traces out a circular path. Find the radius of the circular path.



Suppose, in the last example above, the charged particle has a velocity component parallel to the magnetic field. This component is not affected by the magnetic field, so the charged particle keeps moving in the direction of the magnetic field while undergoing circular motion. This produces a spiral motion (helix) as shown in the figure below.

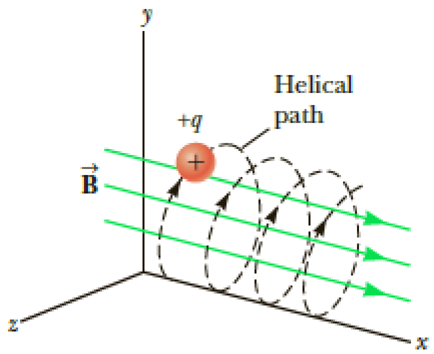


Figure 11-7: Spiral (helical) motion of a charged particle

Exercise

1. A charged particle enters a uniform magnetic field at a speed of $\mathbf{1.79 \times 10^6}$ m/s. It subsequently moves in a circular orbit with a radius of 16.0 cm. The uniform magnetic field has a magnitude of 0.350 T and is directed perpendicular to the particle's velocity. Find the particle's mass-to-charge ratio.

11.6.1.5 Magnetic Force on a Current-Carrying Conductor

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges.

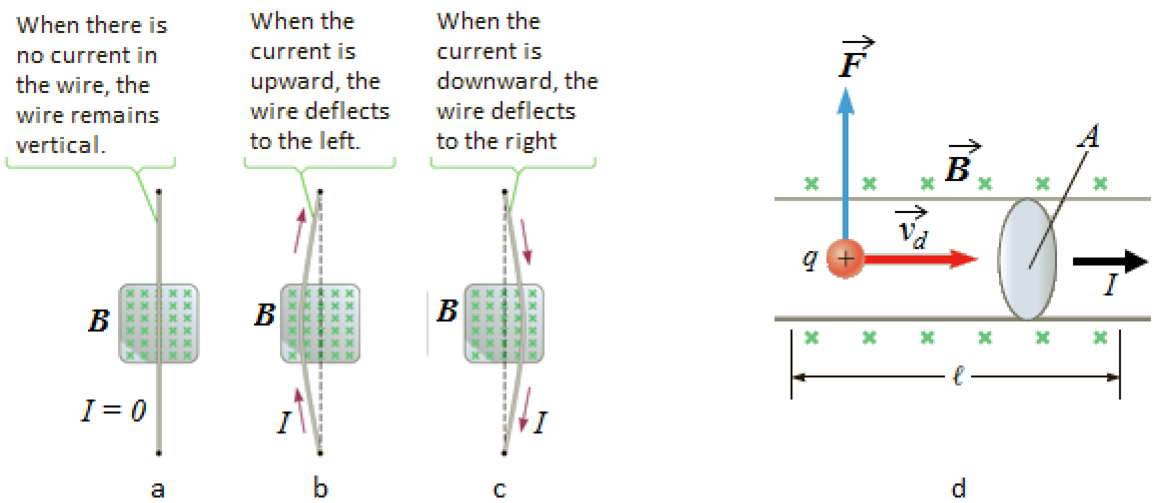


Figure 11-8: A conductor placed in a magnetic field experiences (a) no magnetic force if it carries no current (b & c) a magnetic force perpendicular to both the wire and the **B**-field if it carries a current. (d) A section of the wire magnified to show charges moving past a cross-section.

The force on an individual charge moving at the drift velocity $\boldsymbol{v_d}$ is given by (1.1):

$$F_1 = qv_dB\ sin\ \theta \tag{11-2}$$

Taking **B** to be uniform over a length l of the wire and zero elsewhere, the total magnetic force on the wire is then

$$F = NF_1 = Nqv_d\ sin\ \theta \tag{11-3}$$

where N is the number of charge carriers in the section of wire of length l . Now,

$N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the B-field. Noting that $= Al$, where A is the cross-sectional area of the wire, then the force on the wire is

$$F = (nqAv_d)(lB\ sin\ \theta) \tag{11-4}$$

The first parentheses give the current I in the wire (see Current),

$$F = IlB\ sin\ \theta \tag{11-5}$$

Error! Reference source not found.) is the equation for the magnetic force on a length l of wire arrying a current I in a uniform magnetic field B, as shown in Figure 11-8 (above). If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field:

$$\frac{F}{l} = IB\ sin\ \theta \tag{11-6}$$

The direction of this force is given by the right-hand rule – place your fingers in the direction of the current, curl them in the direction of the B-field, the thumb then points in the direction of the force, see Figure 11-8d, above.

Examples

1. Calculate the force on the wire shown in Figure 11-8, given B = 1.50 T, l = 5.00 cm, and I = 20.0 A.

Solution

Entering the given values into F=$IlBsin\ \theta$ ***(11-5)*** yields

$$F = IlB \sin\theta = (20.0\ A)(0.0500\ m)(1.50\ T)(\sin 90°)$$
$$F = 1.50\ N$$

2. In a lightning strike, there is a rapid movement of negative charge from a cloud to the ground. In what direction is a lightning strike deflected by Earth's magnetic field?

Solution

The downward flow of negative charge in a lightning strike is equivalent to a current moving upward. The magnetic field is from south to North. According to right-hand rule, the lightning strike would be deflected toward the west.

Exercise

1. A wire carries a current of 22.0 A from west to east. Assume the magnetic field of Earth at this location is horizontal and directed from south to north and it has a magnitude of $\mathbf{0.500} \times \mathbf{10^4}$ T. (a) Find the magnitude and direction of the magnetic force on a 36.0-m length of wire. (b) Calculate the gravitational force on the same length of wire if it's made of copper and has a cross-sectional area of $\mathbf{2.50} \times \mathbf{10^{-6}\ m^2}$.

11.6.1.6 Magnetic Torque

Motors have loops of wire in a magnetic field. When current passes through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. In the process, electrical energy is converted to mechanical work (Figure 11-9.)

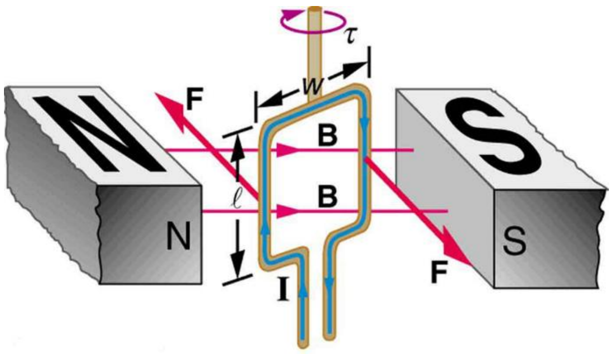


Figure 11-9: A current loop of wire attached to a vertical shaft feels a magnetic torque that produces a clockwise torque as viewed from above.

The force on each segment of the loop in Figure 11-9 can be determined using $\boldsymbol{F}$=$\boldsymbol{IlB\sin\theta}$ *(11-5)*. We take the magnetic field to be uniform over the rectangular loop, which has width $\boldsymbol{w}$ and height . The magnetic forces on the top and bottom segments are parallel to the shaft, equal in magnitude and opposite in direction, and therefore, produce no torque and no net force on the loop. The magnetic forces on the other two parallel sides of the loop (denoted as **F**) are perpendicular to the shaft, and produce a torque that rotates the loop as shown in Figure 11-9. The torque of each force is

$$\boldsymbol{\tau_1} = \left(\frac{\boldsymbol{w}}{\mathbf{2}}\right)\boldsymbol{F\sin\theta}, \quad (11\text{-}7)$$

where θ is the angle between $\boldsymbol{w}$ and F. The total torque is $\boldsymbol{\tau} = \mathbf{2}\boldsymbol{\tau_1}$.

$$\boldsymbol{\tau} = \boldsymbol{wF\sin\theta} \quad (11\text{-}8)$$

As seen in Figure 11-9, the right-hand rule gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. The magnitude of the forces can be determined by **Error! Reference source not found.**). Since the sides are perpendicular to the B-field, in θ in **Error! Reference source not found.**) becomes one. Therefore,

$$F = IlB \tag{11-9}$$

Combining Equations (1.8) and (1.9), one gets

$$\tau = I(wl)B\sin\theta \tag{11-10}$$

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, noting that the area of the loop is $A = wl$, the expression for the torque becomes

$$\tau = NIAB\sin\theta \tag{11-11}$$

The quantity NIA is defined as the magnitude of a vector μ called the magnetic moment of the coil. The magnetic moment μ always points perpendicular to the plane of the loop(s). Its direction is given by the right-hand rule as shown in Figure 11-10: if the fingers of the right hand point in the direction of the current, the thumb points in the direction of μ. The magnetic torque can now ne written in terms of the magnetic moment as

$$\tau = \mu B\sin\theta \tag{11-12}$$

Clearly, the angle θ lies between the directions of the magnetic moment μ and the magnetic field **B**. This equation is valid for a loop of any shape.

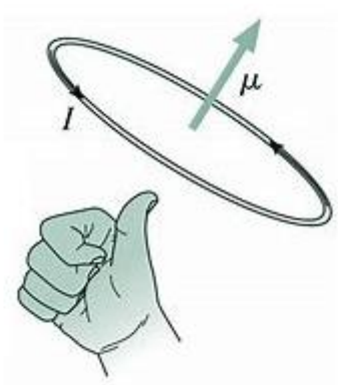


Figure 11-10: Employing the right-hand rule to determine the direction of the magnetic dipole moment of a current loop.

Examples

1. Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Solution

The maximum torque corresponds to sin θ = 1, so $\tau_{max} = NIAB$. Substituting given values yields:

$$\tau_{max} = (100)(15.0\ A)(0.100\ m^2)(2.00\ T) = 30.0\ Nm.$$

2. A circular wire loop of radius 1.00 m is placed in a magnetic field of magnitude 0.500 T. The normal to the plane of the loop makes an angle of 30.0° with the magnetic field (Figure 11-11(a)). The current in the loop is 2.00 A in the direction shown. (a) Find the magnetic moment of the loop and the magnitude of the torque at this instant. (b) The same current is carried by the rectangular 2.00-m by 3.00-m coil with three loops shown in Figure 11-11(b). Find the magnetic moment of the coil and the magnitude of the torque acting on the coil at that instant.

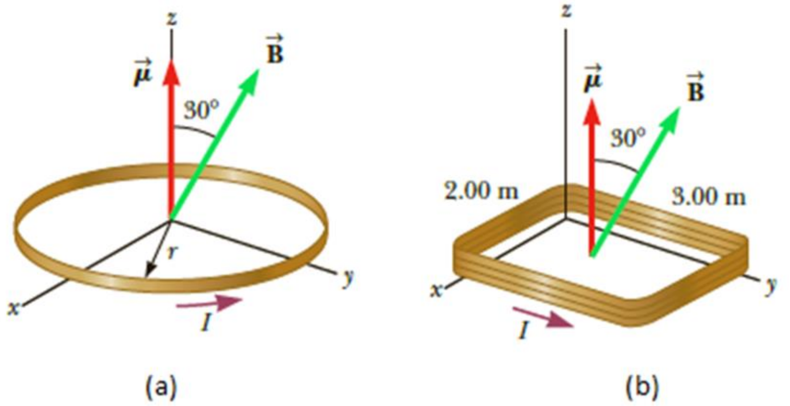


Figure 11-11: (a) A circular current loop lying in the xy-plane in an external magnetic field **B**. (b) A rectangular coil lying in the xy-plane in the same B-field.

Solution

(a) $\mu = NIA = (1)(2.00A)(\pi\ 1.00^2) = 6.28\ A.m^2$

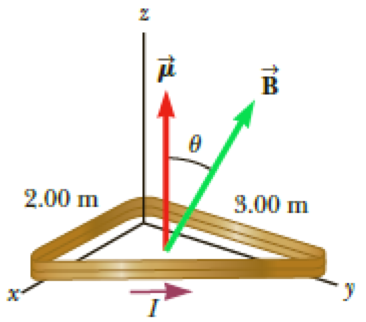
$$\tau = \mu B \sin\theta = (6.28\ A.m^2)(0.500\ T)\sin 30° = 1.57\ N.m$$

(b) $\mu = NIA = (3)(2.00A)(2.00m \times 3.00m) = 36.0\ A.m^2$

$$\tau = \mu B \sin\theta = (36.0\ A.m^2)(0.500\ T)\sin 30° = 9.00\ N.m$$

Exercise

1. Suppose a right triangular coil with base of 2.00 m and height 3.00 m having two loops carries a current of 2.00 A as shown in the figure. Find the magnetic moment and the torque on the coil. The magnetic field is again 0.500 T and makes an angle of 30.0° with respect to the normal direction.



11.6.1.7 Magnetic Fields Produced by Currents

We have seen Section 1.1.1.2 that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

11.6.1.7.1 Magnetic Field Created by a Long Straight Current-Carrying Wire

The magnetic field strength produced by a long straight current-carrying wire is found to be

$$B = \frac{\mu_0 I}{2\pi r} \tag{11-13}$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ Tm/A$ is the permeability of free space. Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

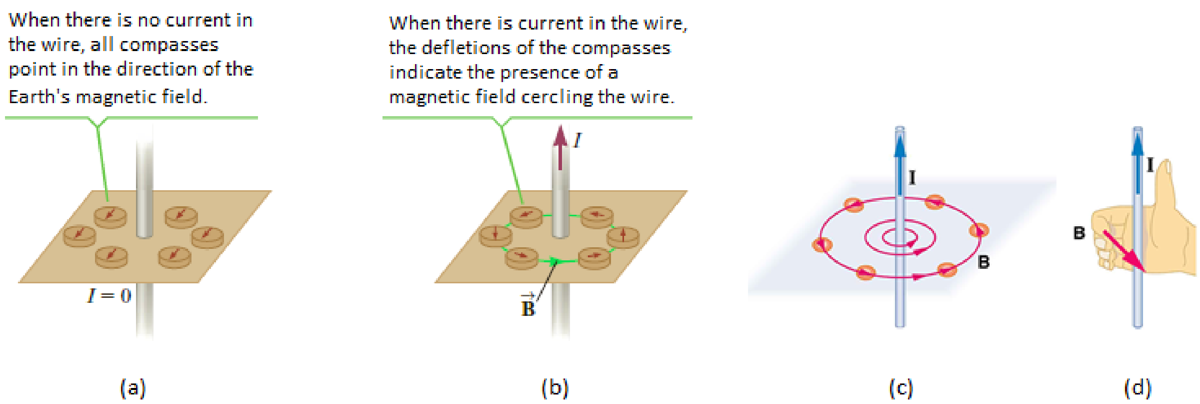


Figure 11-12: (a), (b) Compasses show the effects of the current in a wire. (c) the gap between the circles shows the variation of the field with distance from the wire. (d) the right-hand rule shows the direction of the field.

To determine the direction of the magnetic field around the straight wire, point the thumb of your right hand along the wire in the direction of positive current, as in Figure 11-12(d). Your fingers then naturally curl in the direction of the magnetic field **B**.

Example

1. Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's field at a distance of 5.0 cm from the wire. $B_E \cong 5.0 \times 10^{-5}$ T.

Solution

Solving $\boldsymbol{B}= \boldsymbol{\mu_0 I 2\pi r}$ (***11***-***13***)
for the current I, we get

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi r}{\mu_0}(2B_E)$$

Substituting numerical values: $I = \frac{2\pi(5.0\times10^{-2}m)}{\ }(2 \times 5.0 \times 10^{-5}T) = 25\ A$

Exercise

1. Two straight long parallel wires, separated by a distance of 1.0 m, carry a current of 4.00 A each. A third wire is arranged perpendicular to the two parallel wires. If the magnetic field at a point equidistant from all the wires is zero, find the current in the third wire.

11.6.1.7.2 Magnetic Field at the center of a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in Figure 11-13. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. There is, however, a simple formula for the magnetic field strength at the center of a circular loop. It is

$$\boldsymbol{B} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2R}}\ (\boldsymbol{at\ center\ of\ loop}) \qquad (11\text{-}14)$$

where $\boldsymbol{R}$ is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid only at the center of a circular loop of wire. One way to get a stronger field is to have N loops; then, the field is $\boldsymbol{B = N\mu_0 I/(2R)}$. Note that the larger the diameter of the loop, the smaller the field at its center, because the current is farther away.

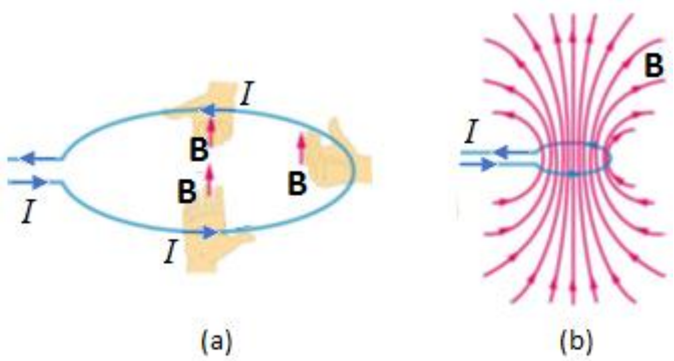


Figure 11-13: The magnetic field of a current loop

11.6.1.7.3 Magnetic Field of a Current-Carrying Solenoid

A solenoid is a long coil of wire with many turns or loops. Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coil is nearly zero. Figure 11-14 shows how the field looks and how its direction is given by the right-hand rule.

The interior magnetic field a solenoid with closely spaced turns is very uniform in direction and magnitude and strong. Near the ends it begins to weaken and change direction. The field outside has similar complexities to single loops and bar magnets. The magnetic field strength inside a long and tightly wound solenoid is simply

$$B = \mu_0 \left(\frac{N}{l}\right) I = \mu_0 n l \quad (11\text{-}15)$$

where N is the number of loops and l, the length of the solenoid and $n = N/l$ is the number of turns per unit length of the solenoid. Fields spread over a large volume are possible with solenoids, as the Example below implies.

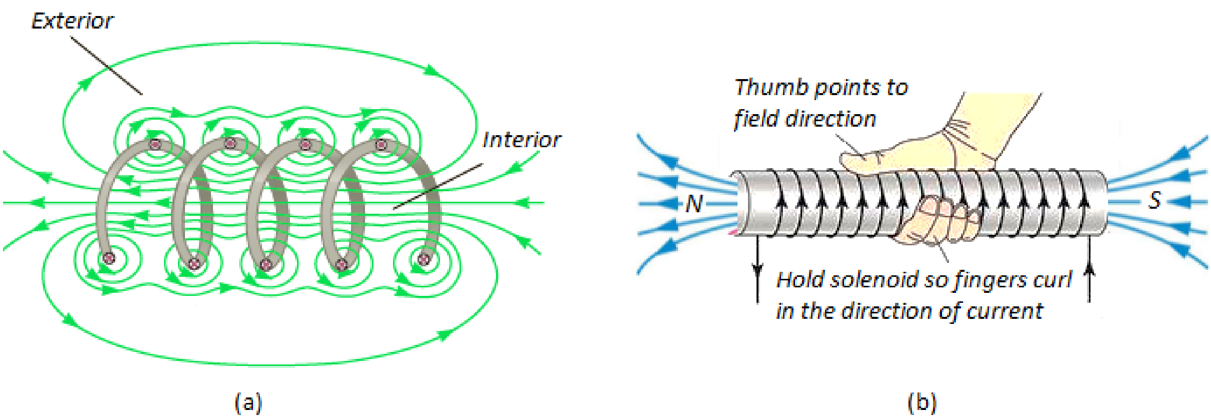


Figure 11-14: The solenoid. (a) The interior magnetic field is stronger and the exterior field is much weaker. (b) The direction of the magnetic field is determined by the right-hand rule.

Example

3. What is the field inside a 2.00-m-long solenoid that has 2000 turns and carries a 1600-A current?

Solution

Substituting known values into Equation 1.15, we get

$$B = \mu_0 \left(\frac{N}{l}\right) I = 4\pi \times 10^{-7}\, T \left(\frac{2000\ turns}{2.00\ m}\right)(1600\ A) = 2.01\ T.$$

Exercises

1. Suppose you have a 32.0 - m length of copper wire. If the wire is wrapped into a solenoid 0.240 m long and having a radius of 0.040 0 m, how strong is the resulting magnetic field in its center when the current is 12.0 A?
2. A certain solenoid consists of 100 turns of wire and has a length of 10.0 cm. (a) Find the magnitude of the magnetic field inside the solenoid when it carries a current of 0.500 A. (b) What is the momentum of a proton orbiting inside the solenoid in a circle with a radius of 0.020 m? The axis of the solenoid is perpendicular to the plane of the orbit. (c) Approximately how much wire would be needed to build this solenoid? Assume the solenoid's radius is 5.00 cm.

11.6.1.8 Magnetic Force between Two Parallel Conductors

The force between two long straight and parallel conductors separated by a distance $\boldsymbol{r}$ can be found by applying what we have developed in preceding sections. Figure 11-15 shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 ($\boldsymbol{F_2}$). by Equation 1.13, the field due to $\boldsymbol{I_1}$ at a distance $\boldsymbol{r}$ is

$$\boldsymbol{B_1} = \frac{\mu_0 I_1}{2\pi r} \qquad (11\text{-}16)$$

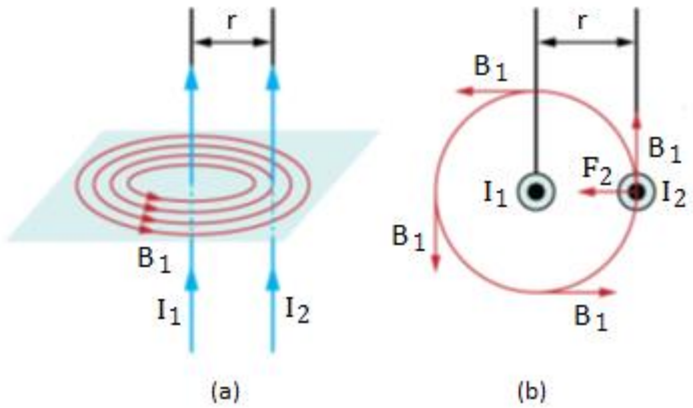


Figure 11-15: (a) Current I_1 produces a magnetic field B_1 at the position of current I_2, which experiences a magnetic force F_2. (b) The same currents as seen from the top.

This field is uniform along wire 2 and perpendicular to it, and so the force $\boldsymbol{F_2}$ it exerts on wire 2 is given by $\boldsymbol{F} = \boldsymbol{IlB}\sin\boldsymbol{\theta}$ with sin θ = 1:

$$\boldsymbol{F_2} = \boldsymbol{I_2 l B_1} \qquad (11\text{-}17)$$

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write $\boldsymbol{F}$ for the magnitude of $\boldsymbol{F_2}$. (Note that $\boldsymbol{F_1} = -\boldsymbol{F_2}$.) Since the wires are very long, it is convenient to think in terms of $/\boldsymbol{l}$, the force per unit length. Substituting the expression for $\boldsymbol{B_1}$ into the last equation and rearranging terms gives

$$\frac{F}{l} = \mu_0 \frac{I_1 I_2}{2\pi r} \qquad (11\text{-}18)$$

The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Examples

1. Two wires, each having a weight per unit length of 1.00×10^{-4} N/m, are parallel with one directly above the other. Assume the wires carry currents that are equal in magnitude and opposite in direction. The wires are 0.10 m apart, and the sum of the magnetic force and gravitational force on the upper wire is zero. Find the current in the wires. (Neglect Earth's magnetic field.)

Solution

The net force on the upper wire is $\boldsymbol{F}_g + \boldsymbol{F}_m = 0$

$$-mg + \frac{\mu_0 I_1 I_2}{2\pi r} l = 0$$

It is given that $I_1 = I_2 = I$. Then

$$I = \sqrt{\frac{2\pi r}{\mu_0}\left(\frac{mg}{l}\right)}$$

Substituting numerical values:

$$I = \sqrt{\frac{2\pi \times 0.10m}{4\pi \times 10^{-7}\, Tm} \times (1.00 \times 10^{-4}\, N/m)} = 7.07\text{ A}$$

Exercise

1. In the example above, if the current in each wire is doubled, how far apart should the wires be placed if the magnitudes of the gravitational and magnetic forces on the upper wire are to be equal?

Definition of the ampere

The operational definition of the ampere is based on the force between current-carrying wires. Consider two parallel wires separated by 1 meter and each carrying current $\boldsymbol{I}$. Suppose the force per meter on each wire due to their magnetic interaction is measured to be $\mathbf{2} \times \mathbf{10^{-7} N/m}$. Now we can determine the current I in each wire using Equation (1.18):

$$2 \times 10^{-7} N/m = 4\pi \times 10^{-7} Tm/A \times \frac{I^2}{2\pi \times 1\,m}$$

Solving for I we get exactly 1 ampere. This is the basis of the operational definition of the ampere:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, is the amount of current that causes a force of exactly 2×10^{-7} *N/m on each conductor.*

11.6.2 Magnetic Flux

Magnetic flux is an important concept in the study of electromagnetism. To get some insight into the concept of magnetic flux consider Figure 11-16, below.

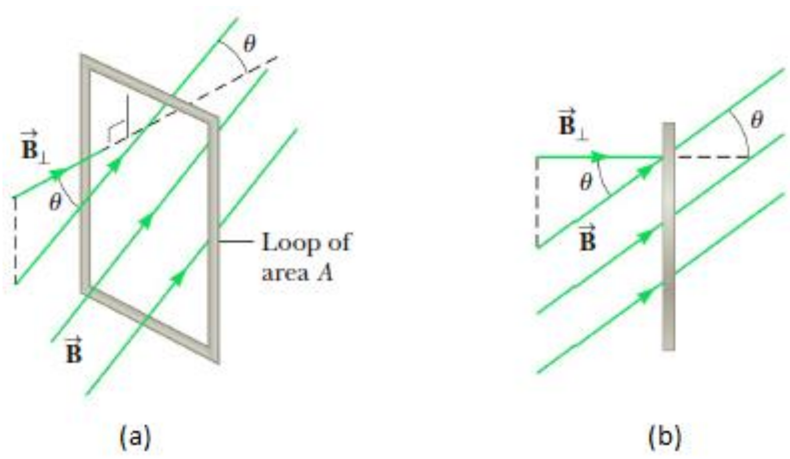


Figure 11-16: (a) A uniform magnetic field **B** crosses a rectangular wire loop of area A making an angle θ with the direction normal to the plane of the loop. (b) A side view clearly showing how the field passes through the loop.

Since the magnetic field lines pass through the loop, we say there is a ***magnetic flux*** through the loop. The magnetic field has components perpendicular and parallel to the plane of the loop. The parallel components do not contribute to the magnetic flux because they go parallel to the plane of the loop NOT through it. The perpendicular components constitute the flux as they penetrate through the plane of the loop. From Figure 11-16, above, the perpendicular component is given by

$$B_\perp = B\cos\theta \tag{11-19}$$

The magnetic flux (Φ_B) through the loop is now defined as the product of $\mathbf{B}_\perp$ and the area A of the loop, that is,

$$\Phi_B = B_\perp A = BA\cos\theta \tag{11-20}$$

Using this definition and Figure 11-16(b), we see that when the magnetic field is totally perpendicular to the plane of the loop, θ = 0° and the magnetic flux becomes maximum (see also Figure 11-17, below). When the magnetic field is totally parallel to the plane of the loop, θ = 90° and the flux is zero. The flux can also be negative. For example, when θ = 180°, the flux is equal to $-BA$. The SI unit of flux is Tm^2, or weber (Wb).

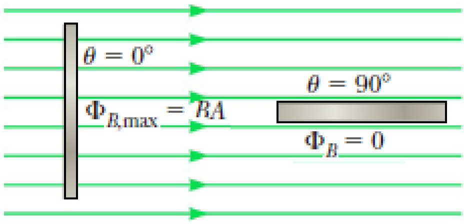


Figure 11-17: When the field lines are perpendicular to the plane of the loop, the magnetic flux through the loop is maximum; When the field lines are parallel to the plane of the loop, the magnetic flux through the loop is zero.

Example

1. A conducting circular loop of radius 0.250 m is placed in the xy-plane in a uniform magnetic field of 0.360 T that points in the positive z-direction, the same direction as the normal to the plane. (a) Calculate the magnetic flux through the loop. (b) Suppose the loop is rotated clockwise around the x-axis, so the normal direction now points at a 45.0° angle with respect to the z-axis. Recalculate the magnetic flux through the loop. (c) What is the change in flux due to the rotation of the loop?

Solution

(a)
$$\Phi_B = BA\cos\theta = B\,(\pi r^2)\cos 0°$$
$$\Phi_B = (0.360\ T)(\pi \times 0.250^2\ m^2) = 0.0706\ \text{Wb}$$

(b)
$$\Phi'_B = BA\cos\theta = B\,(\pi r^2)\cos 45°$$
$$\Phi'_B = (0.360\ T)(\pi \times 0.250^2\ m^2)(\sqrt{2}/2) = 0.0499\ \text{Wb}$$

(c)
$$\Delta\Phi_B = \Phi'_B - \Phi_B = 0.0706\ \text{Wb} - 0.0499\text{Wb} = -0.0207\ \text{Wb}$$

Exercises

1. The loop in the example above, having rotated by 45°, rotates clockwise another 30°, so the normal to the plane points at an angle of 75° with respect to the direction of the magnetic field. Find (a) the magnetic flux through the loop when θ = 75° and (b) the change in magnetic flux during the rotation from 45° to 75°.

2. Find the flux of Earth's magnetic field of magnitude $\mathbf{5.00 \times 10^5}$ T through a square loop of area 20.0 $\mathbf{cm^2}$ (a) when the field is perpendicular to the plane of the loop, (b) when the field makes a 30.0° angle with the normal to the plane of the loop, and (c) when the field makes a 90.0° angle with the normal to the plane.

11.7 Electromagnetic Induction

The concept of magnetic flux is useful to understand the basic idea of electromagnetic induction. Consider a wire loop connected to an ammeter as in Figure 11-18. When a bar magnet is moved toward the loop, the ammeter reads a current in one direction, as in Figure 11-18(a). When the bar magnet is held stationary [Figure 11-18(b)], the ammeter reads zero current. When the bar magnet is moved away from the loop, the ammeter reads a current in the opposite direction [Figure 11-18(c)]. If the magnet is held stationary and the loop is moved either toward or away from the magnet, the ammeter also reads a current.

From these observations, it can be concluded that a current is established in the circuit as long as the magnetic flux through the loop changes irrespective of the relative motion between the magnet and the loop. We call such a current ***induced current*** and the potential difference producing it, ***induced emf***.

The magnitude of the induced emf and its direction are determined, respectively, by Faraday's law and Lenz's law which are discussed below.

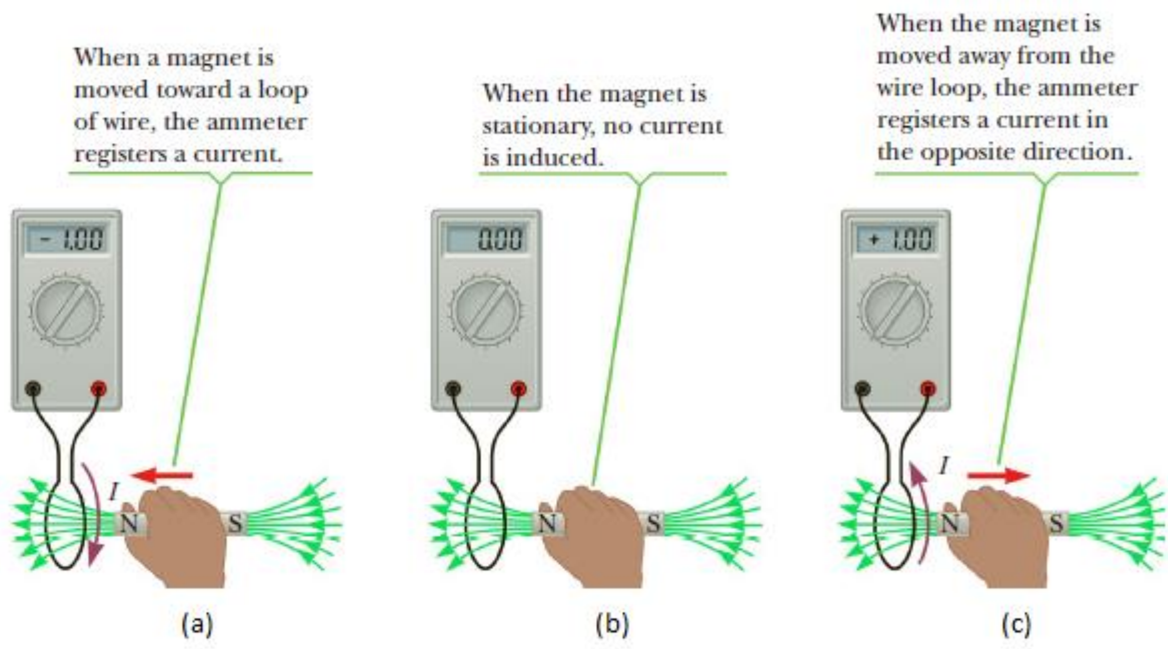


Figure 11-18: A change in the magnetic flux through the loop is produced by moving the bar magnet which, in turn, induces a current in the loop as indicated by the ammeter.

11.7.1.1 Faraday's Law of Induction: Lenz's Law

Faraday's law of magnetic induction states that:

If a circuit contains N tightly wound loops and the magnetic flux through each loop changes by the amount $\boldsymbol{\Delta\Phi_B}$ during the time interval $\boldsymbol{\Delta t}$, the average emf induced in the circuit is

$$\boldsymbol{\mathcal{E}} = -\boldsymbol{N}\frac{\boldsymbol{\Delta\Phi_B}}{\boldsymbol{\Delta t}} \text{ (Faraday's Law)} \qquad (11\text{-}21)$$

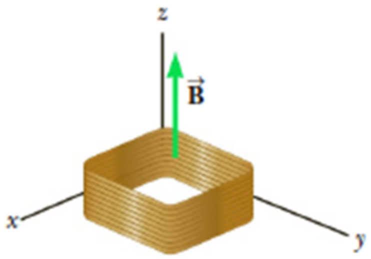
The minus sign in Equation (1-21) is included to indicate the polarity of the induced emf. This polarity determines the direction of the current in the loop and is given by Lenz's law:

The current caused by the induced emf is in a direction that creates a magnetic field with flux opposing the change in the original flux through the circuit.

Example

1. A coil with 25 turns of wire is wrapped on a frame with a square cross section 1.80 cm on a side. Each turn has the same area, equal to that of the frame, and the total resistance of the coil is 0.350 V. An applied uniform magnetic field is perpendicular to the plane of the coil, as in the figure below. (a) If the field changes uniformly from 0.00 T to 0.500 T in 0.800 s, what is the induced emf in the coil while the field is changing? Find (b) the magnitude and (c) the direction of the induced current in the coil while the field is changing.

Solution



(a) Area of loop: $A = (1.80 \times 10^{-2}\ \text{cm})^2$

$A = 3.24 \times 10^{-4}\ \text{m}^2$

Flux at $t = 0$: $\Phi_{B,i} = B_i A = 0.00\ Wb$

Flux at t=0.800s:

$$\Phi_{B,f} = B_f A = 0.500\ \text{T} \times 3.24 \times 10^{-4}\ \text{m}^2$$

$$\Phi_{B,f} = 1.62 \times 10^{-4}\ \text{Wb}$$

Change in flux: $\Delta\Phi_B = 1.62 \times 10^{-4}\ \text{Wb}$

Induced emf:

$$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -25\ turns\ \frac{1.62 \times 10^{-4}\ Wb}{0.800\ s}$$

$$\mathcal{E} = -5.06 \times 10^{-3}\ V$$

(b) Induced current:

$$I = \frac{\mathcal{E}}{R} = \frac{5.06 \times 10^{-3}\ V}{0.350\ \Omega} = 1.45 \times 10^{-2}\ \text{A}$$

(c) The current must be in a clockwise direction as viewed from above the coil. This is because, by Lenz's law, the flux is positive and increasing. This is opposed by a negative flux from a downward magnetic field that could exist only if the induced current is clockwise as seen from above.

Exercise

1. In the example above, suppose the magnetic field changes uniformly from 0.500 T to 0.200 T in the next 0.600 s. Compute (a) the induced emf in the coil and (b) the magnitude and direction of the induced current.

11.7.1.2 Motional emf

A straight conductor of length $\boldsymbol{l}$, is moving with constant velocity $\boldsymbol{v}$ along two conducting rails and perpendicular to a uniform magnetic field directed into the paper, as in Figure 11-19.

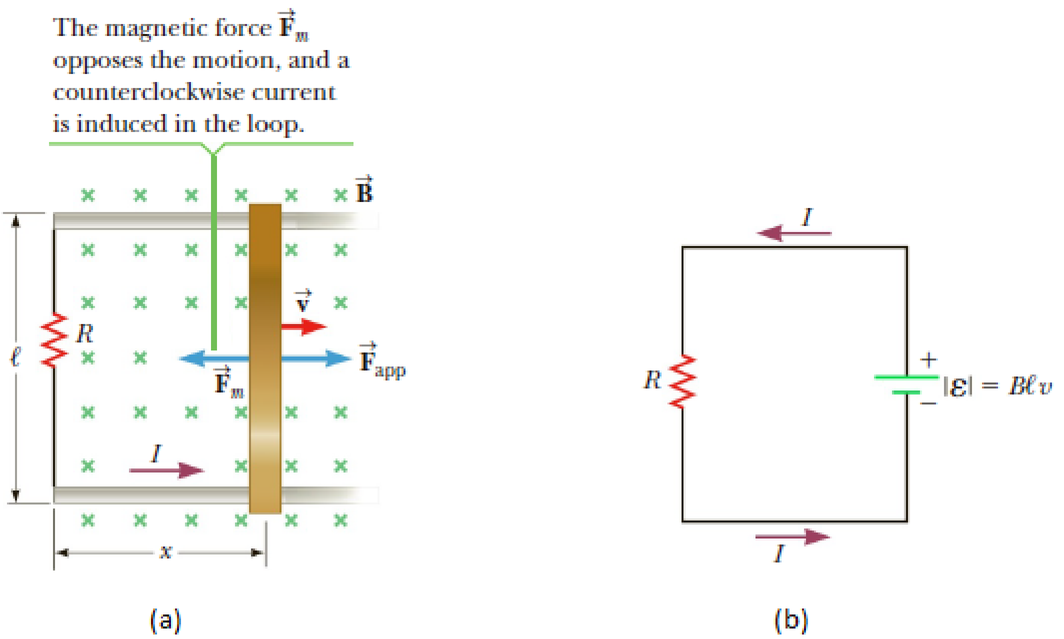


Figure 11-19: (a) An emf is induced when a straight conductor slides with velocity $\boldsymbol{v}$ along two conducting rails under the action of an applied force $\boldsymbol{F}_{app}$. (b) The equivalent circuit shows a battery to represent motional emf in (a).

A magnetic force of magnitude $\boldsymbol{F_m = qvB}$, directed downward, acts on the electrons in the conductor. Because of this magnetic force, the free electrons move to the lower end of the conductor resulting in a separation of charges that makes the upper end more positive than lower end of the conductor. The charge separation produces an electric field which exerts an upward force $\boldsymbol{qE}$ on the electrons. Eventually, the downward magnetic force is balanced by the upward electric force, $\boldsymbol{qE = qvB}$ or $\boldsymbol{E = vB}$.

$$\boldsymbol{E = vB} \tag{11-22}$$

The uniform electric field produced in the conductor is related to the potential difference across the ends created by the charge separation by the formula:

$$\boldsymbol{\Delta V = El} \tag{11-23}$$

Combining Equations (1-22) and (1-23) yields

$$\boldsymbol{\Delta V = Blv} \tag{11-24}$$

This potential difference is due to the motion of the conductor in a magnetic field and hence the name *motional emf*. The direction of the motional emf in Figure 11-19 can be determined by Lenz's law or the right-hand rule:

Using Lenz's law: *The magnetic flux increases into the page. To oppose this change, a magnetic field is induced out of the page which corresponds to an induced current counterclockwise. The motional emf is then from the lower to the upper end of the conductor*.

Using the right-hand rule: *Stretch your right-hand fingers in the direction of the velocity and curl your fingers in the direction of the magnetic field, the thumb will point in the direction of the motional emf*.

The motional emf can be derived using Faraday's law as follows: In Figure 11-19, the area of the loop, $A = lx$, increases as the conductor moves away from the resister. The magnetic flux through the loop, therefore, changes as

$$\Delta\Phi_B = B\Delta A = Bl\Delta x$$

The magnitude of the motional emf is then

$$\boldsymbol{\mathcal{E} = \frac{\Delta\Phi_B}{\Delta t} = Bl\frac{\Delta x}{\Delta t} = Blv} \tag{11-25}$$

If the resistance of the circuit is R, the magnitude of the induced current in the circuit is

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$

Example

3. An airplane with a wingspan of 30.0 m flies due north at a location where the downward component of Earth's magnetic field is 0.600×10^4 T. There is also a component pointing due north that has a magnitude of 0.470×10^4 T. (a) Find the difference in potential between the wingtips when the speed of the plane is 2.50×10^2 m/s. (b) Which wingtip is positive?

Solution

(a) The potential difference across the wingtips

$$\mathcal{E} = Blv = (0.600 \times 10^4 \text{ T})(30.0 \text{ m})(2.50 \times 10^2 \text{ m/s}) = 0.450\ V$$

(b) identify the positive wingtip

Apply the right-hand rule: Point the fingers of your right hand north, in the direction of the velocity, and curl them down, in the direction of the magnetic field. Your thumb points west. Therefore, the west wingtip is positive.

Exercise

1. Suppose a space station is in orbit where the magnetic field is parallel to Earth's surface, points north, and has magnitude $\mathbf{1.80 \times 10^{-4}}$ T. A metal cable attached to the space station stretches radially outwards 2.50 km. (a) Estimate the potential difference that develops between the ends of the cable if it's traveling eastward around Earth at a speed of $\mathbf{7.70 \times 10^{3}}$ m/s. (b) Which end of the cable is positive, the lower end or the upper end?

11.8 Semiconductors and Diodes

Learning outcomes

After completing this section, students are expected to:

- Recognize the basic structure of semiconductors and how they conduct current
- Distinguish between conductors, semiconductors, and insulators
- Describe the structure of a silicon crystal.
- List the two types of carriers and name the type of impurity that causes each to be a majority carrier.
- Explain the conditions that exist at the pn junction of an unbiased diode, a forward-biased diode, and a reverse-biased diode.
- Describe the characteristics and biasing of a diode
- Analyze the operation of a half-wave rectifier and a full-wave rectifier

Solids can be divided into three groups based on their ability to conduct electricity. These groups are

1. *Conductors:* These are solid materials with high electrical conductivity (with $\sigma \sim 10^{-2}\ \Omega$.m). Example: metals.
2. *Insulators:* These are solids having very low conductivity ((with $\sigma \sim 10^{-11}\ \Omega$.m). Examples: plastics and wood.
3. *Semiconductors:* These are solids with conductivity intermediate to metals and insulators (with $\sigma \sim 10^{-5}\ \Omega$.m). Examples: Silicon (Si) and Germanium (Ge)

Metals conduct very well and semiconductors don't. One very interesting difference is that metals conduct less as they become hotter but semiconductors conduct more.

A semiconductor is a solid, crystalline substance that conducts electric current better than an insulator but not as well as a conductor. For example, semiconductors conduct electric current better than paper, wood, and air but not as well as copper wire.

The most widely used semiconductors in electronics are the elemental semiconductors silicon (Si) and germanium (Ge). There are also compound semiconductors formed from (a) the III-V from the periodic table such as CdS, GaAs, CdSe, and InP; and (b) the II-VI such as. ZnS, and HgS.

Further, semiconductors may be classified into two groups: intrinsic and extrinsic semiconductors.

11.8.1 Intrinsic Semiconductors:

Intrinsic semiconductors are one that has no impurities. They are those semiconductors that have been carefully refined to reduce the impurities to a very low level-essentially as pure as can be made available.

In intrinsic semiconductors, the number of free electrons, n_e is equal to the number of holes, n_h.

Semiconductors possess the unique property in which, apart from electrons, the holes also move.

Remark:

Intrinsic (pure) semiconductors are neither good conductors nor good insulators. However, their conductivity can be increased by a process known as doping.

11.8.2 Extrinsic Semiconductors

A semiconductor is a solid whose electrical properties can be modified by a process known as doping. Doped semiconductors are known as extrinsic semiconductors. Extrinsic semiconductors are classified into two main categories: n-type and p-type.

Definition: Doping

Doping is the deliberate addition of impurities to a pure semiconductor material to change its electrical properties.

Semiconductors are often the Group IV elements in the periodic table. The most common semiconductor elements are silicon (Si) and germanium (Ge). The most important property of Group IV elements is that they 4 valence electrons. So, if we look at the arrangement of for example Si atoms in a crystal, they would look like that shown in Figure 11.8.1.

I) N-type Semiconductor

A surplus of electrons is created by adding an element that has more valence electrons than Si to the Si crystal. This is known as *n-type doping* and elements used for n-type doping usually come from Group V in the periodic table. Elements from Group V have 5 valence electrons, one more than the Group IV elements.

A common n-type dopant (substance used for doping) is arsenic (As). The combination of a semiconductor and an n-type dopant is known as an *n-type semiconductor*. A Si crystal doped with Sb is shown in Figure (a). When Sb is added to a Si crystal, the 4 of the 5 valence electrons in Sb bond with the 4 Si valence electrons. The fifth Sb valence electron is free to move around.

It takes only a few Sb atoms to create enough free electrons to allow an electric current to flow through the silicon. Since n-type dopants 'donate' their free atoms to the semiconductor, they are known as *donor atoms*.

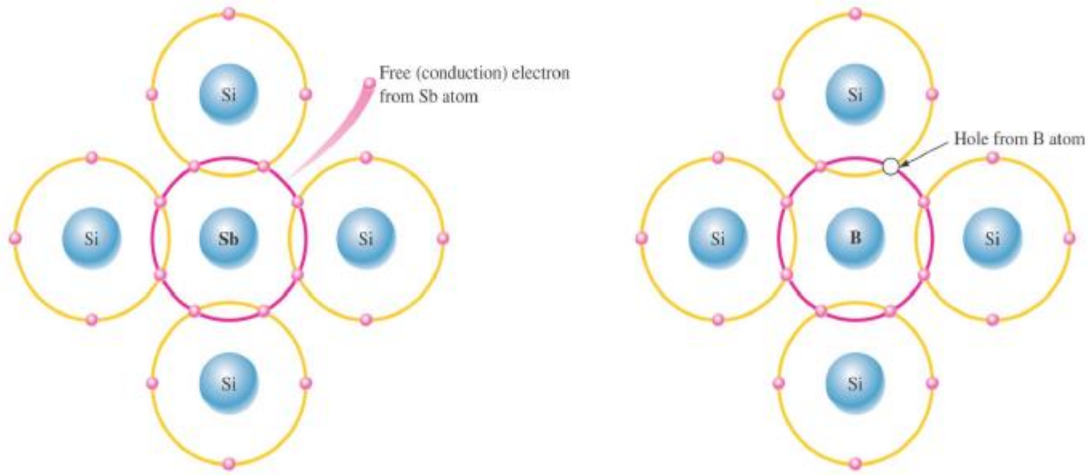


Fig. 11.8.1 Si crystal doped with (a) Antimony (Sb) and (b) Boron (B).

II) P-type Semiconductor

A deficiency of electrons is created by adding an element that has less valence electrons than Si to the Si crystal. This is known as p-type doping and elements used for p-type doping usually come from Group III in the periodic table. Elements from Group III have 3 valence electrons, one less than the semiconductor elements that come from Group IV. A common p-type dopant is boron (B). The combination of a semiconductor and a p-type dopant is known as an p-type semiconductor. A Si crystal doped with B is shown in Figure 11.8.1. When B is mixed into the silicon crystal, there is a Si valence electron that is left unbonded.

The lack of an electron is known as a hole and has the effect of a positive charge. Holes can conduct current. A hole happily accepts an electron from a neighbour, moving the hole over a space. Since p-type dopants 'accept' electrons, they are known as acceptor atoms.

Questions:

1. What is meant by the term intrinsic?
2. How are holes created in an intrinsic semiconductor?
3. How is an n-type semiconductor formed?
4. How is a p-type semiconductor formed?
5. What are majority carriers?
6. Explain the process of doping using detailed diagrams for p-type and n-type semiconductors.
7. Draw a diagram showing a Ge crystal doped with As. What type of semiconductor is this?
8. Draw a diagram showing a Ge crystal doped with B. What type of semiconductor is this?
9. Explain how doping improves the conductivity of semiconductors.
10. Would the following elements make good p-type dopants or good n-type dopants? B, P, Ga, As, In, and Bi.

11.8.3 The pn-junction

When p-type and n-type semiconductors are placed in contact with each other, a p-n junction is formed. Near the junction, electrons and holes combine to create a depletion region.

Fig. 11.8.2: The p-n junction forms between p- and n-type semiconductors.

The free electrons from the n-type material combine with the holes in the p-type material near the junction. There is a small potential difference across the junction. The area near the junction is called the depletion region because there are few holes and few free electrons in this region.

11.8.3.1 pn-junction with no bias

In a pn-junction, without an external applied voltage (no bias), an equilibrium condition is reached in which a potential difference is formed across the junction.

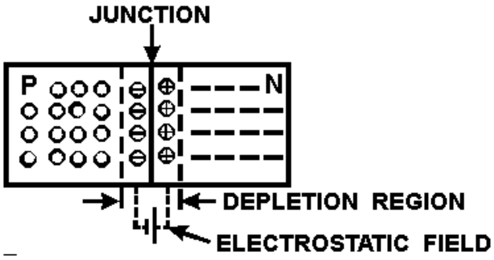


Fig.11.8.3 pn-junction with no bias

Initially, when you put the n-type and p-type semiconductors together to form a junction, holes near the junction tends to 'move' across to the n-region, while the electrons in the n-region drift across to the p-region to 'fill' some holes. This current will quickly stop as the potential barrier is built up by the migrated charges. So in steady state no current flows.

11.8.3.2 Forward biased pn-junction

Forward-bias occurs when the p-type semiconductor material is connected to the positive terminal of a battery and the n-type semiconductor material is connected to the negative terminal. The electric field from the external potential different can easily overcome the small internal field created by the initial drifting of charges. The external field then attracts more electrons to flow from n-region to p-region and more holes from p-region to n-region and you have a forward biased situation. As a result a large current flows across the pn-junction.

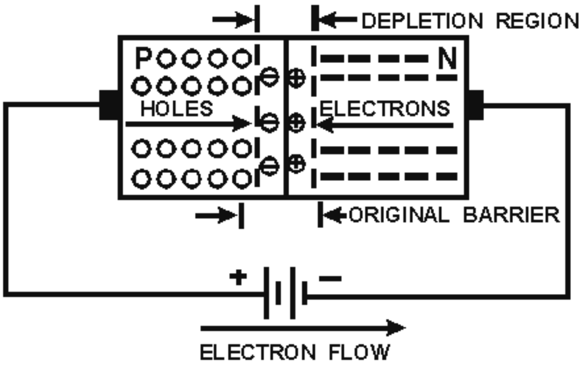


Fig. 11.8.4: Forward biased pn-junction (narrow depletion region).

11.8.3.3 Reverse biased pn-junction

Reverse bias occurs when the p-type semiconductor material is connected to the negative terminal of a battery and the n-type semiconductor material is connected to the positive terminal. In this case the external field pushes electrons back to the n-region while more holes into the p-region. Therefore, you get no current flow. Only the small number of thermally released minority carriers (holes in the n-type region and electrons in the p-type region) will be able to cross the junction and form a very small current.

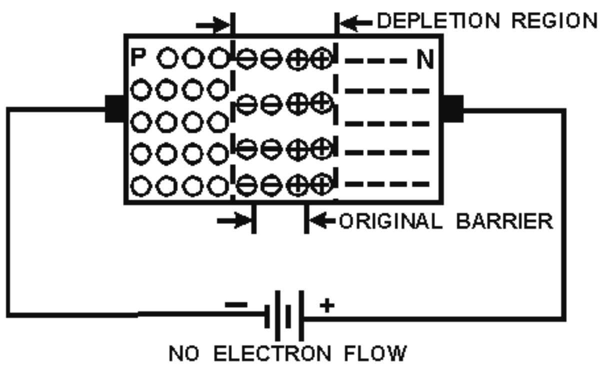


Fig. 11.8.5: Reverse pn-junction (broad depletion region).

Exercise:

1. Compare p- and n-type semiconductors.
2. Explain how a pn-junction works using a diagram.

11.8.4 Active Circuit Elements

Electric circuits that consists of only resistors, capacitors or/and inductors are called passive components. They do not change their behaviour in response to changes in voltage or current. Active components are quite different. Their response to changes in input allows them to make amplifiers, calculators and computers.

11.8.4.1 The Diode

A diode is an electronic device that allows current to flow in one direction only. A diode consists of two doped semi-conductors joined together so that the resistance is low when connected one way and very high the other way.

The diode consists of two semiconductor blocks attached together. Neither block is made of pure silicon - they are both doped. In short, p-type semiconductor has fewer free electrons than normal semiconductor. ‘p’ stands for ‘positive’, meaning a lack of electrons, although the material is actually neutral. The locations where electrons are missing are called holes.

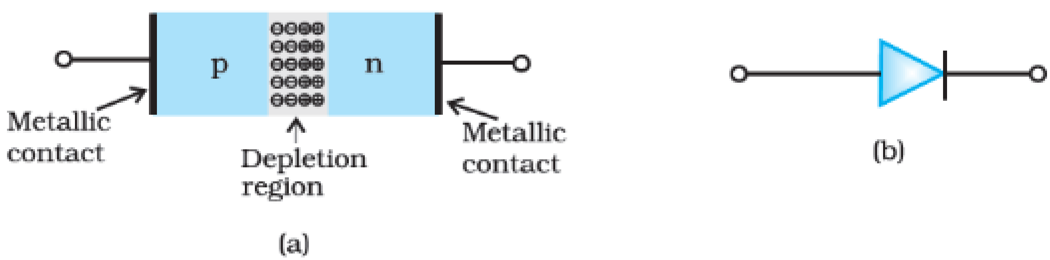


Fig. 11.8.6: (a) Semiconductor diode, (b) Symbol for pn-junction diode.

In n-type semiconductor, the situation is reversed. The material has more free electrons than normal semiconductor. ‘n’ stands for ‘negative’, meaning an excess of electrons, although the material is actually neutral.

When a p-type semiconductor is attached to an n-type semiconductor, some of the free electrons in the n-type move across to the p-type semiconductor. They fill the available holes near the junction. This means that the region of the n-type semiconductor nearest the junction has no free electrons (they've moved across to fill the holes). This makes this n-type semiconductor positively charged. It used to be electrically neutral, but has since lost electrons.

If the diode is reverse-biased, the + terminal of the battery is connected to the n-type semiconductor. This makes it even more negatively charged. It also removes even more of the free electrons near the depletion band. At the same time, the − terminal of the battery is connected to the p-type silicon. This will supply free electrons and fill in more of the holes next to the depletion band. Both processes cause the depletion band to get wider. The resistance of the diode (which was already high) increases. This is why a reverse-biased diode does not conduct.

The reverse current is dependent primarily on the junction temperature and not on the amount of reverse-biased voltage. A temperature increase causes an increase in reverse current. If the external reverse-bias voltage is increased to a large enough value, reverse breakdown occurs.

Most diodes normally are not operated in reverse breakdown and can be damaged if they are. However, a particular type of diode known as a zener diode is specially designed for reverse-breakdown operation.

On the other hand, if the diode is forward biased, the depletion region is made narrower. The negative charge on the p-type silicon is cancelled out by the battery. The greater the voltage used, the narrower the depletion region becomes. Eventually, when the voltage is about 0.6 V (for silicon) the depletion region disappears. Once this has occurred, the diode conducts very well.

The existence of the positive and negative ions on opposite sides of the pn junction creates a barrier potential across the depletion region, as indicated in Figure 11.8.7. The barrier potential, $V_{B,}$ is the amount of voltage required to move electrons through the depletion region. At 25^{0}C, it is approximately 0.7 V for silicon and 0.3 V for germanium. As the junction temperature increases, the barrier potential decreases, and vice versa.

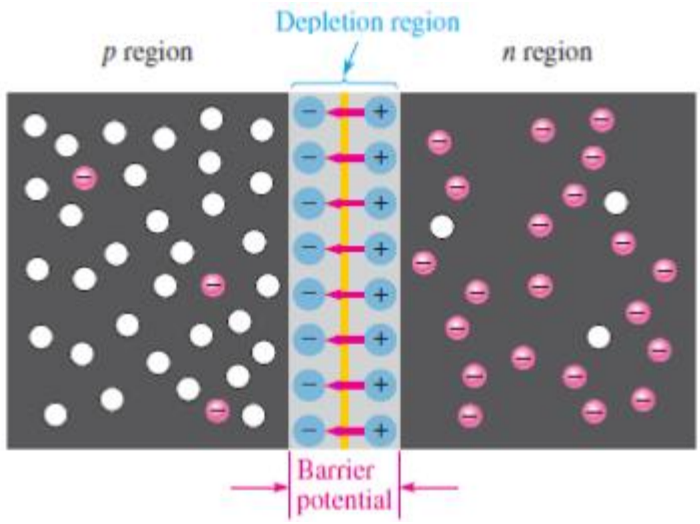


Fig. 11.8. 7: For every electron that diffuses across the junction and combines with a hole, a positive charge is left in the *n* region and a negative charge is created in the *p* region, forming a barrier potential. This action continues until the voltage of the barrier repels further diffusion.

The effect of the barrier potential in the depletion region is to oppose forward bias. This is because the negative ions near the junction in the p region tend to prevent electrons from moving through the junction into the p region.

Exercise

1. What is a diode?
2. What is a diode made of?
3. What is the term which means that a diode is connected the 'wrong way' and little current is flowing?
4. Why is a diode able to conduct electricity in one direction much more easily than the other?
5. When p and n regions are joined, a depletion region forms. Describe the characteristics of the depletion region.

11.8.4.2 Diode Characteristic Curve

Now that we have seen how a junction diode operates, it is time to examine some of its electrical characteristics. This takes into account such things as voltage, current, and temperature. Since these characteristics vary a great deal in an operating circuit, it is best to look at them graphically.

A graph of diode voltage versus current, known as a V-I characteristic curve, is shown in Fig. 11.8.8. The upper right quadrant of the graph represents the forward-biased condition. As you can see, there is very little forward current (I_F) for forward voltages (V_F) below the barrier potential. As the forward voltage approaches the value of the barrier potential, the current begins to increase. Once the forward voltage reaches the barrier potential, the current increases drastically and must be limited by a series resistor. The voltage across the forward-biased diode remains approximately equal to the barrier potential.

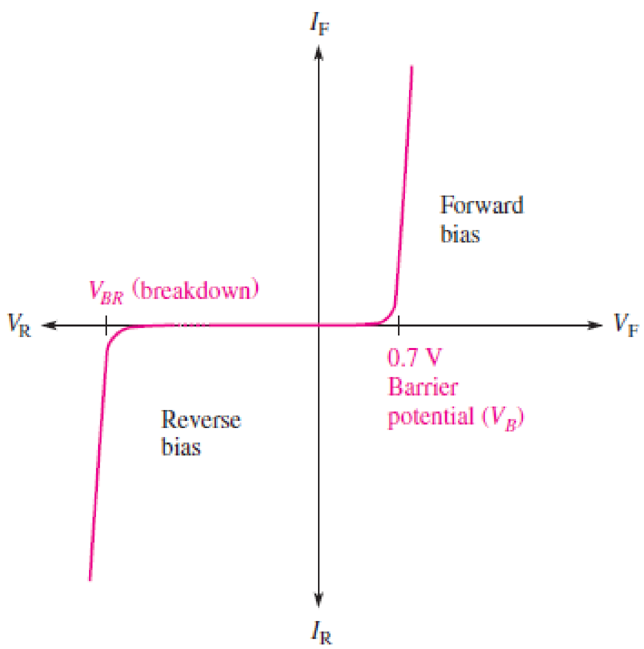


Fig. 11.8.8: General diode V-I characteristic curve

The lower left quadrant of the graph represents the reverse-biased condition. As the reverse voltage (V_R) increases to the left, the current remains near zero until the breakdown voltage (V_{BR}) is reached. When breakdown occurs, there is a large reverse current (I_R) which, if not limited, can destroy the diode. Typically, the breakdown voltage is greater than 50 V for most rectifier diodes. Remember that most diodes should not be operated in reverse breakdown.

11.8.4.3 Diode Rectifiers

Because of their ability to conduct current in one direction and block current in the other direction, p-n diodes can be used in rectifier circuits to convert AC to DC, as required in many electronic circuits. The conversion of AC voltage to DC is called rectification and there are two types of rectifications.

(i) The Half-Wave Rectifier

Figure 11.8.9 illustrates the process called half-wave rectification. In part (a), a diode is connected to an ac source that provides the input voltage V_{in}, and to a load resistor R_L, forming a half-wave rectifier. Keep in mind that all ground symbols represent the same point electrically. Let's examine what happens during one cycle of the input voltage using the ideal model for the diode. When the sinusoidal input voltage goes positive, the diode is forward-biased and conducts current through the load resistor, as shown in part (b). The current produces an output voltage across the load, which has the same shape as the positive half-cycle of the input voltage.

When the input voltage goes negative during the second half of its cycle, the diode is reverse-biased. There is no current, so the voltage across the load resistor is zero, as shown

in Fig. 11.8.9(c). The net result is that only the positive half-cycles of the ac input voltage appear across the load. Since the output does not change polarity, it is a pulsating dc voltage, as shown in part (d).

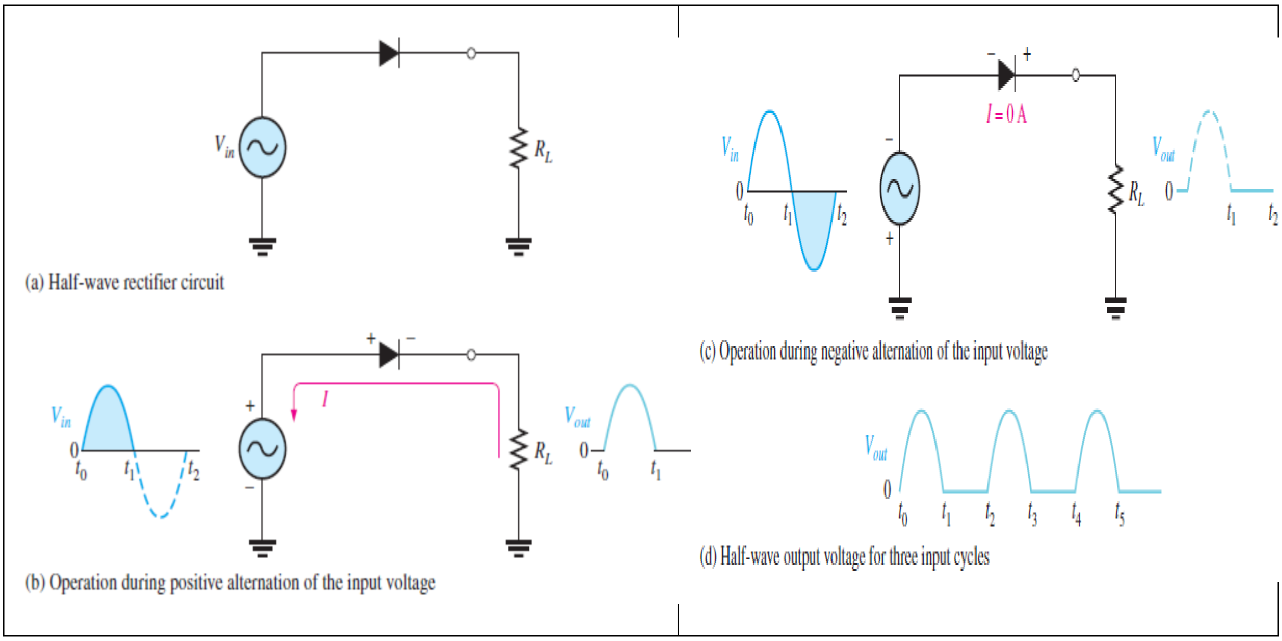
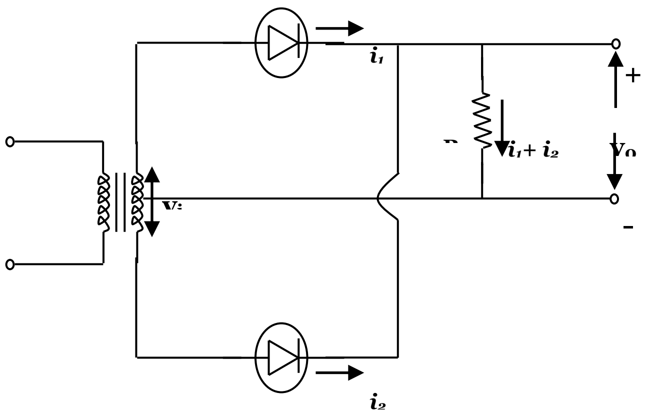


Fig. 11.8.9: Operation of half-wave rectifier

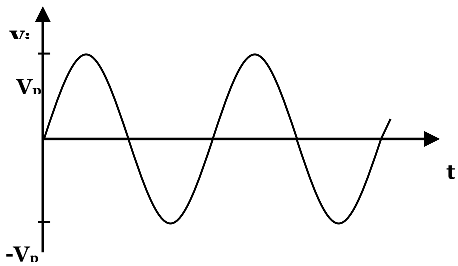
(ii) Full-wave rectifier

The difference between full-wave and half-wave rectification is that a full-wave rectifier allows unidirectional current to the load during the entire input cycle and the half-wave rectifier allows this only during one-half of the cycle.

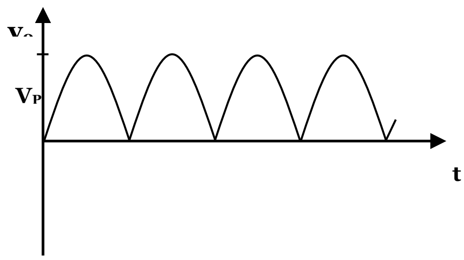
By arranging two diodes, as shown in Fig.11.8.10(a), so that each diode conducts in an alternate half-cycles, full-wave rectification results. The center-tapped transformer in the full-wave rectifier circuit supplies current to both half-cycles of the input voltage. Each diode in the full-wave rectifier must withstand the full end-to-end voltage of the transformer windings. The output voltage is only one-half (minus the voltage drop across the diodes) the total voltage of the transformer secondary.



(a): Full-wave rectifier circuit.



(b): Input voltage.



(c): Output voltage.

Fig. 11.8. 10: Full wave rectification

Rectifier circuits are used when the incoming electrical energy, for example, from a wall outlet, is AC but needs to be changed into DC. Recall that our audio systems (CD players, FM radio tuners, etc.), TVs, microwave ovens, computers, and other electronic appliances use DC.

Rectifier circuits are also important in the type of fuel cell technology in which AC must be changed to DC to separate hydrogen from oxygen in water. To separate the hydrogen and oxygen molecules in water, DC is applied to chemical cells in a process called electrolysis. In these cells, the hydrogen molecules collect at the negative terminals, and the oxygen molecules collect at the positive terminals. The oxygen gas is vented to the atmosphere, but the hydrogen gas is compressed and stored in tanks to be used in cars, buses, electrical appliances, and so on. This process allows electrical energy from the AC grid to be used at off-peak hours to produce clean-burning hydrogen for many uses.

11.8.4.4 Transistors

Like diodes, transistors are made of semiconductors and are found in almost all electronic devices. A transistor is a control device that amplifies a small input signal into a larger output signal. It consists of one type of semiconductor sandwiched between two regions of the opposite type of semiconductor. Such types of transistors are also called bipolar junction transistors (BJT). As shown in Figure 11.8.11, there are two choices for this type of transistor: p-n-p and n-p-n transistors.

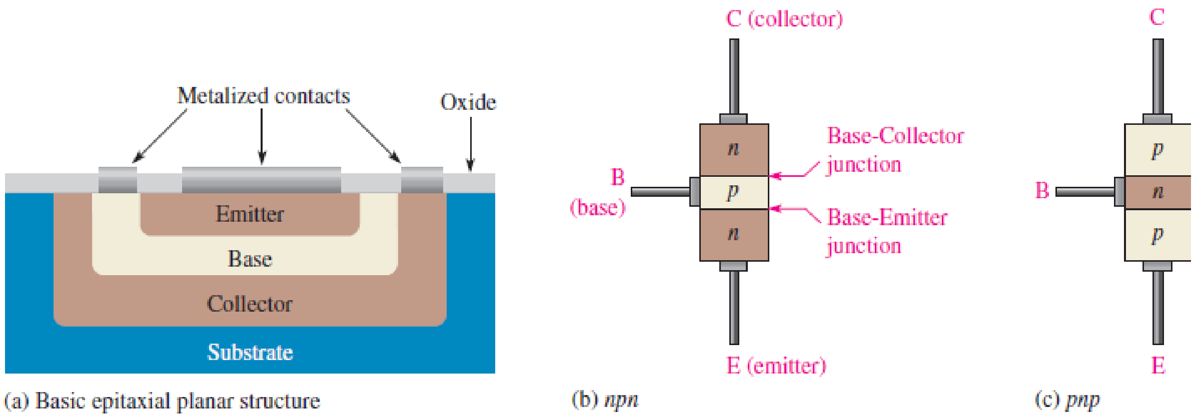


Fig. 11.8.11: Basic construction of bipolar junction transistors.

The centre section of a transistor is the base; it is extremely thin, often about$\mathbf{1\mu m(1\times 10^{-6}m)}$. The outer sections compose the emitter, which is the thinner of the two, and the collector, which is the thicker one.

The pn junction joining the base region and the emitter region is called the base-emitter junction. The junction joining the base region and the collector region is called the base collector junction, as indicated in Figure 11.8.11(b). A wire lead connects to each of the three regions, as shown. These leads are labeled E, B, and C for emitter, base, and collector, respectively. The base material is lightly doped and very narrow compared to the heavily doped emitter and collector materials.

Figure 11.8.12 shows the schematic symbols for the *npn* and *pnp* bipolar transistors. Notice that the emitter terminal has an arrow. The term bipolar refers to the use of both holes and electrons as charge carriers in the transistor structure.

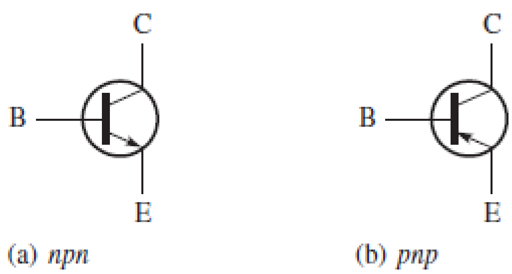


Fig. 11.8.12: Transistor symbols

Transistor Biasing

In order for a transistor to operate properly as an amplifier, the two *pn* junctions must be correctly biased with external dc voltages. Figure 11.8.13 shows the proper bias arrangement for both *npn* and *pnp* transistors. Notice that in both cases the base-emitter (BE) junction is forward-biased and the base-collector (BC) junction is reverse-biased. This is called *forward-reverse bias.*

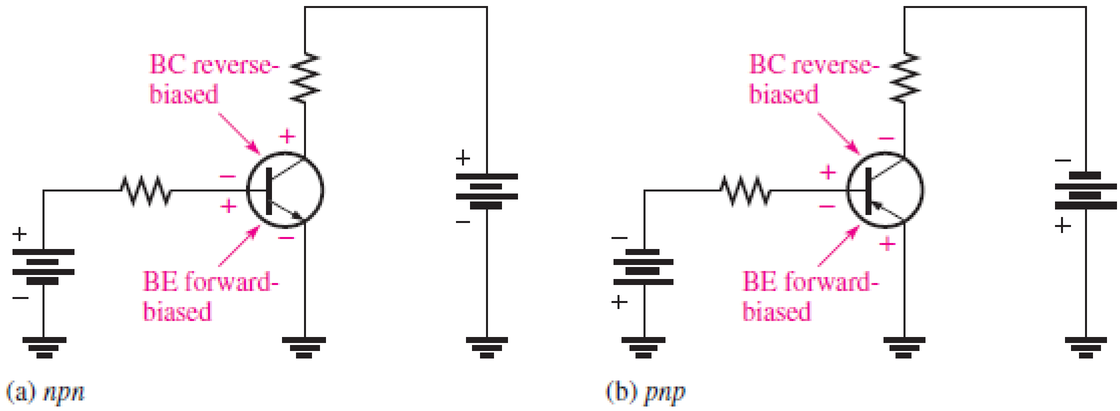


Fig. 11.8.13: Forward-reverse bias of a BJT.

Transistor Currents

The directions of current in an *npn* and a *pnp* transistor are as shown in Figure 11.8.14(a) and (b), respectively. An examination of these diagrams shows that the emitter current is the sum of the collector and base currents, expressed as follows:

$$I_E = I_C + I_B$$

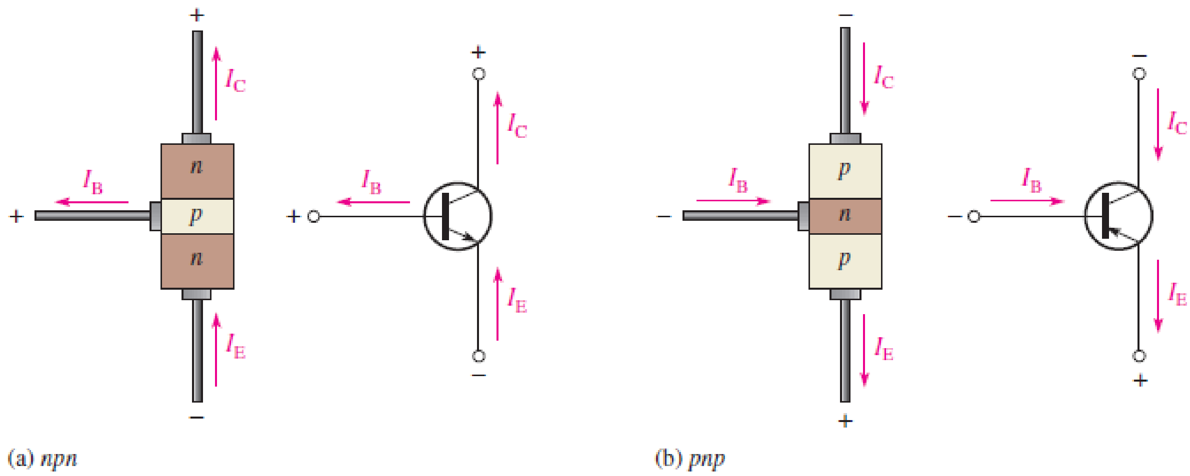


Fig. 11.8.14: Transistor currents

Transistor Voltages

The three dc voltages for the biased transistor in Figure 5 are the emitter voltage (VE), the collector voltage (VC), and the base voltage (VB). These voltages are with respect to ground.

The collector voltage is equal to the dc supply voltage, VCC, less the drop across RC.

$$V_C = V_{CC} - I_C R_C$$

The base voltage is equal to the emitter voltage plus the base-emitter junction barrier potential (VBE), which is about 0.7 V for a silicon transistor.

$$V_B = V_E + V_{BE}$$

In the configuration of Figure 11.8.15, the emitter is the common (grounded) terminal, so VE = 0 V and VB = 0.7 V.

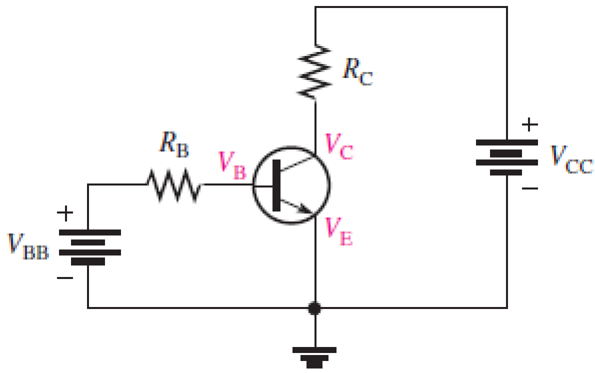


Fig. 11.8.15: Bias voltages

Transistor Application

Sound systems for musicians are a specific use of a transistor amplifier. A small voltage signal can be picked up from an electric guitar, amplified in a transistor circuit, and sent to an output speaker. Figure 11.8.16 shows a p-n-p transistor circuit with a variable input signal and the output signal across the resistor to the loudspeaker. Small changes in the voltage input to the transistor in the emitter circuit result in large changes in the voltage output in the collector circuit. The output is a loudspeaker with a resistance, R. The transistor amplifier shown is a voltage amplifier. Amplification occurs because the collector voltage is larger than the emitter voltage.

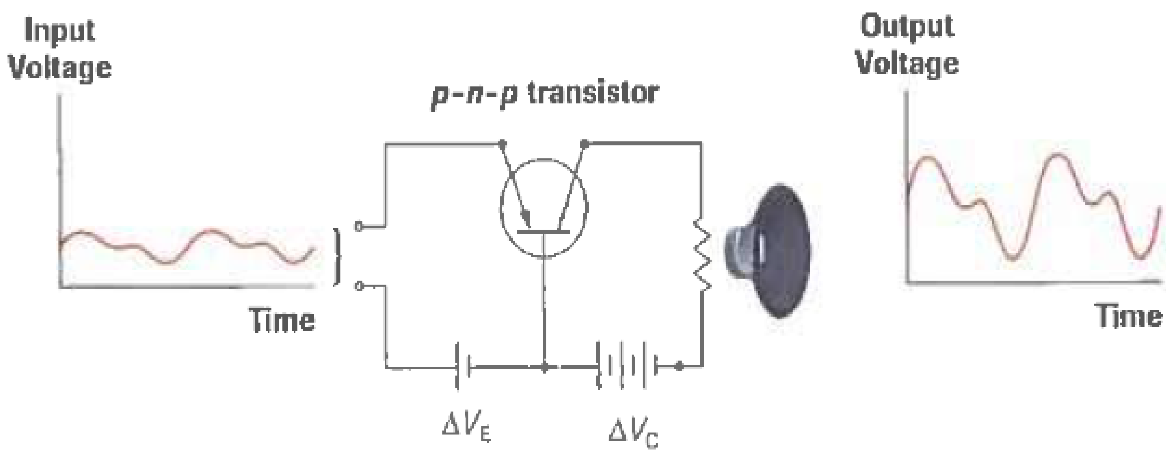


Fig.11.8.16: Transistor amplifies small input voltage into large output voltage

11.9 Summary

Coulomb's law describes the electrostatic force (or electric force) between two charged articles. If the particles have charges q_1 and q_2, are separated by distance r, and are at rest then the magnitude of the force acting on each due to the other is given by

$$\vec{F} = \frac{k_e |q_1||q_2|}{r^2}$$

The *electric field* is defined as $\vec{E} = \vec{F} / q_0$

The electric field vector $\vec{E}$ is tangent to the electric field lines at every point.

Electric Potential The electric potential *V* at a point *P* in the electric field of a charged object is

$$V = \frac{-W_\infty}{q_0} = \frac{PE}{q_0}$$

The average electric current I_{av} in a conductor is defined as $I_{av} = \Delta Q / \Delta t$.

Resistance of a Conductor The resistance *R* of a conductor is defined as $R = V / I$.

Ohm's law describes many conductors for which the applied voltage is directly proportional to the current it causes. The proportionality constant is the resistance:

$$V = IR$$

The power delivered to a resistor can be expressed as

$$P = I^2 R = V^2 / R.$$

The terminal voltage $V = IR$ is given by

$$V = \varepsilon - Ir$$

For N resistors connected in series the equivalent resistance is given by

$$R_{eq} = \sum_{i=1}^{N} R_i$$

For N resistors connected in parallel the equivalent resistance is given by

$$1/R_{eq} = \sum_{i=1}^{N} 1/R_i$$

Kirchhoff's rules

1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
2. The sum of the potential differences across all the elements around any closed circuit loop must be zero.

Semiconductor atoms have four valence electrons. Both germanium (Ge) and silicon (Si) are examples of semiconductor materials.

A pure semiconductor material with only one type of atom is called an intrinsic semiconductor. An intrinsic semiconductor is neither a good conductor nor a good insulator.

An extrinsic semiconductor is a semiconductor with impurity atoms added to it through a process known as doping. Doping increases the conductivity of a semiconductor material.

A diode is a unidirectional device that allows current to flow through it in only one direction.

Majority current carrier is the dominant type of charge carrier in a doped semiconductor material. In an n –type semiconductor, free electrons are the majority current carriers, whereas in a p -type semiconductor, holes are the majority current carriers.

Minority current carrier is the type of charge carrier that appears sparsely throughout a doped semiconductor material. In an n -type semiconductor, holes are the minority current carriers, whereas free electrons are the minority current carriers in a p –type semiconductor

n -type semiconductor is a semiconductor that has been doped with pentavalent impurity atoms. The result is a large number of free electrons throughout the material. Since the electron is the basic particle of negative charge, the material is called n -type semiconductor material.

p -type semiconductor is a semiconductor that has been doped with trivalent impurity atoms. The result is a large number of holes in the material. Since a hole exhibits a positive charge, the material is called p -type semiconductor material.

A diode is forward-biased by making its anode positive relative to its cathode. A diode is reverse-biased by making its anode negative relative to its cathode.

A forward-biased diode has relatively low resistance, whereas a reverse biased diode has very high resistance.

A bipolar junction transistor (BJT) consists of three regions: emitter, base, and collector. A terminal is connected to each of the three regions.

The base is a very thin and lightly doped region that is sandwiched between the emitter and collector regions.

The emitter region is the most heavily doped region in a transistor. Its function is to emit or inject current carriers into the base region.

The collector region is moderately doped and is the largest of all three transistor regions. Most of the current carriers injected into the base are attracted into the collector region rather than flowing out from the base lead.

11.10 Conceptual Questions

1. Figure 11-34 shows four situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?

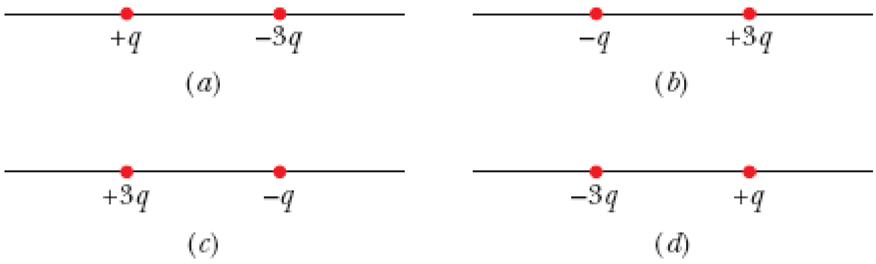


Fig.11.34

2. In fair weather, there is an electric field at the surface of the Earth, pointing down into the ground. What is the sign of the electric charge on the ground in this situation?
3. Figure 11.35 shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point *A* and is then accelerated through point *B* by the electric field. Points *A* and *B* have equal separations in the three arrangements. Rank the arrangements according to the linear momentum of the proton at point *B*, greatest first.

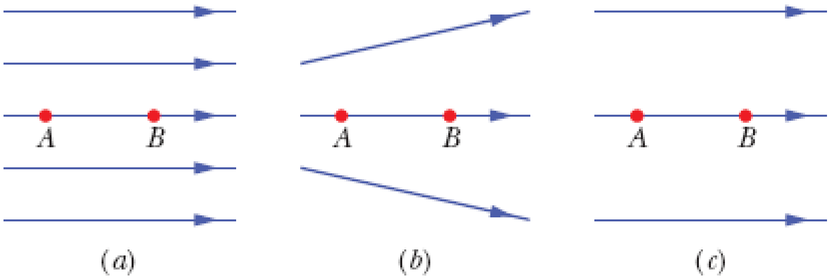
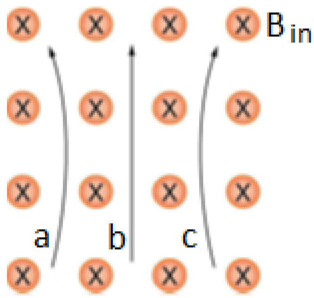


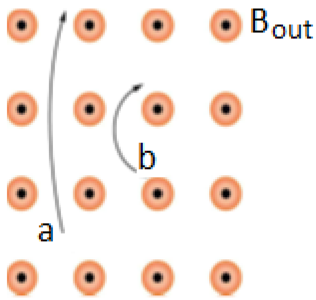
Fig.11.35

4. True or False: If a proton and electron both move through the same displacement in an electric field, the change in potential energy associated with the proton must be equal in magnitude and opposite in sign to the change in potential energy associated with the electron.
5. When is more power delivered to a light bulb, immediately after it is turned on and the glow of the filament is increasing or after it has been on for a few seconds and the glow is steady?
6. Suppose the energy transferred to a dead battery during charging is *W*. The recharged battery is then used until fully discharged again. Is the total energy transferred out of the battery during use also *W*?
7. List the ways in which magnetic field lines and electric field lines are similar.
8. Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

9. If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?
10. How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?
11. What are the signs of the charges on the particles (a, b, c) in the figure below.

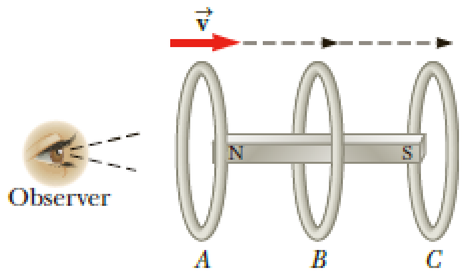


12. Which of the particles (a or b) in the figure below has the greatest velocity, the greatest mass, assuming they have identical charges and masses?



13. While operating, a TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.
14. If you have three parallel wires in the same plane with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.
15. Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right-hand rules.
16. Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?
17. Does dropping a magnet down a copper tube produce a current in the tube? Explain.
18. A bar magnet is held stationary while a circular loop of wire is moved toward the magnet at constant velocity at position A as shown in the figure below. The loop passes over the magnet's center at position B and moves away from the magnet at position C. Viewing the loop from the left as indicated in the figure, find the direction of the induced current in the loop (a) at position

A and (b) at position C. (c) What is the induced current in the loop at position B? Indicate the directions as either CW (for clockwise) or CCW (for counterclockwise).

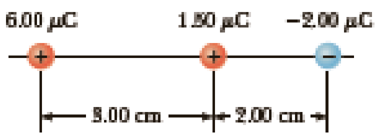


19. What is meant by the term intrinsic?
20. How are holes created in an intrinsic semiconductor?
21. Why is current established more easily in a semiconductor than in an insulator?
22. How is an n-type semiconductor formed? How is a p-type semiconductor formed?
23. What are majority carriers?
24. What is a pn junction?
25. When p and n regions are joined, a depletion region forms. Describe the characteristics of the depletion region.
26. Name the two bias conditions.
27. Which bias condition produces majority carrier current?
28. Which bias condition produces a widening of the depletion region?
29. Minority carriers produce the current during reverse breakdown. (True or False)

11.11 Problems

1. Suppose that in a lightning flash the potential difference between a cloud and the ground is $1.0 \times 10^9 V$and the quantity of charge transferred is 30 C. (a) what is the change in energy of that transferred charge? (b) If all the energy released could be used to accelerate a 1000 kg car from rest, what would be its final speed?
2. Calculate the magnitude and direction of the Coulomb force on each of the three charges shown in Figure 11.36.

Fig. 11.36



3. An electric field of magnitude $5.25 \times 10^5 N/C$ points due south at a certain location. Find the magnitude and direction of the force on a $26.0 \mu C$ charge at this location.

4. A particle of mass $1.0 \times 10^{-9} kg$ and charge $3.0 pC$ is moving in a vacuum chamber where the electric field has magnitude $2.0 \times 10^{3} N/C$ and is directed straight upward. Neglecting other forces except gravity, calculate the particle's (a) acceleration and (b) velocity after 2.00 s if it has an initial velocity of 5.00 m/s in the downward direction.
5. Two 20.0Ω resistors are connected in parallel and this group is connected in series with a 4.0Ω resistor. What is the total resistance of the circuit?
6. Four resistors are connected to a battery with a terminal voltage of 12 V, as shown in Figure 11.37. (a) How would you reduce the circuit to an equivalent single resistor connected to the battery? Use this procedure to find the equivalent resistance of the circuit. (b) Find the current delivered by the battery to this equivalent resistance. (c) Determine the power delivered by the battery. (d) Determine the power delivered to the 50.0Ω resistor.

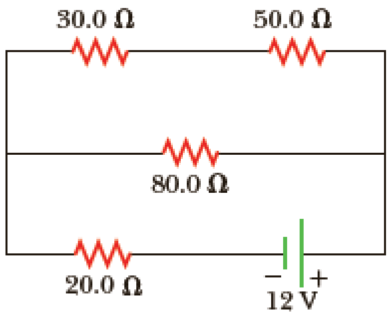


Fig. 11.37

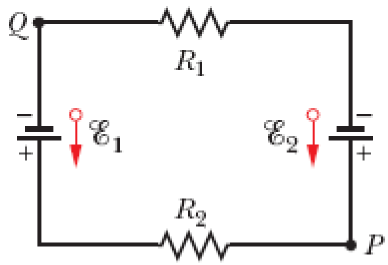
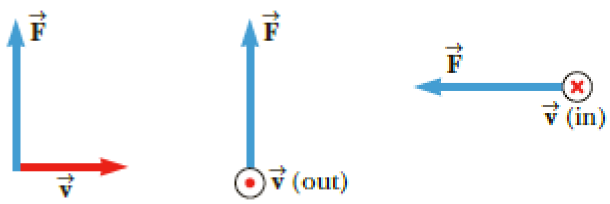


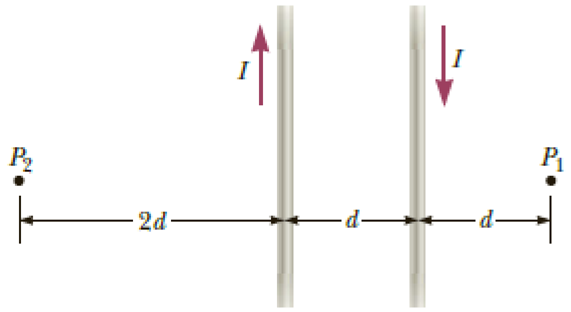
Fig.11.38

7. In Fig. 11.38, the ideal batteries have $\varepsilon_1 = 150.0V$ and $\varepsilon_2 = 50.0V$ and the resistances are $R_1 = 3.0\Omega$ and $R_2 = 2.0\Omega$. If the potential at P is 100 V, what is it at Q?
8. Four 18.0Ω resistors are connected in parallel across a 25.0 V ideal battery. What is the current through the battery?
9. Two 560.0Ω resistors are placed in series across a 400 V supply. Calculate the current drawn.
10. When four identical hotplates on a cooker are all in use, the current drawn from a 240 V supply is 33 A. Calculate (a) the resistance of each hotplate, (b) the current drawn when only three plates are switched on. The hotplates are connected in parallel.
11. Find the direction of the magnetic field acting on the positively charged particle moving in the various situations shown in the figures below if the direction of the magnetic force acting on it is as indicated.



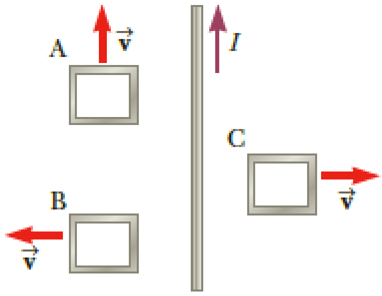
12. A proton moves perpendicular to a uniform magnetic field **B** at a speed of 1.00×10^{7} m/s and undergoes an acceleration of 2.00×10^{13} m/s^2 in the positive x-direction when its velocity is in the positive z-direction. Determine the magnitude and direction of the field.

13. A proton (charge 1e, mass m_p), a deuteron (charge 1e, mass $2m_p$), and an alpha particle (charge 12e, mass $4m_p$) are accelerated from rest through a common potential difference ΔV. Each of the particles enters a uniform magnetic field **B** with its velocity in a direction perpendicular to **B**. The proton moves in a circular path of radius r_p. In terms of r_p, determine (a) the radius r_d of the circular orbit for the deuteron and (b) the radius r_α for the alpha particle.
14. A current $I = 15$ A is directed along the positive x-axis and perpendicular to a magnetic field. A magnetic force per unit length of 0.12 N/m acts on the conductor in the negative y-direction. Calculate the magnitude and direction of the magnetic field in the region through which the current passes.
15. A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?
16. A wire is formed into a circle having a diameter of 10.0 cm and is placed in a uniform magnetic field of 3.00 mT. The wire carries a current of 5.00 A. Find the maximum torque on the wire.
17. The two wires shown in the figure are separated by $d = 510.0$ cm and carry currents of $I = 5\,5.00$ A in opposite directions. Find the magnitude and direction of the net magnetic field (a) at a point midway between the wires; (b) at point P_1, 10.0 cm to the right of the wire on the right; and (c) at point P_2, $2d = 20.0$ cm to the left of the wire on the left.



18. A wire with a weight per unit length of 0.080 N/m is suspended directly above a second wire. The top wire carries a current of 30.0 A, and the bottom wire carries a current of 60.0 A. Find the distance of separation between the wires so that the top wire will be held in place by magnetic repulsion.
19. It is desired to construct a solenoid that will have a resistance of 5.00 V (at 20°C) and produce a magnetic field of 4.00×10^2 T at its center when it carries a current of 4.00 A. The solenoid is to be constructed from copper wire having a diameter of 0.500 mm. If the radius of the solenoid is to be 1.00 cm, determine (a) the number of turns of wire needed and (b) the length the solenoid should have.
20. A long, straight wire lies in the plane of a circular coil with a radius of 0.010 m. The wire carries a current of 2.0 A and is placed along a diameter of the coil. (a) What is the net flux through the coil? (b) If the wire passes through the center of the coil and is perpendicular to the plane of the coil, what is the net flux through the coil?
21. A 2.00 - m length of wire is held in an east–west direction and moves horizontally to the north with a speed of 15.0 m/s. The vertical component of Earth's magnetic field in this region is 40.0 mT directed downward. Calculate the induced emf between the ends of the wire and determine which end is positive.

22. Three loops of wire move near a long straight wire carrying a current as in the figure. What is the direction of the induced current, if any, in (a) loop A, (b) loop B, and (c) loop C?



12 Geometrical Optics

Learning outcome

After completing this section, students are expected to know about:

- the Ray Aspect of Light
- the Law of Reflection
- the Law of Refraction
- Image Formation by Lenses
- Image Formation by Mirrors

Introduction

We know that visible light is the type of electromagnetic waves to which our eyes respond. That knowledge still leaves many questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behaviour of visible light and other electromagnetic waves. In particular, optics is concerned with the generation and propagation of light and its interaction with matter. In this chapter we will concentrate on the propagation of light and its interaction with matter.

12.1 The Ray Aspect of Light

Learning outcome

After completing this section, students are expected to

- list the ways by which light travels from a source to another location.

There are three ways in which light can travel from a source to another location. (See Figure 12.1.) It can come directly from the source through empty space, such as from the Sun to Earth. Or light can travel through various media, such as air and glass, to the person. Light can also arrive after being reflected, such as by a mirror. In all of these cases, light is modelled as traveling in straight lines called rays. Light may change direction when it encounters objects (such as a mirror) or in passing from one material to another (such as in passing from air to glass), but it then continues in a straight line or as a ray. The word ray comes from mathematics and here means a straight line that originates at some point. It is acceptable to visualize light rays as laser rays (or even science fiction depictions of ray guns).

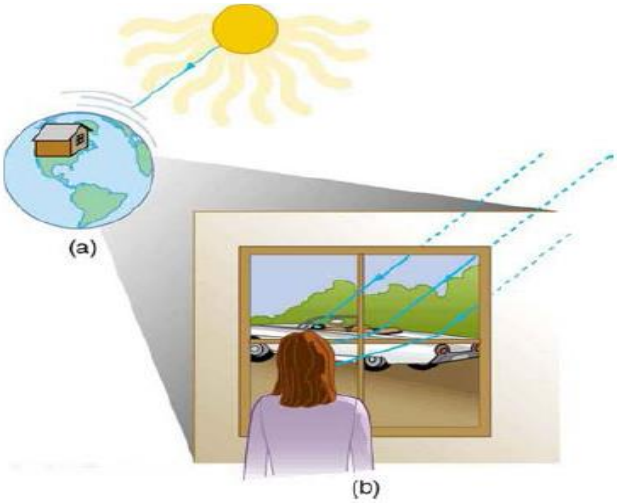


Figure 12.1: Three methods for light to travel from a source to another location. (a) Light reaches the upper atmosphere of Earth traveling through empty space directly from the source. (b) Light can reach a person in one of two ways. It can travel through media like air and glass. It can also reflect from an object like a mirror. In the situations shown here, light interacts with objects large enough that it travels in straight lines, like a ray.

When light interacts with objects or a medium, such as glass or water, it displays certain properties: it can be reflected, refracted, absorbed or transmitted.

12.2 Reflection and Refraction

12.2.1 Reflection

Learning outcome

After completing this section, students are expected to:

- explain reflection of light from polished and rough surfaces.
- understand the Law of reflection

Reflection of light is the bouncing back of light when it falls on opaque bodies. To describe the reflection of light, we will use the following terminology. The incoming light ray is called the incident ray. The light ray moving away from the surface is the reflected ray. The most important characteristic of these rays is their angles in relation to the reflecting surface. These angles are measured with respect to the normal of the surface. The normal is an imaginary line perpendicular to the surface. The angle of incidence, θ_i is measured between the incident ray and the surface normal. The angle of reflection, θ_r is measured between the reflected ray and the surface normal. This is shown in Figure 12.2.

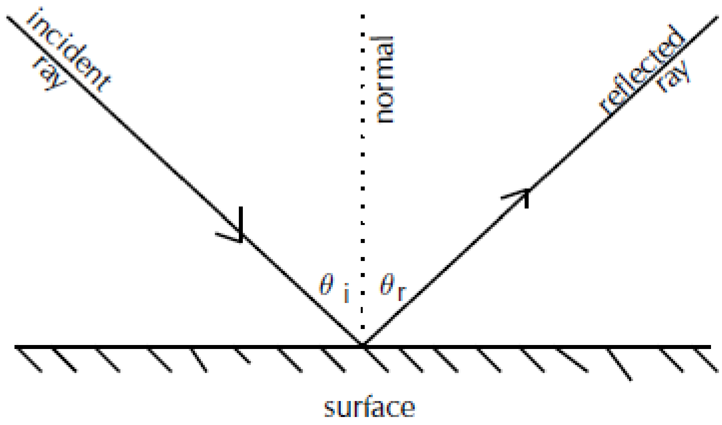


Figure 12.2: The angles of incidence and reflection are measured with respect to the normal to the smooth surface.

Definition: Law of Reflection

The angle of incidence is equal to the angle of reflection

$$\theta_i = \theta_f \qquad 12.1$$

and all the incident ray, reflected ray and the normal lie in the same plane.

We also expect to see reflections when rays strikes from rough surfaces, at different angles, it is reflected in many different directions, or diffused. Figure 12.3 illustrates how a rough surface reflects light. Here, the angle of incidence may or may not be equal to the angle of reflection.

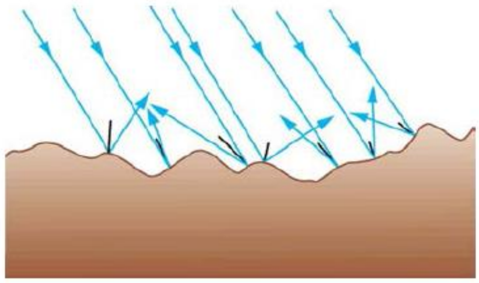


Figure 12.3: Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.

The simplest example of the law of reflection is if the angle of incidence is 0^0. In this case, the angle of reflection is also 0^0. You see this when you look straight into a mirror.

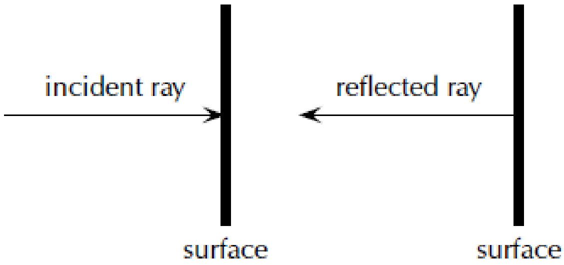


Figure 12.4: When a light ray strikes a surface at right angles to the surface, then the ray is reflected directly back.

Real world applications of reflection

A parabolic reflector is a mirror or dish (e.g. a satellite dish) which has a parabolic shape. Some examples of very useful parabolic reflectors are car headlamps, spotlights, telescopes and satellite dishes. In the case of car headlights or spotlights, the outgoing light produced by the bulb is reflected by a parabolic mirror behind the bulb so that it leaves as a collimated beam (i.e. all the reflected rays are parallel). The reverse situation is true for a telescope where the incoming light from distant objects arrives as parallel rays and is focused by the parabolic mirror to a point, called the focus, where an image can be made. The surface of this sort of reflector has to be shaped very carefully so that the rays all arrive at the same focal point.

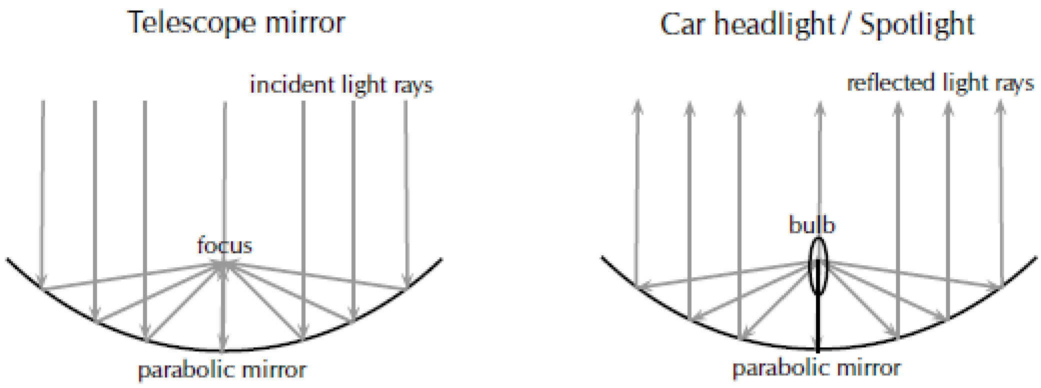
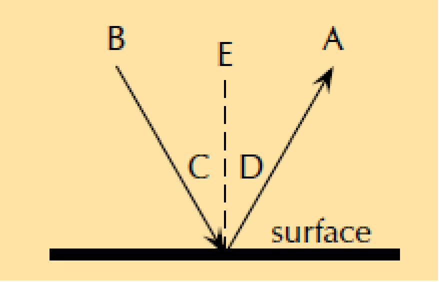


Figure 12.5: On the left is a ray diagram showing how a telescope mirror works to collect incoming incident light (parallel rays) from a distant object such as a star or galaxy and focus the rays to a point where a detector e.g. a camera, can make an image. The diagram on the right shows how the same kind of parabolic reflector can cause light coming from a car headlight or spotlight bulb to be collimated. In this case the reflected rays are parallel.

Exercise 12 – 1: Rays and Reflection

1. Are light rays real? Explain.
2. Which of the labels, A–H, in the diagram, correspond to the following:



a) normal

b) angle of incidence

c) angle of reflection

d) incident ray

e) reflected ray

3. State the Law of Reflection. Draw a diagram, label the appropriate angles and write a mathematical expression for the Law of Reflection.
4. A ray of light leaves a surface at 45_ to the normal to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
5. A ray of light strikes a surface at 25_ to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
6. A ray of light leaves a surface at 65_ to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.

12.2.2 Refraction

Learning outcome

After completing this section, students are expected to:

- determine the index of refraction, given the speed of light in a medium
- explain the phenomenon of total internal reflection.
- describe the workings and uses of fiber optics.

When light moves from one medium into another (for example, from air to glass), the speed of light changes. If the light ray hits the boundary of the new medium (for example the edge of a glass block) at any angle which is not perpendicular to or parallel with the boundary, the light ray will change its direction through the next medium, or appear to 'bend'. This is called refraction of light. It is important to note that while the speed of the light changes when it passes into the new medium, the frequency of the light remains the same.

The speed of light and therefore the degree of bending of the light depends on the refractive index of material through which the light passes. The refractive index (symbol n) is the ratio of the speed of light in a vacuum to its speed in the material.

$$n = \frac{c}{v} \qquad 12.2$$

Where,

n = refractive index (no unit)

c = speed of light in a vacuum (3 × 108 m·s−1)

v = speed of light in a given medium (m·s−1)

If the refractive index, n, increases, the speed of light in the material, v must decrease. Therefore light travels slower through materials of high refractive index, n.

Table 12.1 shows refractive indices for various materials. Light travels slower in any material than it does in a vacuum, so all values for n are greater than 1.

Table 12.1: Refractive indices of some materials. n_{air} is calculated at standard temperature and pressure (STP).

Medium	**Refractive Index**
Vacuum	1
Air*	1.0002926
Carbon dioxide	1.00045
Water: Ice	1.31
Water: Liquid (20^{o}C)	1.333
Ethyl Alcohol (Ethanol)	1.36
Glycerine	1.4729
Rock salt	1.516
Crown Glass	1.52
Sodium chloride	1.54
Glass (typical)	1.5 to 1.9
Diamond	2.419
Silicon	4,01
Zircon	1.923

Representing Refraction with Ray diagrams

Before we draw the diagrams we need to define the following important concepts.

Definitions:

The **normal** to a surface is the line which is perpendicular to the plane of the surface.

Incident Ray is the path along which the light is in the first medium.

Refracted Ray is the path along which the light is in the second medium.

Denser medium is a medium in which the speed of light is less.

Less Dense medium is a medium in which the speed of light is greater

The **angle of incidence** is the angle defined between the normal to a surface and the incoming (incident) light ray.

The **angle of refraction** is the angle defined between the normal to a surface and the refracted light ray.

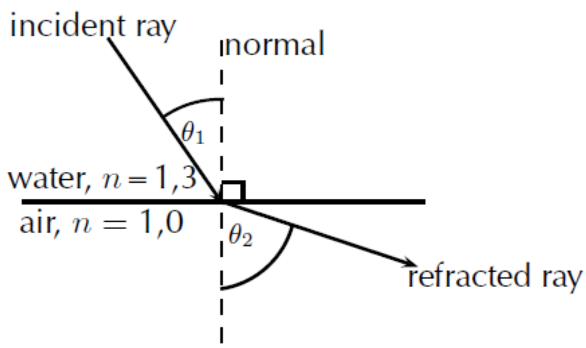


Figure 12.6: The diagram shows the boundary between two media: water (Denser medium) and air (Less Dense medium). An incoming light ray is refracted when it passes through the surface of the water into the air. The angle of incidence is Θ_1 and the angle of refraction is Θ_2.

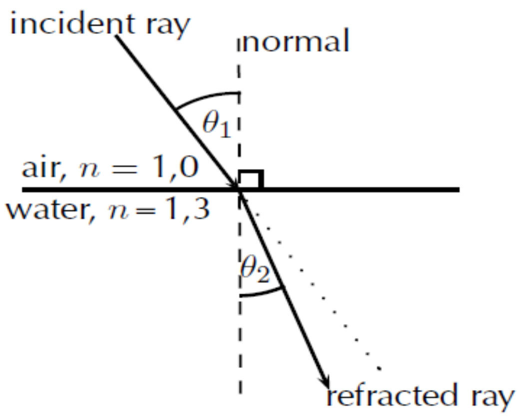


Figure 12.7: Light is moving from an optically less dense medium to an optically denser medium.

Example

Calculate the speed of light in zircon, a material used in jewellery to imitate diamond.

Strategy

The speed of light in a material, v, can be calculated from the index of refraction n of the material using the equation n = c / v.

Solution

Rearranging Eq. 12.2 to determine v gives

$$v = \frac{c}{n}.$$

The index of refraction for zircon is given as 1.923 in Table 12.1, and c is the speed of light. Entering these values in the last expression gives

$$v = \frac{3.00\ x\ 10^{8} m^{-1}}{1.923} = 1.56\ x\ 10^{8} m^{-1}$$

Snell's Law

Now that we know that the degree of bending, or the angle of refraction, is dependent on the refractive index of a medium, how do we calculate the angle of refraction? The answer to this question was discovered by a Dutch physicist called Willebrand Snell in 1621 and is now called Snell's Law or the Law of Refraction.

DEFINITION: Snell's law

$$n_1 \sin \Theta_1 = n_2 \sin \Theta_2 \qquad 12.3$$

Where

n_1 = Refractive index of material 1

n_2 = Refractive index of material 2

Θ_1 = Angle of incidence

Θ_2 = Angle of refraction

If $n_2 > n_1$, then from Snell's Law, $\sin \Theta_1 > \sin \Theta_2$. For angles smaller than 90°, $\sin \Theta_1$ increases as Θ_1 increases. Therefore, $\Theta_1 > \Theta_2$. This means that the angle of incidence is greater than the angle of refraction and the light ray is bent toward the normal.

Similarly, if $n_2 < n_1$ then from Snell's Law, $\sin \Theta_1 < \sin \Theta_2$. For angles smaller than 90°, $\sin \Theta_2$ increases as Θ_2 increases. Therefore, $\Theta_1 < \Theta_2$. This means that the angle of incidence is less than the angle of refraction and the light ray is bent away from the normal.

Example

Light is refracted at the boundary between water and an unknown medium. If the angle of incidence is 25°, and the angle of refraction is 20.6°, calculate the refractive index of the unknown medium and use Table 12.1 to identify the material.

Solution:

$$n_1 \sin \Theta_1 = n_2 \sin \Theta_2$$

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2}$$

$$n_2 = 1.333 \times \frac{\sin 25°}{\sin 20.6°}$$

$$n_2 = 1.66$$

According to Table 5.1, typical glass has a refractive index between 1.5 to 1.9. Therefore, the unknown medium is typical glass.

Critical angles and total internal reflection

DEFINITION: Critical angle

The critical angle is the angle of incidence where the angle of refraction is 90°. The light must travel from an optically denser medium to an optically less dense medium.

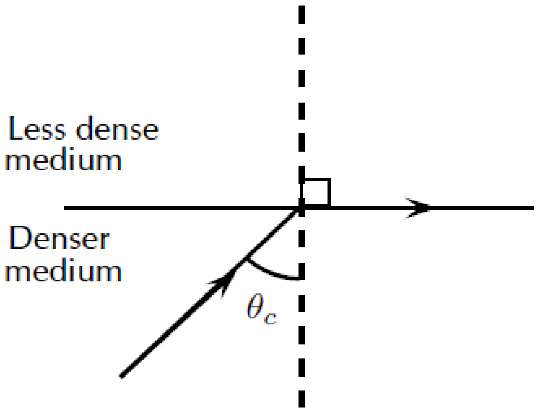


Figure 12.8: When the angle of incidence is equal to the critical angle, the angle of refraction is equal to 90°.

Total internal reflection

If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called total internal reflection.

The conditions for total internal reflection are:

1. light is travelling from an optically denser medium (higher refractive index) to an optically less dense medium (lower refractive index).
2. the angle of incidence is greater than the critical angle.

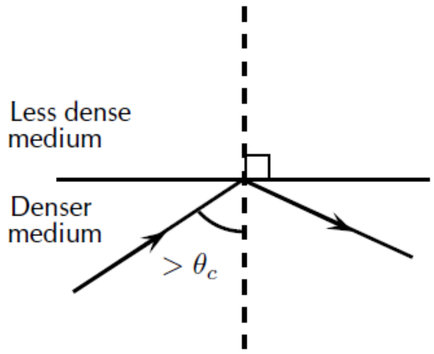


Figure 12.9: When the angle of incidence is greater than the critical angle, the light ray is reflected at the boundary of the two media and total internal reflection occurs.

Example

Given that the refractive indices of air and water are 1.00 and 1.33 respectively, find the critical angle.

Given:

$$n_2 = 1.00, \qquad n_1 = 1.33, \qquad \theta_2 = 90°, \qquad \theta_1 = \theta_c$$

Calculate $\theta_c = \theta_1$.

Solution:

sin Θc = n2 sin 90o

= n2 sin 90o / n1

Θc= sin-1(n2 sin 90o / n1)

= sin-1(1 /1.33)

= 48.8o

For incident angles smaller than 48.8o refraction will occur. For incident angles greater than 48.8°, total internal reflection will occur. For incident angles equal to 48.8° refraction will occur at 90°. The following ray diagrams show the path of light in each situation and also the situation where light is travelling from an optically less dense medium (higher refractive index) to an optically dense medium (lower refractive index).

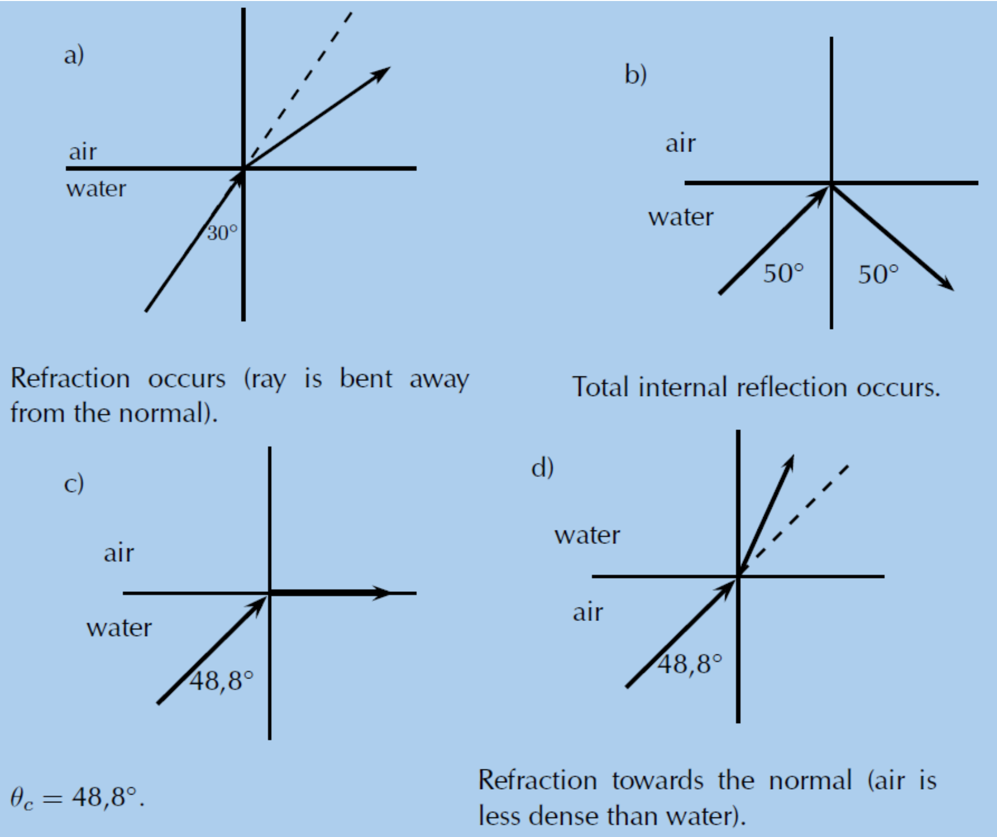


Figure 12.10: Ray diagrams show the path of light based on the above example and also the situation where light is travelling from an optically less dense to an optically dense medium.

Total internal reflection is a very useful natural phenomenon since it can be used to confine light. One of the most common applications of total internal reflection is in fibre optics. An optical fibre is a thin, transparent fibre, usually made of glass or plastic, for transmitting light. Optical fibres are usually thinner than a human hair! The construction of a single optical fibre is shown in Figure 12.11.

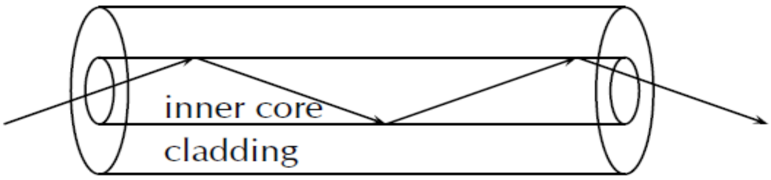


Figure 12.11: Structure of a single optical fiber.

Optical fibres are most common in telecommunications, because information can be transported over long distances, with minimal loss of data. This gives optical fibres an advantage over conventional cables. Optic fibres are also used in medicine in endoscopes.

12.3 Image formation by thin Lenses and Mirrors

Learning outcomes

After completing this section, students are expected to:

- describe the formation of images using the rules of ray tracking.
- determine the power of a lens given the focal length.
- illustrate image formation in mirrors.
- determine focal length and magnification of a lens.

This section covers the formation of images when plane and spherical light waves fall on plane and spherical surfaces. Images can be formed by reflection from mirrors or by refraction through lenses. In our study of mirrors and lenses, we continue to assume light travels in straight lines (the ray approximation), ignoring diffraction.

12.3.1 Image Formation by Lenses

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera' s zoom lens. In this section, we will use the law of refraction to explore the properties of lenses and how they form images.

The Converging (or convex) lens

The shape of which is similar to the convex lens in Figure 12.12. The convex lens shown has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. (The axis is defined to be a line normal to the lens at its center, as shown in Figure 12.12.) Such a lens is called a converging (or convex) lens for the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown, to illustrate how the ray changes direction both as it enters and as it leaves the lens. Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) Due to the lens' s shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined to be the focal point F of the lens. The distance from the center of the lens to its focal point is defined to be the focal length f of the lens.

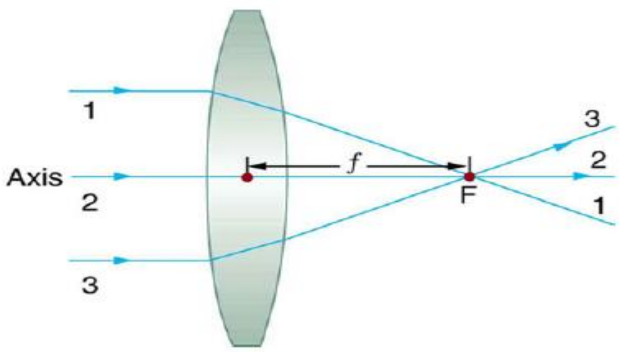


Figure 12.12: Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to it and will have a smaller focal length than a weak lens. The light will also focus into a smaller and more intense spot for a more powerful lens. The power P of a lens is defined to be the inverse of its focal length. In equation form, this is

$$P = \frac{1}{f} \qquad 12.4$$

Where, f is the focal length of the lens, which must be given in meters (and not cm or mm). The power of a lens P has the unit diopters (D), provided that the focal length is given in meters. That is, 1 D = 1 / m. (Note that this power (optical power, actually) is not the same as power in watts defined in Work, Energy, and Energy Resources. It is a concept related to the effect of optical devices on light.) Optometrists prescribe common spectacles and contact lenses in units of diopters.

The diverging (or concave) lens

The concave lens is a diverging lens; because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so that all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of

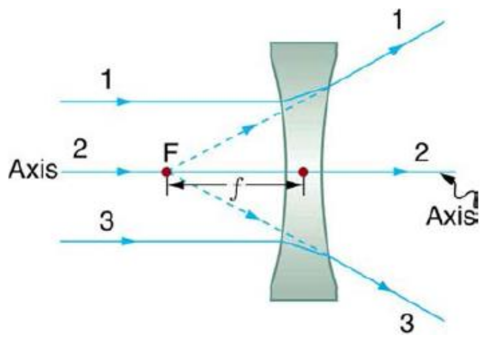


Figure 12.13: Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point F. The dashed lines are not rays–they indicate the directions from which the rays appear to come. The focal length f of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Thin Lenses

A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.

A thin lens is defined to be one whose thickness allows rays to refract, as illustrated in Figure 12.12, but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See Figure 12.14.) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in Figure 12.15.

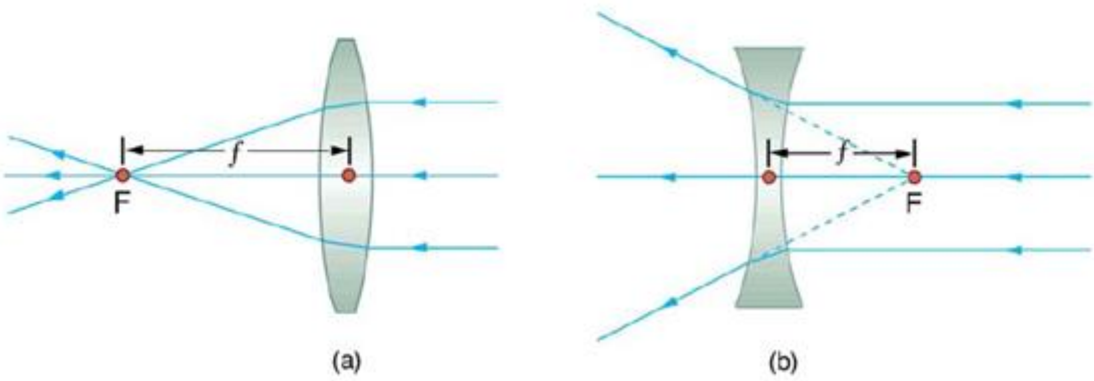


Figure 12.14: Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.

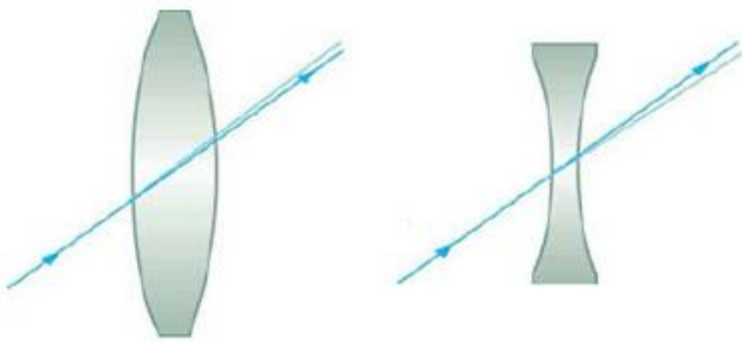


Figure 12.15: The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge parallel to its original path (shown as a shaded line).

Rules for Ray Tracing

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side.
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F.
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its axis.

Image Formation by Thin Lenses

In some circumstances, a lens forms an obvious image, such as when a movie projector casts an image onto a screen. In other cases, the image is less obvious. Where, for example, is the image formed by eyeglasses? We will use ray tracing for thin lenses to illustrate how they form images, and we will develop equations to describe the image formation quantitatively.

Image Formation by a converging lens

Case 1: when an object is held farther to a converging lens than its focal length

Consider an object some distance away from a converging lens, as shown in Figure 12.15. To find the location and size of the image formed, we trace the paths of selected light rays originating from one point on the object, in this case the top of the person's head. The figure shows three rays from the top of the object that can be traced using the ray tracing rules given above. (Rays leave this point going in many directions, but we concentrate on only a few with paths that are easy to trace.) The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). The three rays cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. All rays that come from the same point on the top of the person's head are refracted in such a way as to cross at the point shown. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in Figure 12.16, only two are necessary to locate the image. It is best to trace rays for which there are simple ray tracing rules. Before applying ray tracing to other situations, let us consider the example shown in Figure 12.16 in more detail.

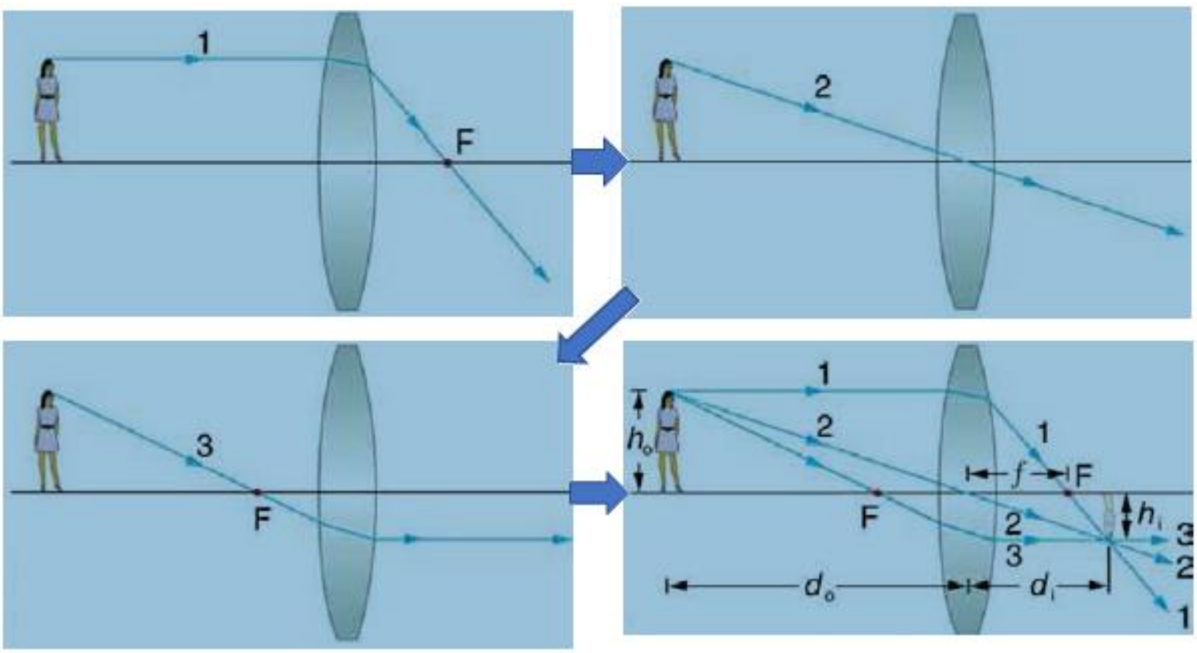


Figure 12.16: Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced–the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image–one that can be projected on a screen–is formed.

The image formed in Figure 12.16 is a real image, meaning that it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye.

Several important distances appear in Figure 12.16. We define d_o to be the object distance, the distance of an object from the center of a lens. Image distance d_i is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o and h_i, respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in Figure 12.15, we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of equations that can be derived from a geometric analysis of ray tracing for thin lenses. The thin lens equations are

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad 12.5$$

and

$$\frac{d_i}{d_o} = -\frac{d_o}{d_i} = m \qquad 12.6$$

Where m is defined as the magnification and is equal to the ratio of image height to object height (h_i / h_o). (The minus sign in the equation above will be discussed shortly.) The thin lens equations are broadly applicable to all situations involving thin lenses (and "thin" mirrors, as we will see later). We will explore many features of image formation in the following worked examples.

Case 2: when an object is held closer to a converging lens than its focal length

The image formed when an object is held closer to a converging lens than its focal length is upright, magnified, virtual image (see Figure 12.18)

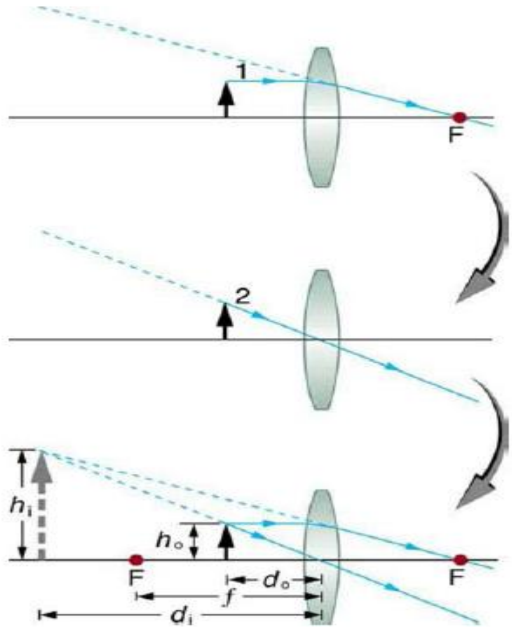


Figure 12.18: Ray tracing predicts the image location and size for an object held closer to a converging lens than its focal length. Ray 1 enters parallel to the axis and exits through the focal point on the opposite side, while ray 2 passes through the center of the lens without changing path. The two rays continue to diverge on the other side of the lens, but both appear to come from a common point, locating the upright, magnified, virtual image. This is a case 2 image.

Case 3: when image is formed by a diverging

A third type of image is formed by a diverging or concave lens. The image that is upright but smaller than the object. This means that the magnification is positive but less than 1. The ray diagram in Figure 12.19 shows that the image is on the same side of the lens as the object and, hence, cannot be projected–it is a virtual image. Note that the image is closer to the lens than the object.

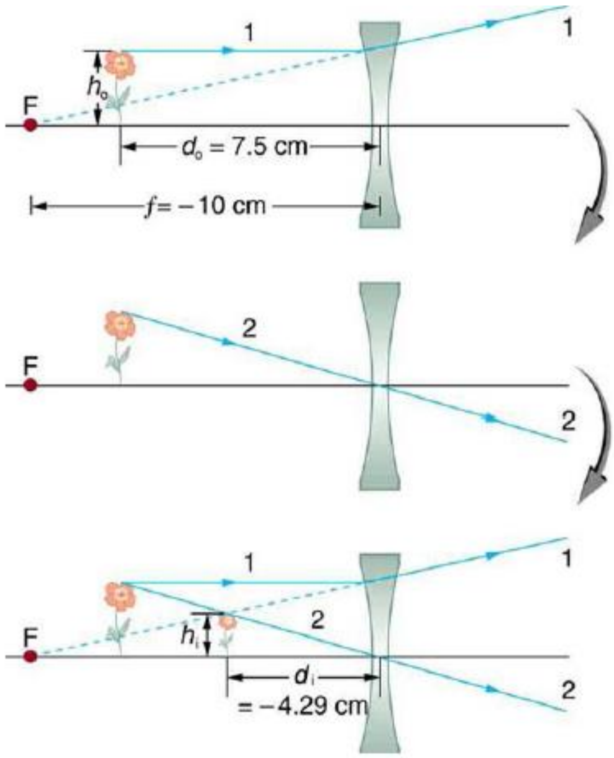


Figure 12.19: Ray tracing predicts the image location and size for a concave or diverging lens. Ray 1 enters parallel to the axis and is bent so that it appears to originate from the focal point. Ray 2 passes through the center of the lens without changing path. The two rays appear to come from a common point, locating the upright image. This is a case 3 image, which is closer to the lens than the object and smaller in height.

Table 12.2: summarizes the three types of images formed by single thin lenses. These are referred to as case 1, 2, and 3 images. Convex (converging) lenses can form either real or virtual images (cases 1 and 2, respectively), whereas concave (diverging) lenses can form only virtual images (always case 3).

Case	Formed When	Image Type	d_i	m
1	f positive, $d_o > f$	real	positive	negative
2	f positive, $d_o < f$	virtual	negative	Positive $m > 1$
3	F negative	virtual	negative	Positive $m < 1$

12.3.2 Image Formation by Mirrors

Images in Plane Mirrors

Figure 12.19 helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and being reflected into the observer's eye. The rays can diverge slightly, and both still get into the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, locating the image. (The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror.) Using the law of reflection–the angle of reflection equals the angle of incidence–we can see that the image and object are the same distance from the mirror $d_{i,} = d_{o.}$ Also, the image and object heights are the same and hence the magnification, M, which is the ratio of image size to object size becomes one.

The image formed by a flat mirror is a virtual, since it cannot be projected–the rays only appear to originate from a common point behind the mirror. Obviously, if you walk behind the mirror, you cannot see the image, since the rays do not go there. But in front of the mirror, the rays behave exactly as if they had come from behind the mirror, so that is where the image is situated.

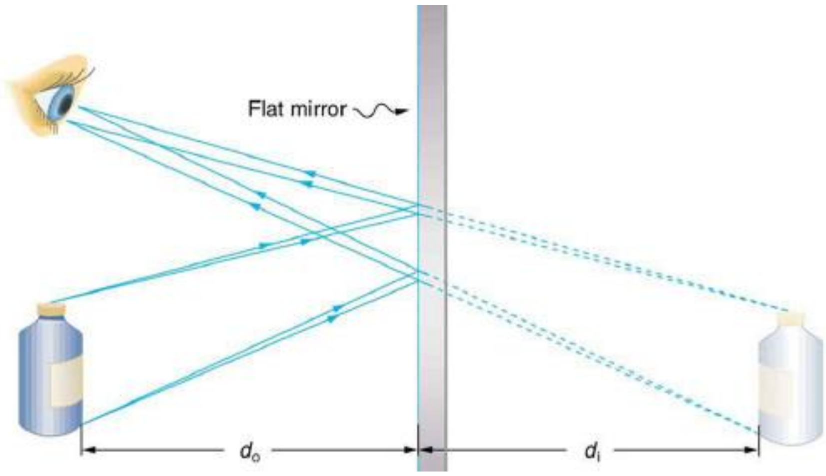


Figure 12.19: Two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer. The reflected rays seem to originate from behind the mirror, locating the virtual image.

Spherical Mirrors

Consider the focal length of a mirror–for example, the concave spherical mirrors in Figure 12.20. Rays of light that strike the surface follows the law of reflection. For a mirror that is large compared with its radius of curvature, as in Figure 12.20(a), we see that the reflected rays do not cross at the same point, and the mirror does not have a well-defined focal point. If the mirror had the shape of a parabola, the rays would all cross at

a single point, and the mirror would have a well-defined focal point. But parabolic mirrors are much more expensive to make than spherical mirrors. The solution is to use a mirror that is small compared with its radius of curvature, as shown in Figure 12.20(b). (This is the mirror equivalent of the thin lens approximation.) To a very good approximation, this mirror has a well-defined focal point at F that is the focal distance f from the center of the mirror. The focal length f of a concave mirror is positive, since it is a converging mirror.

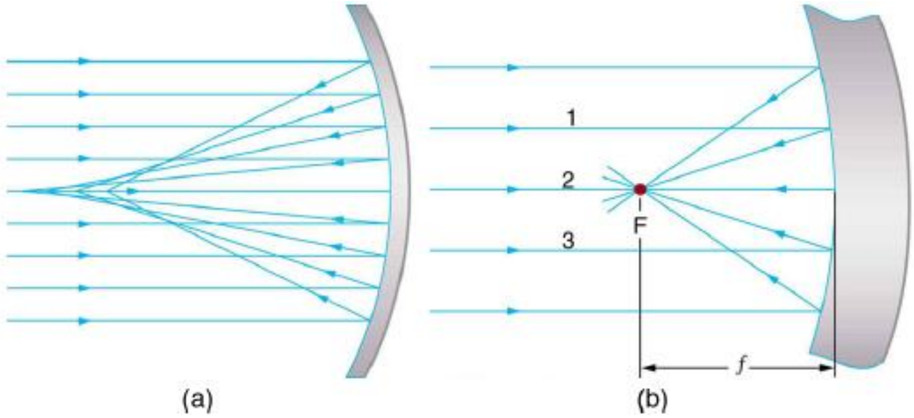


Figure 12.20: (a) Parallel rays reflected from a large spherical mirror do not all cross at a common point. (b) If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point.

The distance of the focal point from the center of the mirror is its focal length f. Since this mirror is converging, it has a positive focal length. Just as for lenses, the shorter the focal length, the more powerful the mirror; thus, P = 1 / f for a mirror, too. A more strongly curved mirror has a shorter focal length and a greater power. Using the law of reflection and some simple trigonometry, it can be shown that the focal length is half the radius of curvature, or

$$f = \frac{R}{2}. \qquad 12.7$$

Where, R is the radius of curvature of a spherical mirror. The smaller the radius of curvature, the smaller the focal length and, thus, the more powerful the mirror is.

The convex mirror shown in Figure 12.21 also has a focal point. Parallel rays of light reflected from the mirror seem to originate from the point F at the focal distance f behind the mirror. The focal length and power of a convex mirror are negative, since it is a diverging mirror.

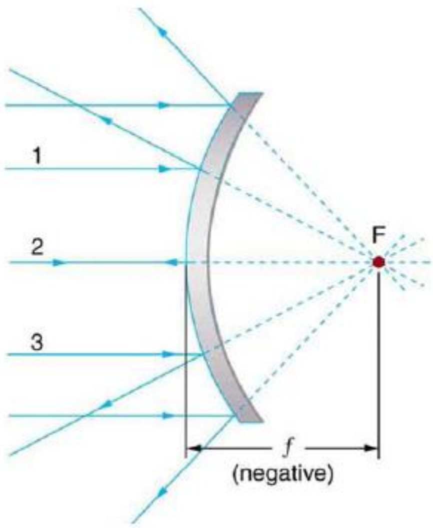


Figure 12.21: Parallel rays of light reflected from a convex spherical mirror (small in size compared with its radius of curvature) seem to originate from a well-defined focal point at the focal distance f behind the mirror. Convex mirrors diverge light rays and, thus, have a negative focal length.

Ray tracing is as useful for mirrors as for lenses. The rules for ray tracing for mirrors are based on the illustrations just discussed:

1. A ray approaching a concave converging mirror parallel to its axis is reflected through the focal point F of the mirror on the same side. (See rays 1 and 3 in Figure 12.20(b).)
2. A ray approaching a convex diverging mirror parallel to its axis is reflected so that it seems to come from the focal point F behind the mirror. (See rays 1 and 3 in Figure 12.21.)
3. Any ray striking the center of a mirror is followed by applying the law of reflection; it makes the same angle with the axis when leaving as when approaching. (See ray 2 in Figure 12.22.)
4. A ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (The reverse of rays 1 and 3 in Figure 12.20.)
5. A ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis. (The reverse of rays 1 and 3 in Figure 12.21.)

We will use ray tracing to illustrate how images are formed by mirrors, and we can use ray tracing quantitatively to obtain numerical information. But since we assume each mirror is small compared with its radius of curvature, we can use the thin lens equations for mirrors just as we did for lenses.

Consider the situation shown in Figure 12.22, concave spherical mirror reflection, in which an object is placed farther from a concave (converging) mirror than its focal length. That is, f is positive and do > f, so that we may expect an image similar to the case 1 real image formed by a converging lens. Ray tracing in Figure 12.22 shows that the rays from a common point on the object all cross at a point on the same side of the mirror as the object. Thus a real image can be projected onto a screen placed at this location. The image distance is positive, and the image is inverted, so its magnification is negative. This is a case 1 image for mirrors. It differs from the case 1 image for lenses only in that the image is on the same side of the mirror as the object. It is otherwise identical.

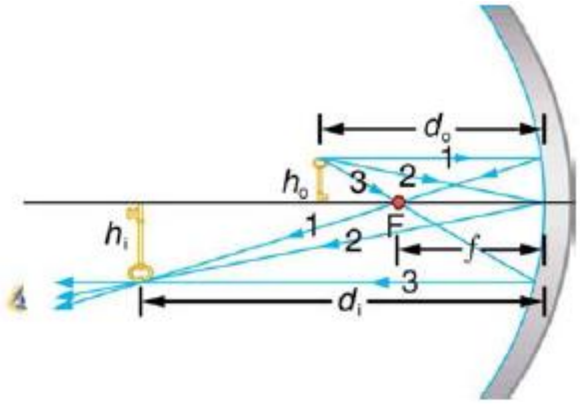


Figure 12.22: A case 1 image for a mirror. An object is farther from the converging mirror than its focal length. Rays from a common point on the object are traced using the rules in the text. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 goes through the focal point on the way toward the mirror. All three rays cross at the same point after being reflected, locating the inverted real image. Although three rays are shown, only two of the three are needed to locate the image and determine its height.

If an object is closer to a concave mirror than its focal length (do < f and f positive), which is a magnifier, rays from a common point on the object are reflected in such a manner that they appear to be coming from behind the mirror, meaning that the image is virtual and cannot be projected. As with a magnifying glass, the image is upright and larger than the object. This is a case 2 image for mirrors and is exactly analogous to that for lenses.

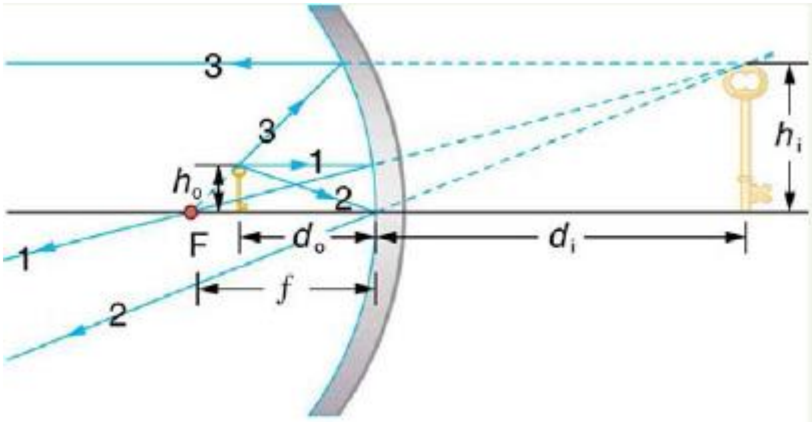


Figure 12.23: Images formed when a converging mirror has an object closer to it than its focal length. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches the mirror as if it came from the focal point.

All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be larger than the object. Makeup mirrors are perhaps the most common use of a concave mirror to produce a larger, upright image.

A convex mirror is a diverging mirror (f is negative) and forms only one type of image. The image formed is upright and smaller than the object, just as for diverging lenses. Figure 25.46(a) uses ray tracing to illustrate the location and size of the case 3 image for mirrors. Since the image is behind the mirror, it cannot be projected and is thus a virtual image. It is also seen to be smaller than the object.

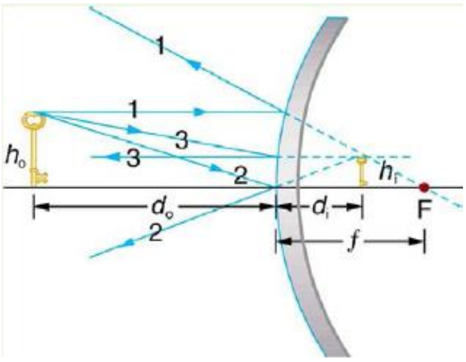


Figure 12.24: Case 3 images for mirrors are formed by any convex mirror. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches toward the focal point. All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be smaller than the object.

Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared to what would be observed for a flat mirror (and hence security is improved).

Example

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in Figure 25.35. Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.

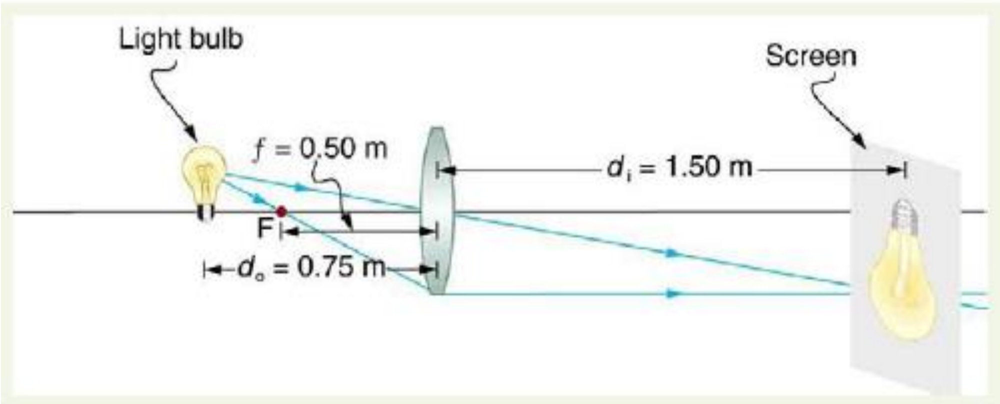


Figure 12.17: A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

Solutions

Rearranging the thin lens equations to isolate d_i gives:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Noting that d_o = 0.750 m and f = 0.500 m.

$$\frac{1}{d_i} = \frac{0.667}{m}$$

This must be inverted to find d_i:

$$d_i = 1.50m$$

The magnification m is then obtained using the values of both d_i and d_o as.

$$m = \frac{d_i}{d_o} = -\frac{1.50m}{0.750m} = -2.00$$

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

12.4 Chapter Summary

The Ray Aspect of Light

- A straight line that originates at some point is called a ray.
- The part of optics dealing with the ray aspect of light is called geometric optics.
- Light can travel in three ways from a source to another location: (1) directly from the source through empty space; (2) through various media; (3) after being reflected from a mirror.

The Law of Reflection

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

The Law of Refraction

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum c = 2.99792458×108 m/s ≈ 3.00×108 m/s.
- Index of refraction $n = \frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as n1 sin θ 1 = n2 sin θ 2.

Total Internal Reflection

- The incident angle that produces an angle of refraction of 90^0 is called critical angle.
- Total internal reflection is a phenomenon that occurs at the boundary between two mediums, such that if the incident angle in the first medium is greater than the critical angle, then all the light is reflected back into that medium.
- Fiber optics involves the transmission of light down fibers of plastic or glass, applying the principle of total internal reflection.

Image formation by Lenses

- Light rays entering a converging lens parallel to its axis cross one another at a single point on the opposite side.
- For a converging lens, the focal point is the point at which converging light rays cross; for a diverging lens, the focal point is the point from which diverging light rays appear to originate.
- The distance from the center of the lens to its focal point is called the focal length f.

- Power P of a lens is defined to be the inverse of its focal length,

$$p = \frac{1}{f}$$

- A lens that causes the light rays to bend away from its axis is called a diverging lens.
- Ray tracing is the technique of graphically determining the paths that light rays take.
- The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.
- Thin lens equations are $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ and $\frac{h_i}{h_o} = -\frac{d_i}{d_o}$ = m (magnification).
- The distance of the image from the center of the lens is called image distance.
- An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Image Formation by Mirrors

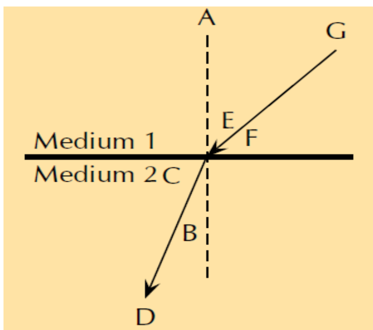
- The characteristics of an image formed by a flat mirror are: (a) The image and object are the same distance from the mirror, (b) The image is a virtual image, and (c) The image is situated behind the mirror.
- Image length is half the radius of curvature.

$$f = \frac{R}{2}$$

- A convex mirror is a diverging mirror and forms only one type of image, namely a virtual image.

12.5 Conceptual Questions

1. Using the law of reflection, explain how powder takes the shine off of a person' s nose. What is the name of the optical effect?
2. Explain refraction in terms of a change of wave speed in different media.
3. In the diagram, label the following:
    a. angle of incidence
    b. angle of refraction
    c. incident ray
    d. refracted ray
    e. normal



4. Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.
5. Why is the index of refraction always greater than or equal to 1?
6. Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.
7. Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?
8. Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?
9. Explain why a person’s legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.
10. A ring with a colorless gemstone is dropped into water. The gemstone becomes invisible when submerged. Can it be a diamond? Explain.
11. A high-quality diamond may be quite clear and colorless, transmitting all visible wavelengths with little absorption. Explain how it can sparkle with flashes of brilliant color when illuminated by white light.
12. It can be argued that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are di and do related?
13. You can often see a reflection when looking at a sheet of glass, particularly if it is darker on the other side. Explain why you can often see a double image in such circumstances.
14. When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?
15. A thin lens has two focal points, one on either side, at equal distances from its center, and should behave the same for light entering from either side. Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they are thin lenses.
16. Will the focal length of a lens change when it is submerged in water? Explain.
17. What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?
18. Can you see a virtual image? Can you photograph one? Can one be projected onto a screen with additional lenses or mirrors? Explain your responses.
19. Is it necessary to project a real image onto a screen for it to exist?
20. At what distance is an image always located—at do, di, or f
21. Under what circumstances will an image be located at the focal point of a lens or mirror?
22. What is meant by a negative magnification? What is meant by a magnification that is less than 1 in magnitude?

12.6 Problems

1. Calculate the speed of light through glycerine which has a refractive index of 1.4729.
2. Use the values given in Table 12.1, and the definition of refractive index to calculate the speed of light in water (ice).
3. Calculate the refractive index of an unknown substance where the speed of light through the substance is 1,974×108 m·s−1. Round off your answer to 2 decimal places. Using Table 12.1, identify what the unknown substance is.
4. A ray of light travels from silicon to water. If the ray of light in the water makes an angle of 69o to the normal to the surface, what is the angle of incidence in the silicon?
5. Light travels from a medium with n = 1.25 into a medium of n = 1.34, at an angle of 27 o from the normal.
6. What happens to the speed of the light? Does it increase, decrease, or remain the same?
7. What happens to the wavelength of the light? Does it increase, decrease, or remain the same?
8. Does the light bend towards the normal, away from the normal, or not at all?
9. Light is refracted at the interface between air and an unknown medium. If the angle of incidence is 53o and the angle of refraction is 37o, calculate the refractive index of the unknown, second medium.
10. Will light travelling from diamond to silicon ever undergo total internal reflection?
11. If a fibre optic strand is made from glass, determine the critical angle of the light ray so that the ray stays within the fibre optic strand.
12. A diamond ring is placed in a container full of glycerin. If the critical angle is found to be 37.4o and the refractive index of glycerin is given to be 1.47, find the refractive index of diamond.
13. A keratometer is a device used to measure the curvature of the cornea, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea is. If the light source is 12.0 cm from the cornea and the image' s magnification is 0.0320, what is the cornea' s radius of curvature?
14. What is the power in diopters of a camera lens that has a 50.0 mm focal length?
15. Your camera' s zoom lens has an adjustable focal length ranging from 80.0 to 200 mm. What is its range of powers?
16. What is the focal length of 1.75 D reading glasses found on the rack in a pharmacy?
17. You note that your prescription for new eyeglasses is −4.50 D. What will their focal length be?
18. How far from the lens must the film in a camera be, if the lens has a 35.0 mm focal length and is being used to photograph a flower 75.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.
19. A certain slide projector has a 100 mm focal length lens. (a) How far away is the screen, if a slide is placed 103 mm from the lens and produces a sharp image? (b) If the slide is 24.0 by 36.0 mm, what are the dimensions of the image? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

20. A doctor examines a mole with a 15.0 cm focal length magnifying glass held 13.5 cm from the mole (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 5.00 mm diameter mole?
21. How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?
22. A camera with a 50.0 mm focal length lens is being used to photograph a person standing 3.00 m away. (a) How far from the lens must the film be? (b) If the film is 36.0 mm high, what fraction of a 1.75 m tall person will fit on it? (c) Discuss how reasonable this seems, based on your experience in taking or posing for photographs.
23. A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm. (a) what is the closest object that can be photographed? (b) What is the magnification of this closest object?
24. Suppose your 50.0 mm focal length camera lens is 51.0 mm away from the film in the camera. (a) How far away is an object that is in focus? (b) What is the height of the object if its image is 2.00 cm high?

(a) What is the focal length of a magnifying glass that produces a magnification of 3.00 when held 5.00 cm from an object, such as a rare coin? (b) Calculate the power of the magnifier in diopters. (c) Discuss how this power compares to those for store-bought reading glasses (typically 1.0 to 4.0 D). Is the magnifier's power greater, and should it be? it be?

25. What is the focal length of a makeup mirror that has a power of 1.50 D?
26. Some telephoto cameras use a mirror rather than a lens. What radius of curvature mirror is needed to replace a 800 mm focal length telephoto lens?

(a) Calculate the focal length of the mirror formed by the shiny back of a spoon that has a 3.00 cm radius of curvature. (b) What is its power in diopters?

27. What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away?
28. A shopper standing 3.00 m from a convex security mirror sees his image with a magnification of 0.250. (a) Where is his image? (b) What is the focal length of the mirror? (c) What is its radius of curvature?
29. An object 1.50 cm high is held 3.00 cm from a person's cornea, and its reflected image is measured to be 0.167 cm high. (a) What is the magnification? (b) Where is the image? (c) Find the radius of curvature of the convex mirror formed by the cornea. (Note that this technique is used by optometrists to measure the curvature of the cornea for contact lens fitting. The instrument used is called a keratometer, or curve measurer.)
30. Ray tracing for a flat mirror shows that the image is located a distance behind the mirror equal to the distance of the object from the mirror. This is stated di = –do, since this is a negative image distance (it is a virtual image). (a) What is the focal length of a flat mirror? (b) What is its power?

13 Cross-Cutting Application of Physics

13.1 Application in Agriculture

Learning outcome

After completing this Chapter, students are expected to:

- compute water contents in soil
- explain the working principles of Principle of Motor and Generator
- describe energy balance in the environment

Why Physics in Agriculture?

The baffling problem of how plants can grow without any immediate obvious source of food supply is still being un-ravelled. Investigations stretching over the last few hundred years have led to an understanding of many processes involved in plant growth, but this increased understanding has at the same time led us on to ask still further questions, in the way that scientific investigation always seems to do. It is not surprising that soil was first thought to be the sole supplier of food for plants. How it came to be realized that plants "feed" chiefly by absorbing carbon dioxide as a gas from the atmosphere in the presence of light, synthesizing more complex products of higher chemical potential energy, is one of the fascinating stories of scientific discovery.

Energy balance concept

We may consider that the atmosphere of the planet Earth is a thermodynamic system, the system Earth, which receives a rate of heat, Q_S; primarily from the Sun, and simultaneously radiates heat, Q_E in all directions. In addition, because of the nuclear reactions that continuously occur inside the core of the planet, an additional quantity of heat power, Q_{int}; is convicted by magma to the surface of the planet. For this analysis, we may identify the atmospheric layer around the surface of the planet of total mass m_E, with average specific heat capacity, c_E , and average temperature T_E .

For this thermodynamic system, which is schematically depicted in Fig 7.1 below, one may write the energy balance equation as follows:

$$m_E c_E \frac{dT_E}{dt} = \sum_i \dot{Q}_i = \dot{Q}_S - \dot{Q}_E + \dot{Q}_{int}$$

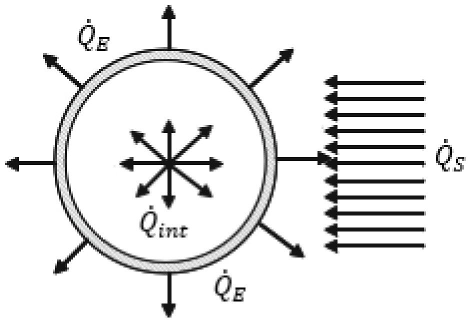


Fig 7.1 The Earth’s surface layer as a closed system and its heat balance.

13.1.1 Energy balance in soils

Application of the First Law of Thermodynamics: Energy Balance

The First Law of Thermodynamics is one of the fundamental and most general principles of science. It defines what is commonly referred to as the energy conservation principle. There are several formulations of the First Law, which are pertinent to the various types of systems and processes used. All formulations may be summarized by the general expression of energy conservation: energy is neither created nor destroyed. It may only be transformed from one form to another.

For a closed Thermodynamic system, the energy balance is best given in terms of a process leading from state 1 to state 2 and may be stated as follows: The heat entering a closed system minus the work produced by this system during a process 1–2 is equal to the difference of the total energy of the system between these two states.

This energy conservation law is depicted schematically in Fig. 7.2

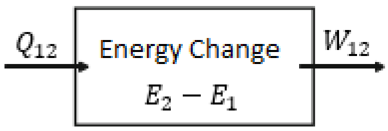


Fig. 7.2 The first law of thermodynamics as an energy balance

In symbolic form we may write:

$$Q_{12} - W_{12} = E_2 - E_1$$

where the total energy of the system E is defined as the sum of the internal energy, U, the potential energy, mgz, the kinetic energy, 1/2 mV 2 and any other forms of energy the system may possess, and which may be described by potential functions as for example, electric charge energy, magnetic energy, surface tension energy, elastic energy, etc. Thus:

$$E_2 - E_1 = (U_2 - U_1) + \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1) + \cdots$$

13.1.2 Physics of soils

The Soil as a Reservoir

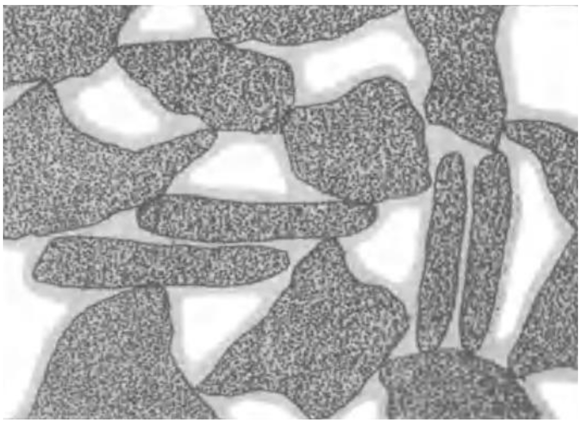
The soil acts as a tremendously large reservoir. Considering the many ways by which water may be removed from the soil, it must be considered as a very efficient storage medium. Fortunately, water can be held by the soil for long periods of time and still be available for plant use when plant growth begins.

Plants use a tremendous amount of water; it is the soil that "holds" this water and "supplies" it to the plants. The combined evaporation and transpiration may be as high as 1 cm per day or about 100,000 kg per ha per day or 214 barrels per acre per day. The average day's evaportranspiration during the growing season is about 50,000 kg per ha per day or 107 barrels per acre per day.

Calculating Soil Water

Solid particles of varying sizes and shapes make up the "skeleton" of the soil. Between these solid particles are interconnected pore spaces that vary continuously in size and shape. In a completely dry soil, all of the pore space would be filled with air; and in a completely saturated soil, water (soil solution) would occupy all of the pore space. Agricultural soils seldom, if ever, exist in either of these extreme conditions.

The physical properties of the soil, including its ability to store water, are highly related to the fraction or percentage of the total soil volume that is occupied by solid and the fraction or percentage that is pore space. For plant growth and development, the fraction or percentage of the pore space that is occupied by water and the fraction or percentage that contains air is of extreme interest (see Fig. 1.1). These concepts can be expressed quantitatively by defining terms such as soil porosity and soil water content. Many of the concepts, however, may be expressed in several ways; hence, several terms are defined for specifying a particular concept. Water content, for example, may be expressed on a volume basis (volume of water per unit volume of moist soil), on a dry mass basis (mass of water per unit mass of soil solids), or on a wet mass basis (mass of water per unit mass of wet soil). Further, the water contents may be expressed as percentages, but are often given as fractions. In converting between mass and volume units, density is used. Consequently, soil density terms must also be defined.



7. 3 Diagram of a cross section of soil, showing solid soil particles (dark areas), water films (light areas), and air spaces (white areas)

Exercise

Given: A cube of soil measures 10 x 10 x 10 cm (D = 10 cm, A = 100 cm^2) and has a total (wet) mass of 1460 g, of which 260 g is water. Assume the density of water, Pw, is 1.00 g/cm^3 and the soil particle density, \rho p, is 2.65 g/cm^3 .

Find: Mass water content, dry mass water percentage, volume water content, volume water percentage, depth of water, soil bulk density, soil porosity, water holding capacity, aeration porosity, and relative saturation.

13.2 Physics and Industries

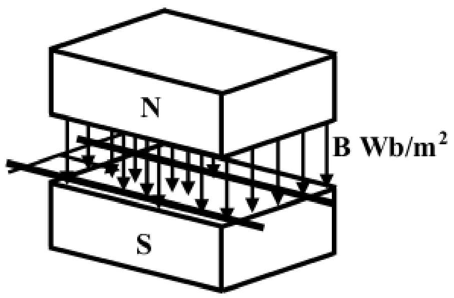
Learning Outcome

After completing this Chapter, students are expected to:

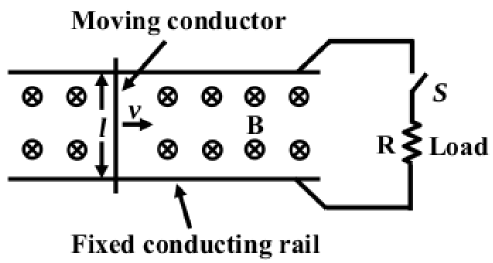
- describe the working principles of generator
- define loading of generator and motor
- explain how motoring an generating actions go side by side

Consider a straight conductor of active length (the length which is under the influence of the magnetic field) l meter is placed over two friction less parallel rails as shown in the figure 7.2.1a. The conductor is moving with a constant velocity v meter/second from left to right in the horizontal plane. In the presence of a vertical magnetic field directed from top to bottom of strength B Wb/m^2, a voltage e = Blv will be induced across the ends of the moving conductor. The magnitude of the voltage will be constant and the polarity will be as shown in the figure 7.2.1b. In other words the moving conductor has become a seat of emf and one can replace it by battery symbol with an emf value equal to Blv Volts.

At no load i.e., (resistance in this case) is connected across the moving conductor, output current hence output power is zero. Input power to the generator should also be zero which can also be substantiated by the fact that no external force is necessary to move a mass with constant velocity over a frictionless surface. The generator is said to be under no load condition. Let us now examine what is going to happen if a resistance is connected across the source. Obviously the conductor starts delivering a current i = e/R the moment
resistance is connected.



7.2.1a Elementary Generator



7.2.1b Top view of figure 7.2.1a

However, we know that a current carrying conductor placed in a magnetic field experiences a force the direction of which is decided by the left-hand rule. After applying this rule one can easily see that the direction of this electromagnetic force will be opposite to the direction of motion i.e., v. As told earlier that to move the conductor at constant velocity, no external force hence prime mover is not necessary. Under this situation let us assume that a load resistance R is connected across the conductor. Without doing any mathematics we can purely from physical reasoning can predict the outcome.

The moment load is connected, the conductor starts experiencing a electromechanical force in the opposite direction of the motion. Naturally conductor starts decelerating and eventually comes to a stop. The amount of energy dissipated in the load must have come from the kinetic energy stored in the conductor.

13.3 Physics in Health Sciences and Medical Imaging

Learning Outcome

After completing this Chapter, students are expected to:

- list some of medical imaging devices
- explain the physics principles in x-ray and MRI
- describe the health effect of radiation
- identify the advantage and disadvantage of radiation.

Introduction

Radiation and radioactive materials are part of our environment. The radiation in the environment comes from both cosmic radiation that originates in outer space, and from radioactive materials that occur naturally in the earth and in our own bodies. Together, these are known as background radiation. Everyone is exposed to background radiation daily. In addition, radiation and radioactive materials are produced by many human activities. Radiation is produced by x-ray equipment and by particle accelerators used in research and medicine. Radioactive materials are produced in nuclear reactors and particle accelerators.

13.3.1 RADIOACTIVITY

Radioactivity may be defined as spontaneous nuclear transformations in unstable atoms that result in the formation of new elements. These transformations are characterized by one of several different mechanisms, including alpha-particle emission, beta-particle and positron emission, and orbital electron capture. Each of these reactions may or may not be accompanied by gamma radiation. Radioactivity and radioactive properties of nuclides are determined by nuclear considerations only and are independent of the chemical and physical states of the radionuclide. Radioactive properties of atoms, therefore, cannot be changed by any means and are unique to the respective radionuclides. The exact mode of radioactive transformation depends on the energy available for the transition. The available energy, in turn, depends on two factors: on the particular type of nuclear instability that is, whether the neutron-to-proton ratio is too high or too low for the particular nuclide under consideration and on the mass–energy relationship among the parent nucleus, daughter nucleus, and emitted particle.

13.3.2 Health Effects of Radiation

Biological effects of radiation are typically divided into two categories. The first category consists of exposure to high doses of radiation over short periods of time producing acute or short term effects. The second category represents exposure to low doses of radiation over an extended period of time producing chronic or long term effects. High doses tend to kill cells, while low doses tend to damage or change them. High doses can kill so many cells that tissues and organs are damaged. This in turn may cause a rapid whole body response often called the Acute Radiation Syndrome (ARS).

Low doses spread out over long periods of time don't cause an immediate problem to any body organ. The effects of low doses of radiation occur at the level of the cell, and the results may not be observed for many years.

Use of high energy EM waves (Radiation) rapidly increasing in Health care industry. Especially in diagnosis with the help of X-ray , computerized tomography (CT), Magnetic resonance imaging (MRI) , and Positron emission tomography (PET) have made drastic revolution in diagnosis application with minimal non-invasive surgeries.

13.3.3 Medical Imaging

X-ray

X-ray imaging depends on the partial translucence of biological tissue with respect to X-ray photons. If a beam of X-rays is directed at the human body, a fraction of the photons will pass through without interaction

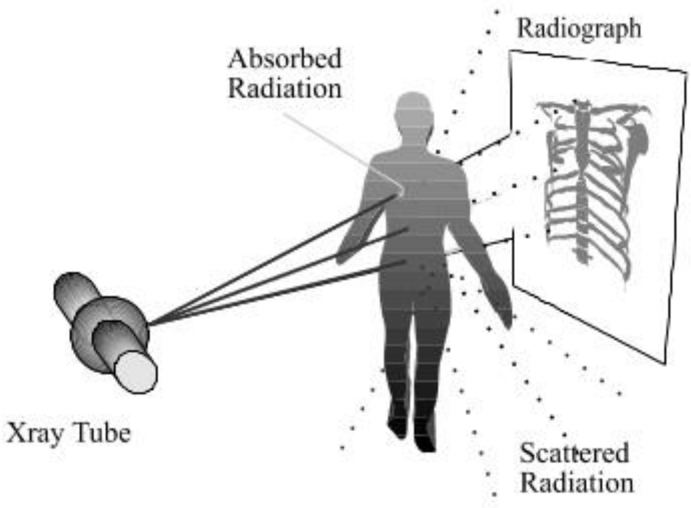
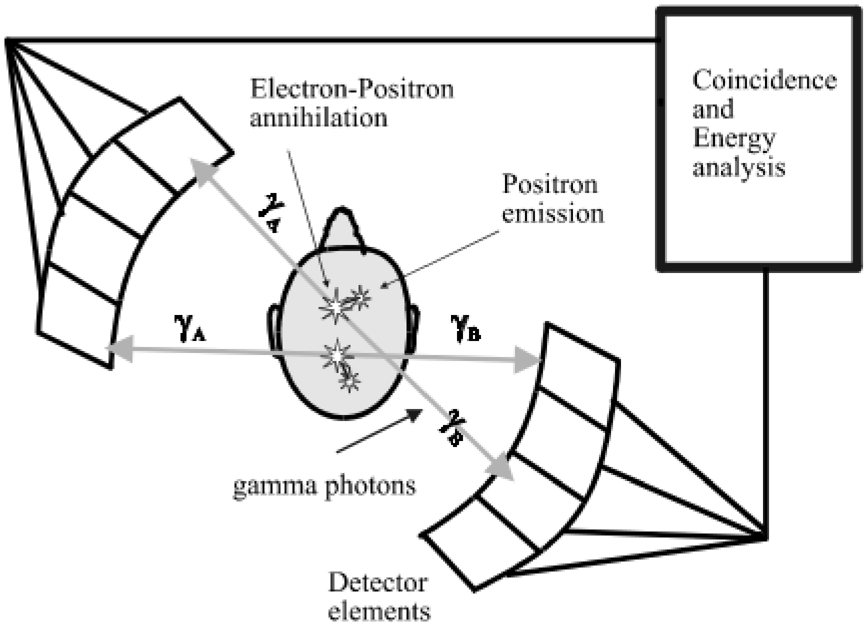


Fig7.3 A typical X-ray radiographic geometry. X-ray photons generated by the tube are directed at the patient. A fraction of the photons pass directly through the body to create a 2-dimensional projection of the exposed anatomy.

Positron Emission Tomography — PET

PET or positron emission tomography is a more recent clinical modification of gamma imaging that makes use of the two γ -rays, emitted simultaneously, when a positron annihilates with an electron. The tracer introduced into the patient is a positron emitter such as 15O, 11C, 18F, bound to a suitable carrier. The radioactive decay produces a positron, e+, with an initial kinetic energy of ~1 MeV. Although it has a high initial kinetic energy, the charged positron has very strong interactions with the surrounding tissue. These interactions transfer the positron kinetic energy to the tissue in a series of scattering events that produce a broad spectrum of X-ray energies (bremmstrahlung radiation) and a shower of photoelectrons. Typically, the positron travels less than 5 mm in biological tissue from its point of emission. The high electron density of biological tissue ensures frequent electron/positron encounters; one of these will result in the disappearance or annihilation of the two particles, replacing them with two γ-rays.

The conservation of energy demands that the energy of the two γ-rays is supplied by the total energy of the positron and the electron. By the time the annihilation takes place, nearly all the initial kinetic energy of the positron has been dissipated in tissue.



Positron Annihilation. The injected tracer is a positron emitter. The emitted positron travels about 5 mm before annihilating with an electron to form two γ-rays which travel away from the annihilation site in opposite directions. Coincidence detection is used to discriminate against spurious background counts and define a line of sight.

MRI

MRI is short for Magnetic Resonance Imaging, a title giving no hint of precisely which of the many possible magnetic quantities might be involved.

It is the nuclei of the hydrogen atoms in water that are involved. MRI is more transparently described as Spatially Localized Nuclear Magnetic Resonance. Each hydrogen nucleus is a proton, which carries a tiny compass needle or magnetic moment. When placed in a large magnetic field, hydrogen nuclei in the human body can be aligned in one direction and then made to absorb just one radio frequency supplied from outside.

Under the right conditions, the nuclear compass needles can be made to oscillate, creating another radio frequency signal that is easily detectable outside the body. The modulation of that signal, just like Virgin Radio or BBC, carries important information about molecular arrangements.

Ultrasound

Ultrasound imaging takes us to a larger scale of tissue structure, the boundaries between organs, well beyond the atomic and the strictly molecular arrangement. Ultrasound imaging depends entirely on changes in the velocity of sound (ultrasound) as we go from one tissue type to another.

Within human soft tissue sound velocities vary by about 10% either side of 1500 m s − 1 , the speed of sound in pure water. Whereas kidney contains 78% water by weight, liver has 75%. This difference, although small, is more than enough to cause the sound wave speed to change sufficiently across the boundary for quite strong sound reflections to be produced. Ultrasound, like MR, is insensitive to atomic type but very sensitive to macroscopic biological structure within soft tissue. The technology of ultrasound is very simple to use, cheap to produce and, at imaging power levels, almost without any hazard to the patient. For these reasons, ultrasound is second only to X-ray radiography in dentistry in the frequency of its use. X-rays, ultrasound and then MR between them cover a very large fraction of the diagnostic role required of imaging in clinical practice. None of these, however, completely satisfactorily addresses metabolic aspects of disease processes

13.4 Physics and Archaeology

Learning Outcome

After completing this Chapter, students are expected to:

- describe the techniques of radioactive dating
- state and explain the application of physics in archaeology

An account is given of the three main chronological applications of physics in archaeology: radiocarbon dating, thermoluminescence dating and archaeomagnetism.

The technique of comparing the abundance ratio of a radioactive isotope to a reference isotope to determine the age of a material is called radioactive dating. Many isotopes have been studied, probing a wide range of time scales.

The isotope ^{14}C, a radioactive form of carbon, is produced in the upper atmosphere by neutrons striking ^{14}N nuclei. The neutron is captured by the ^{14}N nucleus and knocks out a proton. Thus, we have a different element, ^{14}C. The isotope, ^{14}C, is transported as $^{14}CO_2$, absorbed by plants, and eaten by animals. If we were to measure the ratio of ^{14}C to ^{12}C today, we would find a value of about one ^{14}C atom for each one-trillion ^{12}C atoms. This ratio is the same for all living things–the same for humans as for trees or algae.

Once living things die, they no longer can exchange carbon with the environment. The isotope ^{14}C is radioactive, and beta-decays with a half-life of 5,730 years. This means that in 5,730 years, only half of the ^{14}C will remain, and after 11,460 years, only one quarter of the ^{14}C remains. Thus, the ratio of ^{14}C to ^{12}C will change from one in one-trillion at the time of death to one in two trillion 5,730 years later and one in four-trillion 11,460 years later. Very accurate measurements of the amount of ^{14}C remaining, either by observing the beta decay of ^{14}C or by accelerator mass spectroscopy (using a particle accelerator to separate ^{12}C from ^{14}C and counting the amount of each) allows one to date the death of the once-living things.

Industrial Applications

The applications of radioisotopes in industry are numerous. Many types of thickness gauges exploit the fact that gamma rays are attenuated when they pass through material. By measuring the number of gamma rays, the thickness can be determined. This process is used in common industrial applications such as:

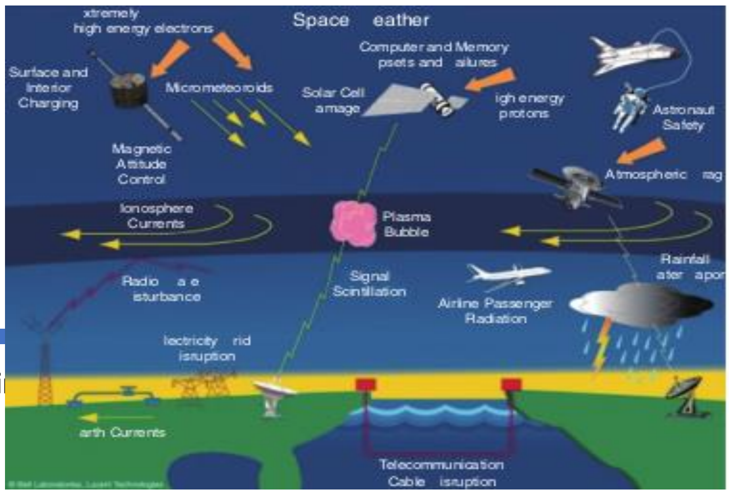
1. the automobile industry–to test steel quality in the manufacture of cars and to obtain the proper thickness of tin and aluminum
2. the aircraft industry–to check for flaws in jet engines
3. construction–to gauge the density of road surfaces and subsurfaces
4. pipeline companies–to test the strength of welds
5. oil, gas, and mining companies–to map the contours of test wells and mine bores, and
6. cable manufacturers–to check ski lift cables for cracks.

The isotope ^{241}Am is used in many smoke detectors for homes and businesses (as mentioned previously), in thickness gauges designed to measure and control metal foil thickness during manufacturing processes, to measure levels of toxic lead in dried paint samples, and to help determine where oil wells should be drilled.

13.5 Application in Earth and Space Sciences

Space weather

Space weather is a term which describes variations in the Sun, solar wind, magnetosphere, ionosphere, and thermosphere, which can influence the performance and reliability of a variety of space-borne and ground-based technological systems and can also endanger human health and safety[Koons et al., 1999]. Space weather has broad, everyday impacts on humans and technology. Spacecraft and astronauts are directly exposed to intense radiation that can damage or disable systems and sicken or kill astronauts. Radio signals from satellites to ground communication and navigation systems, such as the Global Positioning System (GPS), are directly affected by changing space environment conditions. What may be surprising is that many ground systems, such as power transmission grids and pipelines, and landline communication networks, such as transoceanic fiber-optic cables, are also susceptible to space weather impacts. Fig a shows the wide variety of systems that are affected by space weather, including astronauts and commercial airline crew and passengers as well as a host of satellite and radio communication devices. This chapter will describe how space weather affects these systems and describe the impacts space weather-related failures can have on technology and society.



Different system affected by space weather.

Aurora

The aurora borealis and aurora australis often called the northern lights and southern lights are common occurrences at high northern and southern latitudes, less frequent at mid-latitudes, and seldom seen near the equator. The typical aurora is caused by collisions between fast-moving electrons from space with the oxygen and nitrogen in Earth's upper atmosphere. The electrons which come from the Earth's magnetosphere, the region of space controlled by Earth's mag-neticfield transfer their energy to the oxygen and nitrogen atoms and molecules, making them "excited". As the gases return to their normal state, they emit photons, small bursts of energy in the form of light. When a large number of electrons come from the magnetosphere to bombard the atmosphere, the oxygen and nitrogen can emit enough light for the eye to detect, giving us beautiful auroral displays.

The Aurora Borealis as seen in Iceland
Photograph taken by Luka Esenko

13.5.1 Satellite Orbits

We have become dependent on space technology, using satellites for a wide range of Earth-observing (such as weather) and communication (data, voice, television, and radio) purposes. Satellite technology is finding its way into a number of everyday activities. You probably used a satellite today. You did if you watched cable or satellite TV, listened to a nationally syndicated radio program, tracked a package being delivered to you by one of the major courier services, or used a credit card at a gas station pump or at a major retail store. To support these services, there are hundreds of satellites orbiting Earth. These satellites are in a variety of orbits, which means that each satellite has a unique path around Earth.

Satellite orbits are loosely grouped into four categories: low earth orbit (LEO), medium earth orbit (MEO), geosynchronous orbit (GEO), and high earth orbit (HEO). Figure below shows the relationships among the different orbits. The corresponding orbital altitudes (above the earth's surface) are

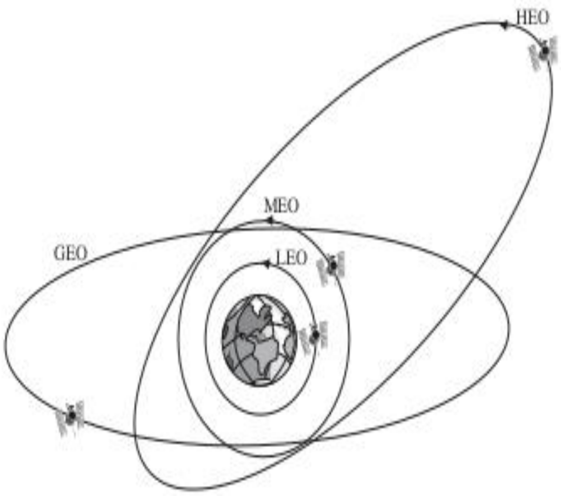
LEO, 500–900 km

MEO, 5000–12,000 km

GEO, 36,000 km*

HEO, 50,000 km

These definitions are not universal and may be applied rather casually. HEO satellites have an elliptical orbit, which may be very near the earth at its low.



Satellite orbits

13.5.2 Application in Power Generation

Geothermal Energy

The term geothermal is a composition of two Greek words: geo, meaning earth, and thermal, meaning heat. Combined, geothermal means heat generated from the earth.

Earth's center is made of molten iron, located at about 6430 km from its crust. Core estimated temperature is about 5000 ^{o}C, the heat would be conducts outward and heats up the outer layers of rock, referred to as the mantel. When mantel melts and spewed out of the crust, it is called magma.

Geothermal energy is energy created by the heat of the Earth. Under the Earth's crust lies a layer of thick, hot rock with occasional pockets of water. This water sometimes seeps up to the surface in the form of hot springs. Even where the water does not travel naturally to the Earth's surface, it is sometimes possible to reach it by drilling. This hot water can be used as a virtually free source of energy either directly as hot water, steam, or heat or as a means of generating power.

According to estimates from field studies, Ethiopia has the potential to generate up to 10,000 MW of electric power from geothermal sources. However, fifty years of geothermal exploration efforts have only resulted in ten companies receiving development licenses so far. Two of these licenses were given to Ethiopian Electric Power (EEP) for the resources at Aluto Langano and Alalobat at Tendaho in the Oromia and Afar regional states.

A geological survey of Ethiopia has identified 24 sites across the Rift Valley that have potential for generating geothermal energy.

Hydro-power

Hydroelectric power captures the energy released from falling water. In the most simplistic terms, water falls due to gravity, which causes kinetic energy to be converted into mechanical energy, which in turn can be converted into a usable form of electrical energy. Hydropower is often used to make electricity, usually at dams. The amount of energy in water depends on its flow or fall.

Hydroelectric power potential

Hydroelectric power plant potential consists of two parameters, namely, the amount of water flow per unit of time and the vertical height, or head, that water can be made to fall. In some instances, water head may be attributed to natural site topography, or it may be created artificially by constructing dams. Water accumulation in a dam depends on the intensity, distribution, and duration of rainfall, as well as direct evaporation, transpiration, ground infiltration, and the field moisture capacity of the basin or reservoir soil. A simple formula for approximating electric power production at a hydroelectric plant is:

$$P = \rho h Q g \eta$$

Where

- P is power in watts,
- ρ is the density of water (1000 kg/m^3),
- h is height in meters,
- $Q = V/t$ is flow rate in cubic meters per second,
- g is acceleration due to gravity of 9.8 m/s^2
- η is a coefficient of efficiency ranging from 0 to 1.

Efficiency is often higher (that is, closer to 1) with larger and more modern turbines.

Environmental impacts of hydroelectric power

Hydropower is better than burning coal, oil or natural gas to produce electricity, as it does not contribute to global warming or acid rain do not result in the risks of radioactive contamination associated with nuclear power plants.

A few recent studies of large reservoirs created behind hydro dams have suggested that decaying vegetation, submerged by flooding, may give off quantities of greenhouse gases equivalent to those from other sources of electricity. If this turns out to be true, hydroelectric facilities such as the James

Bay project in Quebec that flood large areas of land might be significant contributors to global warming. Run of the river hydro plants without dams and reservoirs would not be a source of these greenhouse gases.

13.6 Conceptual Questions

1. Describe how an X-ray image of the bones in the hand is produced
2. Why is hydrogen the most commonly targeted element in the magnetic resonance imaging process?
3. Why is hydrogen the most commonly targeted element in the magnetic resonance imaging process?
4. Explain how surface temperatures and chemical compositions of stars can be determined from their spectra
5. As a soil drains, is it true that small diameter pores drain before larger diameter pores? Explain.
6. For a given soil, is the bulk density a constant? Explain.
7. Explain why soil water would move from a drier to a wetter soil. Or could it?
8. If water exist in soil as films only (that is, water is only coating particle surfaces), explain why you would expect more water in a clay soil than in a sand
9. Which statements are correct about solar activity?
   - A. Solar flares are huge eruptions from the Sun, emitting large amounts of energetic particles and intense radiation.
   - B. The number of sunspots varies with a period for approximately 11 years (or has at least done so for the last 250 years).
   - C. The general level of activity follows the same 11-year period as the sunspots.
10. Which statements are correct about solar wind?
    - A. The electrons generated in the nuclear processes in the sun cause a net negative charge on the sun, repelling electrons from the solar surface and creating the solar wind.
    - B. If it were not for the ionizing radiation from the sun, the ions and electrons in the solar wind would rapidly recombine.
    - C. The solar wind sometimes reaches as far as Earth's orbit, but usually disappears well within the orbit of Venus.
11. Which statements are correct about a spacecraft?
    - A. Rockets need something to push on, and therefore do not work in vacuum, only inside an atmosphere.
    - B. The important parameter for launching a rocket is the total impulse (the time integral of the force): a small force applied during a long time is just as efficient for launching a rocket as a big force during a short time.
    - C. The basic shape of any satellite orbit is an ellipse, though there can be perturbations resulting from non-ideal effects like the non-spherical distribution of mass on the Earth, the gravitational influence of the moon, and air friction.

12. Which statements are correct about aurora?
    A. The most common colour of the aurora is red.
    B. The auroral light is emitted when atoms (sometimes also molecules and/or ions) in the upper atmosphere de-excite after having been excited by electrons in the keV range coming down along the magnetic field lines from the magnetosphere.
    C. The auroral light is mainly emitted at altitudes between 100 and 200 km

13.7 Problems

1. A bucket (20 cm diameter by 10 cm depth) contains a loam soil with a particle density of 2.7 g/cm 3 and a porosity of 40%. The soil is at a volumetric water content of 0.10. If the bucket receives 2.0 cm of rainfall, (a) Determine the soil water content after the rainfall (you may assume that the rainfall mixes uniformly throughout the soil volume). (b) Determine the weight of the bucket of soil after the rainfall (you may disregard the weight of the empty bucket.
2. A cylinder (4 cm diameter by 10 cm long) contains 210.0 g of oven-dry mineral soil. Estimate the grams of water required to fully saturate the soil in the cylinder.
3. A long capillary tube (radius=0.0015 cm) with a semi-permeable membrane on the lower end is oriented vertically and placed in a dilute sodium chloride solution at T = 20 0 C. If the height of rise of water in the tube is 20 cm, what is the solute potential (in head units) of the solution? State assumptions.

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